WORKSHOP CETRARO

Algebraic Combinatorics of the Symmetric Groups and Coxeter Groups

5-9 July 2021

Monday	10.00	Malvenuto
	11.10	Patras
	16.50	Welcome of participants (delayed to the afternoon due to late arrivals monday morning and in the early afternoon)
	17	Brenti
	18.00	Bergeron (remote)
	19.15	Aperitivo
Tuesday	9	Manchon
	10	Coffee break
	10.30	Santocanale
	11.30	Iraci
	17	Papi
	18	Reutenauer (remote)
Wednesday	9	Pepe
	10	Coffee break
	10.30	Hubert
	11.30	Di Trani
Thursday	9	Siconolfi
	10	Coffee break
	10.30	Berger
	11.30	Celestino
	17	Hohlweg (remote)
	18	Procesi
	20.00	Gala diner
Friday	9	Novelli
	10	Coffee break
	10.30	Cassam-Chenaï
	PM	Work in groups/departures

Clemens Berger
On a certain filtration of the universal bundle of a finite Coxeter group

The universal bundles of the symmetric groups assemble into an E-infinity operad which plays an important role in the theory of infinite loop spaces. This E-infinity operad admits a filtration by E_n-suboperads thereby producing combinatorial models for point configurations in n-dimensional Euclidean space. We shall define an analogous filtration for the universal bundle of any finite Coxeter group and study the homotopy types of the resulting filtration stages.

François Bergeron (online)
Irrational Catalan Combinatorics

Following the recent work of Blasiak-Haiman-Morse-Pun-Seelinger: A Shuffle Theorem for Paths under any Line, we extend Catalan Combinatorics (Dyck paths, parking function, Macdonald eigenoperators, elliptic Hall algebra, etc.) to the context of any "triangular" partitions. These are partitions whose Ferrers diagram is the set of cells that sit below a line joining pints (0,s) and (r,0) for any positive real numbers s and r; with the classical case corresponding to r=s=n an integer. We first prove general enumerative results, and then discuss properties of associated (GL_k x S_n) character formulas (if time allows), as well as ties with many other subjects.

Francesco Brenti
Graphs, stable permutations, and Cuntz algebra automorphisms

Stable permutations are a class of permutations that arises in the study of the automorphism group of the Cuntz algebra. In this talk I will present a characterization of stable permutations in terms of certain associated graphs. As a consequence of this characterization we prove a conjecture in [Advances in Math., 381(2021) 107590], namely that almost all permutations are not stable, and we characterize explicitly stable 4 and 5-cycles.
(Joint work with Roberto Conti and Gleb Nenashev)

Patrick Cassam-Chenaï
Invariants and covariants in Quantum Chemistry: applications and open problems

We will review the use of invariants and covariants in Quantum Chemistry from the introduction of these algebraic techniques by A. Schmelzer, J. Murrell, [Int. J. Quantum Chem. 28, 287- 195 (1985)] to their recent combination with artificial intelligence tools [K. Shao, J. Chen, Z. Zhao, and D. H. Zhang, J. Chem. Phys. 145, 071101 (2016)]. A special emphasis will be put on open problems arising from infinite symmetry groups such as SO(3).

Sabino Di Trani
Small Representations in the Exterior Algebra and Generalized Exponents

Let g be a simple Lie algebra over C, and consider the exterior algebra \Lambda g as g-representations. In 1997 Mark Reeder conjectured that it is possible to determine the graded multiplicities in \Lambda g of certain irreducible representations reducing the computations to a combinatorial problem involving suitable Weyl group representations. We will give an idea of the strategy we used to prove the conjecture in the classical cases. Moreover, we will expose how our formulae can be rearranged involving the Generalized Exponents, obtaining a generalization of some classical results for graded multiplicities in the exterior algebra for adjoint and little adjoint representations.

Christophe Hohlweg
Shi arrangements and Garside shadows in Coxeter groups

Given a Coxeter group W, the Shi arrangement is a refinement of the corresponding Coxeter hyperplane arrangement; this arrangement was introduced by Shi to study the Kazhdan-Lusztig cells in the case of an affine Weyl group. Shi showed that each region of a Shi arrangement contains exactly one element of minimal length. Garside shadows were introduced in relation to the word problem in Artin-Tits (braid) group (joint work with Dehornoy and Dyer). In this talk I will discuss the following conjecture: the set of minimal length elements of the regions in a Shi arrangement is a Garside Shadow. In particular, I will outline a proof in the case of affine Weyl groups
(Joint work with Nathan Chapelier)

Evelyne Hubert
Sparse Interpolation in Terms of Multivariate Chebyshev Polynomials

Sparse interpolation refers to the exact recovery of a function as a short linear combination of basis functions from a limited number of evaluations. For multivariate functions, the case of the monomial basis is well studied, as is now the basis of exponential functions. Beyond the multivariate Chebyshev polynomial obtained as tensor products of univariate Chebyshev polynomials, the theory of root systems allows to define a variety of generalized multivariate Chebyshev polynomials that have connections to topics such as Fourier analysis and representations of Lie algebras. We present a deterministic algorithm to recover a function that is the linear combination of at most r such polynomials from the knowledge of r and an explicitly bounded number of evaluations of this function.
(Joint work with M. Singer)

Alessandro Iraci
Delta Conjectures and Theta Operators

Macdonald polynomials are a family of symmetric functions whose interesting combinatorial properties generated many results in algebraic combinatorics, such as the famous shuffle theorem, which gives a combinatorial interpretation for the bigraded

Frobenius characteristic of the module of diagonal harmonics. In this talk, we will discuss the state of the art about the multiple generalisations of the shuffle theorem, give a sketch of its proof (focusing on the combinatorics), and state the currently open problems and some new conjectures related to the Delta and Theta operators.

Martina Lanini
To Be Announced

Claudia Malvenuto
Primitive elements in the Poirier-Reutenauer Hopf Algebra of tableaux

In 1995 Poirier and Reutenauer introduced some algebraic structures, different from the plactic monoid, which induce some products and coproducts of tableaux, with homomorphisms. Their starting point are the two dual Hopf algebras of permutations, introduced by Malvenuto and Reutenauer in 1995. In 2006 Aguiar and Sottile introduced a new basis of this algebra, by Moebius inversion in the poset of weak Bruhat order, to describe the primitive elements of the Hopf algebra of permutations. Using this method, we determine the primitive elements of the Poirier-Reutenauer algebra of tableaux, using a partial order on tableaux defined by Taskin.
(Joint work with Christophe Reutenauer)

Dominique Manchon
Families of algebraic structures

Algebraic structures may come into families, where each operation at hand is replaced by a family of operations indexed by some parameter set, which often bears a semigroup structure. I will introduce Rota-Baxter families, then address other family structures (dendriform, duplicial, pre-Lie,...), and finally give a general account of family algebras over a finitely presented linear operad, this operad together with its presentation naturally defining an algebraic structure on the set of parameters.
(Based on recent joint works with Loïic Foissy, Xing Gao and Yuanyuan Zhang)

Jean-Christophe Novelli
Lagrange inversion, noncrossing partitions and a quasi-symmetric analogue of the Farahat-Higman algebra

We study the structure constants of the noncommutative analog of the Lagrange basis of symmetric functions. This gives rise to analogs of classical constructions and surprising combinatorial results connected to noncrossing partitions.

Paolo Papi
The role of affine Weyl groups in some problems of combinatorics and representation theory

I will discuss some instances of the emergence of affine Weyl groups as a basic tool to solve problems in representation theory and combinatorics. I will consider on some detail the

study of B-orbits on abelian ideals (joint works with Gandini, Maffei and Mosender Frajria) and an analog of Panyushev`s rootlet theory for infinitesimal symmetric spaces.
(Developed in the very recent PhD thesis of my student Federico Stara).

Frédéric Patras
Wick polynomials in non-commutative probability

Wick polynomials and Wick products have classically a rich combinatorics closely related to the one of set partitions. They are studied here in the context of non-commutative probability theory. It is shown that free, boolean and conditionally free Wick polynomials can be defined and related through the action of the group of characters over a particular combinatorial Hopf algebra.
(Joint work with Kurusch Ebrahimi-Fard, Nikolas Tapia, Luca Zambotti)

Valentina Pepe
Widened derangements and generalized Laguerre polynomials

Let D_h, E_k and F_a be sets of size h, k, a respectively, with k smaller or equal to h. We define a strongly widened derangement to be a permutation of D_h \cup E_k \cup F_a such that the elements of D_h are not fixed and the elements of E_k cannot occupy a site originally occupied by an object of the same type or by an object of F_a. We will show a connection between strongly widened derangements and generalized Laguerre polynomials that provides a generalization, for integer values of a, of Even and Gillis (Math Proc Camb Philos Soc, 1976) different from the one presented in Foata and Zeilberger (SIAM J Discrete Math, 1988).
(Joint work with S. Capparelli and A. Del Fra)

Claudio Procesi
First and second fundamental theorems of invariant theory, old and new

These names were given by Hermann Weyl in his book 'Classical groups' and refer to the description by generators and relations for invariants of the direct sum of copies of the fundamental representation and its dual of a classical group. In his book these are the basis for the representation theory of classical groups given by tensor symmetry. After reviewing these ideas we shall give a new form of these theorems which is suggested by quantum information theory.

Luigi Santocanale
Linear orders on involutive quantales

An involutitve quantale Q is a sort of (possibly non commutative) generalized Boolean algebra with a conjunction \otimes and a negation *.
We consider functions \chi : X^2 - Delta_X --> Q such that \chi(x,y) \otimes \chi(y,z) \leq \chi(x,z) and \chi(y,x) = \chi(x,y)^*.

If X is totally ordered and the unit of the quantale is such that 1^* \leq 1, then these functions can be ordered yielding a lattice. If X = [n] and Q = 2, then these functions bijectively correspond to transitive tournaments, that is, linear orders, and the ordering is the weak ordering on permutations. If Q is the Sugihara monoid on the three element chain, then these functions yields pseudo-permutations and the facial weak ordering.
I'll present in this talk general results of these functions, that I call linear orders or skew metrics on an involutitve quantale, present some more examples, and expose then the case where Q is a Sugihara monoid on a finite chain of odd length. I'll give a combinatorial model for the poset of linear orders on these quantales, whose elements turns out to be in bijective correspondence with maximal elements of the intersection poset of affine braid arrangements.

Christophe Reutenauer
The stylic monoid

The stylic monoid Styl(A) is a finite quotient of the plactic monoid. It is obtained by the natural action (left Schensted insertion) of the free monoid A^* on columns over A . It turn out that the elements of Styl(A) are faithfully represented by certain semi-standard tableaux on A, that we call N-tableaux; the bijection is obtained by an insertion algorithm, which is a variant of Schensted`s one. Consequently there is a bijection from Styl(A) onto the set of (set-theoretic) partitions of subsets of A. The cardinality of Styl(A) is therefore B_{n+1}, the Bell number, n=|A|. A presentation of the monoid Styl(A) is obtained by adding to the plactic (Knuth) relations the idempotent relations a^2=a, a in A. The natural involution of A^*, which reverses words and exchanges the order on A, induces an anti-automorphism of Styl(A); it is computed by a variant of evacuation (Schützenberger involution) which works on standard immaculate tableaux (they are naturally in bijection with N-tableaux). The monoid Styl(A) is J-trivial, and the J-order is graded. The monoid Styl(A) is the syntactic monoid of the function mapping each word onto the length of its longest strictly decreasing sequence.
(Joint work with Antoine Abram)

Viola Siconolfi *Ricci curvature, graphs and Coxeter groups*

I will talk about a notion of curvature for graphs introduced by Schmuckenschlueger which is defined as an analogue of Ricci curvature. This quantity can be computed explicitly for various graphs and allows to find bounds on the spectral gap of the graph and isoperimetric-type inequalities. I will present some general results on the computation of the discrete Ricci curvature of any locally finite graph. I will then focus on graphs associated with Coxeter groups: Bruhat graphs, weak order graphs and Hasse diagrams of the Bruhat order.