Algebraic Combinatorics and Finite Groups III WORKSHOP CETRARO
8-12 July 2024
Schedule and Abstracts

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-10:00	Welcoming words Patrick Cassam-Chenai	Bérénice Delcroix-Oger	Frédéric Menous	Cordian Riener	Joachim Kock
10:00-11:00	Silvia Pappalardi	Pierre Catoire	Martina Lanini	Samuele Girdaudo	Jean-Christophe Novelli Closing words
11:00-11:30	Coffee break in the terrace				
11:30-12:30	Mercedes Rosas	Sabino Di Trani	—	Angela Carnevale	—
13:00	Lunch at the beach				
PM	Group Work				
18:00-19:00	Evelyne Hubert	Paolo Papi	Claudio Procesi	Hans Munthe-Kaas	
around 20:30	19:30 Welcome Cocktail Dinner	Dinner Please, note that trousers are mandatory for gentlemen at dinner in the evening			

Angela Carnevale

Coloured shuffle compatibility and Hadamard products

In this talk, I will present recent work on coloured shuffle compatibility of permutation statistics and its applications to zeta functions in algebra. I will discuss how we extended recent work of Gessel and Zhuang, introducing shuffle algebras associated with coloured permutation statistics. Our shuffle algebras provide a natural framework for studying Hadamard products of certain rational generating functions. As an application, we will see how to explicitly compute such products in the context of so-called class- and orbit-counting zeta functions of direct products of suitable groups. This is joint work with V. D. Moustakas and T. Rossmann.

Patrick Cassam-Chenaï

2D-block Geminals: Taming the combinatorics while releasing the strong orthogonality constraint

Quantum Chemistry is mainly concerned with solving the Schrödinger equation for the electrons of a molecule, that is to say, by solving the so-called "molecular electronic structure" problem. Getting inspiration from the Lewis pair model of Classical Chemistry, some quantum chemists have been looking for approximate wave functions in the form of an antisymmetrized product of geminals (APG), the latter term designing wave functions of electron pairs. However, the computational cost of the general APG model, without further constraints, scales exponentially with the size of the system. More precisely, the combinatorics is given by a sum over disjoint cycles related to the "Compositional Formula, permutation version" and the "Exponential Formula, permutation version" of ref.1 (1R. P. Stanley, Enumerative combinatorics Vol. 2, Cambridge University Press, 1999) A new geminal ansatz called antisymmetrized product of 2D-block geminals has been introduced recently to limit the cost of the APG Hamiltonian expectation value calculation (2P. Cassam-Chenaï, Thomas Perez, Davide Accomasso, The Journal of Chemical Physics 158 (2023) 074106). It builds on an antisymmetrized product of strongly orthogonal geminals (APSG) while lifting the "strong orthogonality" and "seniority zero" restrictions. The combinatorics to select the terms, which can be added to each geminal of the product, to break these two constraints, grows exponentially with the number of geminals. However, we will show that the number of terms able to lower significantly the electronic energy in variational calculations, can be drastically reduced. This makes the method quite practical to use. Moreover, in most cases, the APSG geminals can be easily related to classical Lewis structures of chemistry. The improvement of the 2D-block APG model with respect to APSG is illustrated by comparing potential energy curve calculations for diatomic molecules, which which is the quantity driving the vibrational motion of the nuclei.

Pierre Catoire

The free tridendriform algebra, Schroeder trees and Hopf algebras

The notions of dendriform algebras, respectively tridendriform, describe the action of some elements of the symmetric groups called *shuffle*, respectively *quasi-shuffle* over the set of words whose letters are elements of an alphabet, respectively of a monoid. A link between dendriform and tridendriform algebras will be made. Those words algebras satisfy some properties but they are not *free*. This means that they satisfy extra properties like commutativity. In this talk, we will describe the *free tridendriform* algebra. It will be described with planar trees (not necessarily binary) called *Schroeder trees*. We will describe the tridendriform structure over those trees in a non-recursive way. Then, we will build a coproduct on this algebra that will make it a (3,2)-dendriform bialgebra graded by the number of leaves. Once it will be build, we will study this Hopf algebra: duality, quotient spaces, dimensions, study of the primitives elements...

Bérénice Delcroix-Oger

Algebraic combinatorics of the parking trees

Parking functions were first introduced by Konheim and Weiss in the 1960s. They appear naturally in the study of hash functions, more precisely when collisions are resolved by linear probing. Since then, they have appeared in connection with many different combinatorial objects such as Cayley trees, Catalan objects, hyperplane arrangements, non-crossing partition posets, Tamari lattice and diagonal coinvariants. In this talk, I will first recall the historical context of these objects. I will then describe the combinatorial species associated with parking functions, that we called "parking trees" in a joint work with Josuat-Vergès and Randazzo, and compare it to the species of Cayley trees. Finally, I will introduce a new poset on parking trees and state several associated results and conjectures.

Sabino Di Trani

Multipath Matroids, Cohomology and Digraph Colorings

A celebrated result in graph theory links the chromatic polynomial of a graph to the Tutte polynomial of the associated graphic matroid. Helme-Guizon and Rong in 2005 proved that the chromatic polynomial is categorified by a cohomological theory, the chromatic cohomology. In the talk, we describe how to associate a matroid to a directed graph G, called the multipath matroid of G, which encodes relevant combinatorial information about edge orientation. We also show that a specialization of the Tutte polynomial of the multipath matroid of G provides the number of certain "good" digraph colorings. Finally, analogously to what happens with the chromatic polynomial and chromatic cohomology, I will describe how the polynomial expressing the number of "good" digraph colorings is linked to multipath cohomology, introduced in a work with Caputi and Collari in 2021.

Samuele Giraudo

Natural Hopf algebras and polynomial realizations through related alphabets

A polynomial realization of a combinatorial Hopf algebra H is a map from H to a space of noncommutative polynomials on an infinite set of variables. In such a realization, the product of H must translate to a polynomial product, and the coproduct of H must be computable by the *alphabet doubling trick*. Many combinatorial Hopf algebras admit polynomial realizations. Among these are the Malvenuto-Reutenauer Hopf algebra of permutations [Duchamp, Hivert, Thibon, 2002], the Loday-Ronco Hopf algebra of binary trees [Hivert, Novelli, Thibon, 2005], and the Connes-Kreimer Hopf algebras of forests [Foissy, Novelli, Thibon, 2014]. Besides these, there is a construction N introduced by MÃ©ndez, associating a Hopf algebra N-O with a nonsymmetric operad O, called the *natural Hopf algebra* of O. We construct a polynomial realization of NO using alphabets of variables endowed with (binary or otherwise) relations. This work is from the preprint [arXiv:2406.12559].

Evelyne Hubert

Computing fundamental invariants and equivariants of a finite group action

For a finite group, we present an algorithm to compute a generating set of invariant simultaneously to generating sets of basic equivariants, i.e., equivariants for the irreducible representations of the group. The main novelty resides in the exploitation of the orthogonal complement of the ideal generated by invariants; Its symmetry adapted basis delivers the fundamental equivariants. Fundamental equivariants allow to assemble symmetry adapted bases of polynomial spaces of higher degrees, and these are essential ingredients in exploiting and preserving symmetry in computations. They appear within algebraic computation and beyond, in physics, chemistry and engineering. This is joint work with Erick Rodiguez Bazan published as: E. Hubert & E. Rodriguez Bazan. Algorithms for fundamental invariants and equivariants (of finite groups); Mathematics of Computation, volume 91 number 337 pages 2459-2488 (2022) [doi:10.1090/mcom/3749]

Joachim Kock

The universal property of the decomposition space of quasisymmetric functions

The coalgebra QSym of quasisymmetric functions was shown by Aguiar, Bergeron, and Sottile to be the terminal object in the category of graded coalgebras with a zeta function. I'll explain a categorified version of that result, in the framework of simplicial homotopy theory: QSym is the incidence coalgebra of a decomposition space Q of monotone surjections, and its zeta function Z is given by the empty surjection and the connected surjections. We show that for any graded decomposition space X with a zeta function F, there is a unique graded span of decomposition spaces X <-- J--> Q, where the backward map is ikeo (inner Kan and equivalence on objects) and the forward map is culf (conservative and unique lifting of factorisations) inducing F from Z. (Such spans induce coalgebra homomorphisms, and conjecturally all.) In fact, the result turns out to be much more general, leading to a functoriality that also gives the universal property of QSym as a bialgebra, and giving as a side effect Hoffmann's general construction of quasi-shuffles. (Most of the talk will be about simplicial machinery, and a secondary goal is to promote interactions between algebraic combinatorics and homotopy theory/category theory.) This is joint work with Philip Hackney and Jan Steinebrunner

Martina Lanini

From finite dimensional algebras to polyhedral complexes and back

In this talk I will report on joint work with Alessio Cipriani. Given a finite dimensional algebra, it is possible to study its so-called wall and chamber structure, a certain polyhedral complex which is in general rich in combinatorics and applications. Motivated by a geometric question (about the space of Bridgeland stability conditions for the bounded derived category of constructible sheaves on projective spaces) we end up investigating one of these polyhedral complexes for a very special algebra. Determining the number of chambers seems to be hard from a combinatorial viewpoint, but it becomes an easy task once we have in hand some representation theoretical results.

Frédéric Menous

Catalan idempotent(s), from resummation theory to Rota-Baxter algebras and trees.

This talk is a survey on the Catalan idempotent and its recent multivariate version that gives rise to the "small" Catalan idempotents. The Catalan idempotent, first introduced by Jean Ecalle as an operator that "analyse" the singularities of some analytic functions, gives rise to a primitive element of the algebra of noncommutative symmetric function and thus to an idempotent of the descent algebra. This operator is also related to Rota-Baxter algebras and it allows to define a multivariate version of the Catalan idempotents. It appears that the coefficients of this multivariate version, in a specific basis, decomposes in sum of monomials over intervals of the Tamari order of non plane trees, this decomposition being essentially based on the properties of Rota-Baxter algebras. This talk is base on joint works with Loïc Foissy, Jean-Christophe Novelli and Jean-Yves Thibon.

Hans Z. Munthe-Kaas

Groups of formal diffeomorphisms and geometric numerical integration

This talk is a survey on certain algebraic structures appearing in the intersection between differential geometry and numerical analysis. Motivated by the need for analysing geometric algorithms for integrating differential equations, we will discuss algebras of connections on manifolds. We will see that pre-Lie, post-Lie and Lie admissible triple algebras are special cases of such algebras, and present a unified view on such algebras.

Jean-Christophe Novelli

Algebraic and combinatorial constructions about the arc order of Reading

This talk will present combinatorial questions concerning the lattice quotients of the weak order on permutations. We will start with two well-known quotients and then present the general theory of arcs and their forcing order as studied by Reading, and we'll then focus on insertion algorithms on those quotients.

Paolo Papi

Identities involving Weyl groups from representation theory of minimal W-algebras

There are many identities of shape "sum= product" which have a representation theoretic interpretation. The "sum" part is usually related to Weyl groups. Instances of this situation are the Vandermonde determinant (coming from the Weyl character formula), the Jacobi triple product identity (coming from the Weyl-Kac character formula). We will present, along these lines, a number of classical identities and some (possibly) new identities coming from unitary representations of quantum affine minimal W-algebras. This is a joint project with V. Kac and P. Moseneder Frajria.

Silvia Pappalardi

Free Probability Approaches to Quantum Dynamics

Free Probability extends probability theory to non-commuting variables, and it is based on the combinatorics of non-crossing partitions. In this talk, I will present how this framework can be used to rationalize the universality of quantum dynamics. After introducing the current framework for chaotic many-body dynamics, called the Eigenstate Thermalization Hypothesis, I will illustrate its relation to non-crossing partitions and free cumulants.

Claudio Procesi

Special bases for the swap algebras

Given a vector space V we have an action of the symmetric group S_n on V^{\otimes n}$. If n>dim V, the algebra generated by this symmetric group action is a proper quotient of the group agebra. In this talk we will discuss some constructions of a linear basis of this algebra extracted from the symmetric group

Cordian Riener

The wonderful geometry of the Vandermonde map

The main object of this talk is the Vandermonde map - the map that given by evaluating some power sum polynomials - which appears quite naturally in various contexts and thus providing connections between different mathematical domains. We will highlight some aspects of the beautiful geometry of this classical map. One motivation for our considerations is the following question: Suppose that we are given a polynomial expression in traces of powers of symmetric matrices is there an algorithm to decide whether this expression is nonnegative for all symmetric matrices of all sizes? What happens if we replace trace by normalized trace? As one of the results of our work we show that the first (unnormalized) problem is undecidable, while the second one is decidable. The key to the hardness of the unnormalized problem is the fascinating geometry of the image of the probability simplex under the Vandermonde map.

Mercedes Rosas

All linear symmetries of the SU(3) tensor multiplicities

The SU (3) tensor multiplicities are piecewise linear polynomial in their labels. The pieces are the chambers of a complex of cones. I will describe in detail this chamber complex and determine the group of all linear symmetries (of order 144) for these tensor multiplicities. This is joint work with Emmanuel Briand and Stephan Trandafir.