Inter-event times statistic in stationary processes: non linear ARMA modeling of wind speed time series

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The random sequence of inter-event times of a level-crossing is a statistical tool that can be used to investigate time series from complex phenomena. Typical features of observed series as the skewed distribution and long range correlations are modeled using non linear transformations applied to Gaussian ARMA processes. We investigate the distribution of the inter-event times of the level-crossing events in ARMA processes in function of the probability corresponding to the level. For Gaussian ARMA processes we establish a representation of this indicator, prove its symmetry and that it is invariant with respect to the application of a non linear monotonic transformation. Using simulated series we provide evidence that the symmetry disappears if a non monotonic transformation is applied to an ARMA process. We estimate this indicator in wind speed time series obtained from three different databases. Data analysis provides evidence that the indicator is non symmetric, suggesting that only highly non linear transformations of ARMA processes can be used in modeling. We discuss the possible use of the inter-event times in the prediction task.

Keywords: inter-event time; up-crossing; non Gaussian ARMA; symmetry; wind speed; time series

MSC Classification: 60G10; 62M10

I. INTRODUCTION

Time series obtained from measures of wind speed are characterized by high variability and uncertainty and both linear and non linear models have been applied to forecast wind speed and power production (see the reviews[1],[2],[3]). Linear Gaussian autoregressive (AR) and moving average (MA) models were firstly used to model and forecast wind speed series[4]. These models have been extended to improve predictive per-

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formance using different methods: composition of non stationary ARIMA models[5],[6], ARMA-GARCH[7], fractional ARMA[8], bivariate ARMA[9],[10], mixed methods combining ARIMA models, wavelets and neural networks[11]. Among these methods we consider in the present work the application of a non linear transformation to a series obtained from a linear AR model[12].

In the analysis of environmental time series interesting statistical indicators are the inter-event times of the occurrence of a specific event. For instance ramp events, consisting in a sudden increase or decrease, were used to characterize the memory and intermittence of wind series[13],[14]; another example is the sojourn above a threshold in ozone monitoring[15]. A typical event is the crossing of a given level; an application to wind series is[16].

The level-crossing statistic of stationary Gaussian processes has been extensively investigated[17] with particular emphasis on the number of crossings of a given level occurring in a time interval. Much less investigation was devoted to the distribution properties of the related inter-event times. The recurrence Kac's theorem[18],[19], states that the expectation of the inter-event times is inversely related to the probability of the event. The probability of a level-crossing event depends on the bivariate distribution of the process and theoretical results on the distribution of the inter-event times in function of the level are not known also in ARMA processes.

In this work we investigate the inter-event times of ARMA processes and of processes obtained applying to these ones non linear and non invertible transformations. A crucial point is the parametrization of the level for which the inter-event times are computed. We parametrize the level using the corresponding probability p in the univariate distribution of the process. We call the resulting function of p the level-crossing profile. Using this parametrization the expectation of the inter-event times is invariant with respect to monotonic increasing transformations for any stationary process.

For ARMA processes we provide a new result concerning the symmetry properties of the level-crossing profile and provide evidence that if a non monotonic transformation is applied the symmetry disappears. Environmental time series can show several types of asymmetries, reflecting the non invariance with respect to time reversal. Asymmetries in the slopes of local trends[20] and in local maxima and minima[21] were found using different methods as the visibility graph method[22].

In data analysis of wind series we define the levels as the quantiles of the empirical distribution corresponding to a finite set of equidistant probabilities in (0,1), following the same method used in[23]. Using these levels we estimate the level-crossing profile and provide evidence that the profile is non symmetric. This suggests that a non monotonic transformation should be applied to ARMA processes in order to fit the data. In our analysis we use planar wind velocity data (a similar dataset is used in[24]) from the NCEP/NCAR Reanalysis 1 project database, the wind speed data from an Italian wast to energy plant and from a wind farm of Enel Green Power.

In the next section we establish a representation of the level-crossing profile for Gaussian ARMA processes and show its symmetry. In sec. 3 we perform a simulation study in which provide evidence that a non monotonic transformation produces a non symmetric profile. In sec. 4 we apply this methodology to wind data series. In the last section we provide the conclusions.

II. METHODS

Let $..., X_{-1}, X_0, X_1, X_2, ...$ be a stationary sequence of continuous random variables (r.v.) with probability distribution P, whose indexes are referred to as times. For a given event A let T_k denotes the sequence of times at which $X_t \in A$ (event times) and denote $R_k = T_{k+1} - T_k$ the inter-event times. If the stochastic sequence X_t is stationary and ergodic, the r.v. R_k are a stationary sequence (conditional to the event $X_0 \in A$) and the Kac's recurrence theorem[19],[18] states that the expectation of R_0 conditional to A is

$$\mathbb{E}(R_0 \mid A) = \frac{1}{\mathrm{P(A)}} \tag{1}$$

For a given level $x \in \mathbb{R}$ we say that there is an up-crossing at time t if $\{X_{t-1} < x, X_t > x\}$. Let T_k be the random sequence of the up-crossing times and let $R_k = T_{k+1} - T_k$ be the inter-event times. Conditional to the event $\{X_{-1} < x, X_0 > x\}$, time $T_0 = 0$ is an up-crossing time and $R_0 = T_1 - T_0$. From the above theorem the r.v. R_k are a stationary sequence (conditional to the event $\{X_{-1} < x, X_0 > x\}$) and

$$\mathbb{E}(R_0 \mid X_{-1} < x, X_0 > x) = \frac{1}{\mathrm{P}(X_{-1} < x, X_0 > x)} \tag{2}$$

Definition If $F(x) = \mathrm{P}(X_0 \leq x)$ is the univariate distribution function we define the level-crossing profile as the function $U(p), \quad p \in (0,1)$, given by

$$U(p) = \frac{1}{\mathrm{P}(X_{-1} < x, X_0 > x)} \Bigg|_{x=F^{-1}(p)} \tag{3}$$

Remark 1 An important consequence of this definition is that the process $\eta(X_t)$ where $\eta : \mathbb{R} \to \mathbb{R}$ is a monotonic strictly increasing function has the same profile of X_t.

Example Let X_t be a stationary sequence of independent variables with univariate continuous distribution and $p = \mathrm{P}(X_0 < x)$; one has by independence $\mathrm{P}(X_{-1} < x, X_0 > x) = p(1-p)$, hence

$$U(p) = \frac{1}{p(1-p)} \tag{4}$$

In a stationary sequence the probability of the level-crossing event can be computed in terms of the bivariate distribution at two consecutive times. In a sequence of normal variables this distribution depends only on the value of the autocorrelation at lag 1. The level x is related to the probability p by $x = \Phi^{-1}(p)$, where Φ is the univariate normal distribution. We can prove the following.

Proposition In a stationary sequence of normal variables with lag 1 correlation ρ the level-crossing profile $U(p)$ is given by

$$U(p) = \frac{1}{p(1-p) - I(p)} \tag{5}$$

where

$$I(p) = \frac{1}{2\pi} \int_0^{\rho} \frac{1}{\sqrt{1-r^2}} e^{-\frac{\Phi^{-1}(p)^2}{1+r}}\, dr \tag{6}$$

The profile is symmetric with respect to $p = 1/2$ and for $p = 1/2$ it is given by

$$U(\frac{1}{2}) = \frac{4}{1 - \frac{2}{\pi}\arcsin\rho} \tag{7}$$

that tends to infinity as ρ tends to 1 and that attains its minimum 2 at $\rho = -1$.

Proof Let X_t be a stationary sequence of normal variables, that can be assumed without loss of generality to have zero mean, and whose bivariate marginal (X_{-1}, X_0) has correlation ρ. If Φ and Φ_2 denote the univariate and bivariate distributions one has $\mathrm{P}(X_{-1} < x, X_0 > x) = \Phi(x) - \Phi_2(x,x)$. The level-crossing profile is given by

$$U(p) = \frac{1}{\Phi(x) - \Phi_2(x,x)} \bigg|_{x=\Phi^{-1}(p)} \tag{8}$$

Using the integral representation of $\Phi_2(x,x)$ in[25]

$$\Phi_2(x,x) = \Phi(x)^2 + \frac{1}{2\pi} \int_0^{\rho} \frac{1}{\sqrt{1-r^2}} e^{-\frac{x^2}{1+r}}\, dr \tag{9}$$

one has

$$\begin{aligned} \Phi(x) - \Phi_2(x,x) &= \Phi(x)(1-\Phi(x)) \\ &\quad - \frac{1}{2\pi} \int_0^{\rho} \frac{1}{\sqrt{1-r^2}} e^{-\frac{x^2}{1+r}}\, dr \end{aligned} \tag{10}$$

This proves eq.(5) since $\Phi(x) = p$. This quantity is symmetric in x with respect to $x = 0$ and since $x = \Phi^{-1}(p)$ it is symmetric in p with respect to $p = 1/2$. From eq. (9) if $x = 0$ one has

$$\Phi_2(0,0) = \frac{1}{4} + \frac{1}{2\pi}\arcsin\rho$$

$$\Phi(0) - \Phi_2(0,0) = \frac{1}{4}(1 - \frac{2}{\pi}\arcsin\rho)$$

Hence the profile for $p = 1/2$ is given by eq.(7).

Remark 2 In ARMA series the profile can be computed using the value of the autocorrelation at lag 1 in dependence on the parameter of the model. For instance in AR(1) with parameter ϕ_1 one has $\rho = \phi_1$ and in AR(2) with parameters $\phi_1,\ \phi_2$, one has $\rho = \phi_1/(1-\phi_2)$.

Remark 3 Non Gaussian ARMA processes can be obtained from Gaussian ones applying the Tukey g-and-h transformation[12], that is a monotonic increasing transformation, in order to fit the skewed univariate distribution of wind series. By our results for these non Gaussian processes the profile is still symmetric.

If X_t is a simulated or a data series the expectation of inter-event times can only be estimated using ergodicity, i.e. computing the occurrence frequency of the event in the series. For any $p \in (0,1)$ let x be the corresponding empirical quantile of the univariate distribution of the series, and consider the number N of occurrences of the up-crossing of the level x as shown in fig. 1. Denoting $r_1, ..., r_N$ the inter-event times, we estimate their profile in function of p as

$$\hat{U}(p) = \frac{1}{N}\sum_{j=1}^{N} r_j \tag{11}$$

Since $\sum_{j=1}^{N} r_j$ is close to the length L of the series, eq. (11) can be rewritten as $\hat{U}(p) \sim L/N$ i.e. the reciprocal of the empirical frequency of the up-crossing event, according to eq. (1).

III. SIMULATIONS

Simulations and data analysis are performed using the `R` software[26]. We consider the following data generating processes where $\epsilon_t,\ z_t$ are standard Gaussian noise.

AR(1)

$$X_t = \phi_1 X_{t-1} + \epsilon_t \tag{12}$$

monotonic transform of AR(1)

$$X_t = e^{Y_t/5}; \quad Y_t = \phi_1 Y_{t-1} + \epsilon_t \tag{13}$$

non monotonic transform of AR(1)

$$X_t = |Y_t|; \quad Y_t = \phi_1 Y_{t-1} + \epsilon_t \tag{14}$$

AR(2)

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \epsilon_t \tag{15}$$

GARCH(1,1)

$$\begin{aligned} X_t = \phi_1 X_{t-1} + \epsilon_t; \quad \epsilon_t = z_t \sigma_t; \\ \sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \end{aligned} \tag{16}$$

Processes (12), (15) and (16) have been extensively used to model wind speed series. The process (13) is obtained applying a monotonic transformation to (12); (14) is obtained applying a non monotonic transformation to (12), that mimics the modulus of a wind velocity vector computed from its components. According to the theoretical result of sec.2 we expect processes (12), (15), (13) to have a symmetric profile.

We first evaluate the dependence of the level-crossing profile with respect to ϕ_1 in models (12), (14). For simulated series of 10000 values, 20 equispaced values of p in

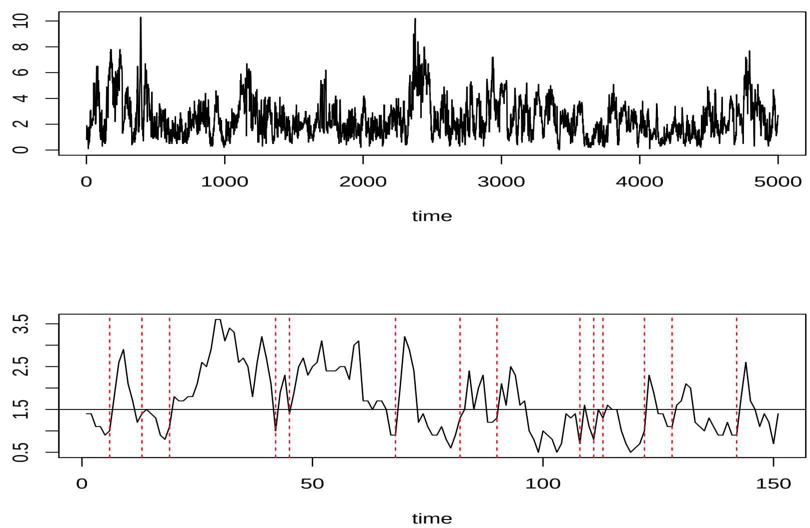


FIG. 1: Top: Example of a time series of wind speed with 10 min aggregation period. Bottom: Window of 150 values; the up-crossing times are marked by red lines.

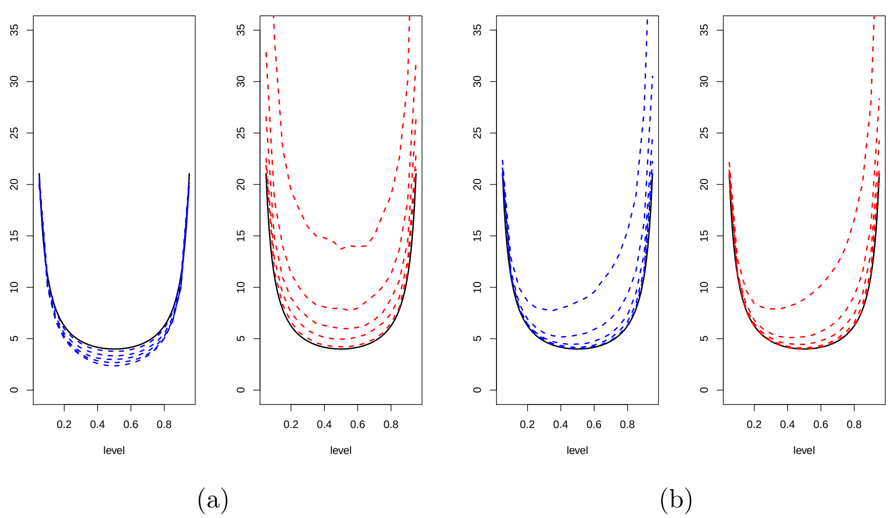


FIG. 2: Dependence of the profile on the parameter ϕ of (A) AR(1) process; (B) Absolute AR(1) process. Simulated series of 10000 elements and 20 levels $p \in (0,1)$, with $\phi_1 < 0$ (left panel, dotted lines, blue online) and $\phi_1 > 0$ (right panel, dashed lines, red online); theoretical curve $1/(p(1-p))$ for independent series (solid line).

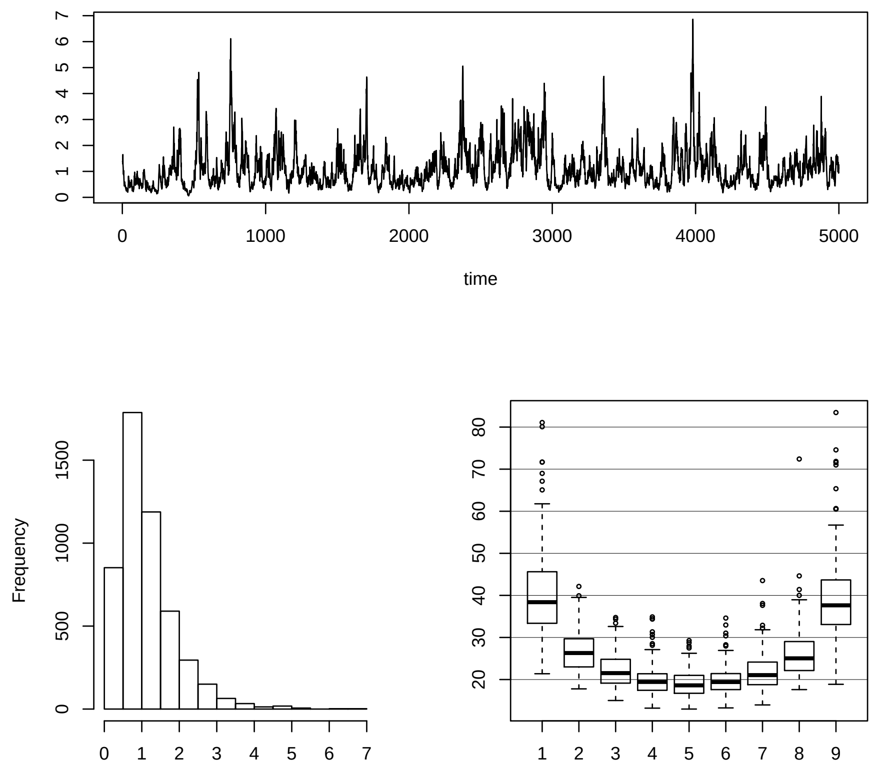


FIG. 3: Top: time series of exponential transformed AR(1) process. Bottom: histogram and profile.

$(0, 1)$, 10 values of ϕ_1 from -0.9 to 0.9, we compute $\hat{U}(p)$ from eq.(11). In case of model (12) the $\hat{U}(p)$ profiles obtained from the series are shown in the panels (A) of fig. 2 with $\phi_1 < 0$ (left panel, blue lines) and the ones with $\phi_1 > 0$ (right panel, red lines). Noticeably the greater is $|\phi_1|$ the larger is the distance from the curve $1/(p(1-p))$ corresponding to the independent series $\phi_1 = 0$. Since the minimum is at $p = 1/2$ corresponding to $x = 0$, the increase of the minimum in function of ϕ_1 is given by the eq. (7), with $\rho = \phi_1$. In case of model (14) the results are summarized in panels (B) of fig. 2. Interestingly the level-crossing profiles for $|\phi_1| \lessapprox 1$ are markedly asymmetric, having their minimum below $p = 1/2$ and all the profiles are greater than the curve $1/(p(1-p))$.

In order to analyze the statistical features of the profiles we have generated 500 realizations of the processes with the following parameters producing long range correlations

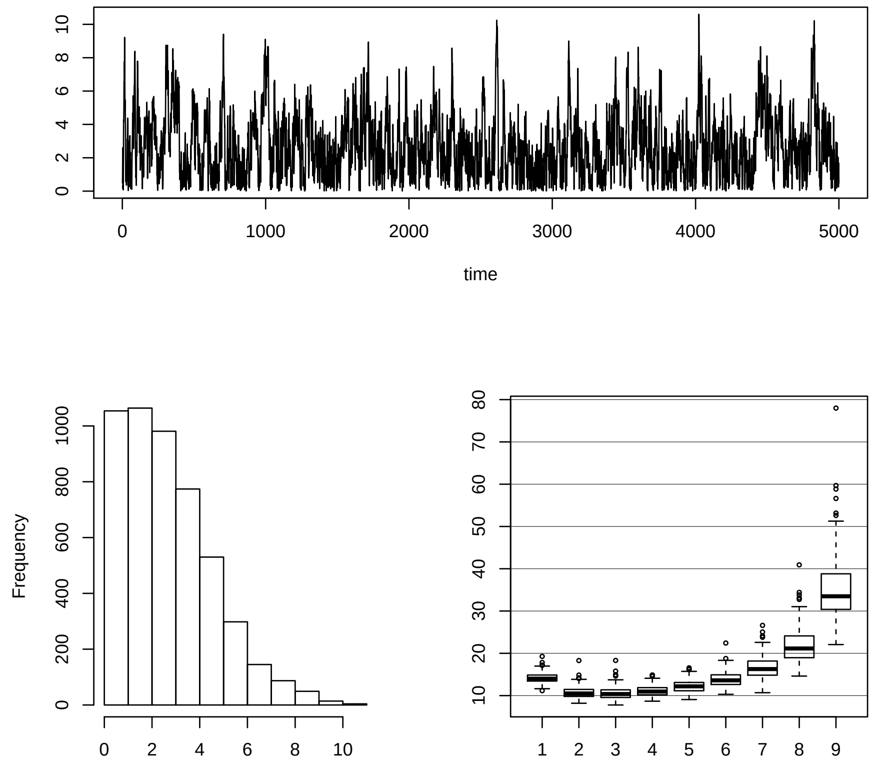


FIG. 4: Top: time series of absolute AR(1) process. Bottom: histogram and profile.

typical of wind speed series:

AR(1), $\phi_1 = 0.95$

AR(2), $\phi_1 = 0.82, \quad \phi_2 = 0.08$

ARMA-GARCH(1,1), $\phi_1 = 0.95, \alpha_0 = 0.3, \alpha_1 = 0.2, \beta_1 = 0.3$

The series have 1000 values and we have estimated the level-crossing profiles in each realization for 7 probability levels; the 500 replicated profiles are globally represented in box plots. In fig. 3 the AR(1) series has been transformed via a monotonic function that produces a non symmetric right skewed histogram and the box plots of the profiles as expected are symmetric. In fig. 4 the AR(1) series has been transformed via a non monotonic transformation. As before the histogram is right skewed, but the profile shows an evident asymmetry.

The results for AR(2) and GARCH(1,1) are shown in box plots of fig. 5 (A) and (B). The AR(2) process as expected shows a symmetric profile. In order to test the symmetry in the GARCH case we have used the Wilcox test of comparison of location parameters for the inter-event times at levels 1 and 7. The test does not reject the symmetry hypothesis.

IV. DATA ANALYSIS

We have performed analysis of wind data from different databases.

A. NCEP/NCAR Reanalysis 1 project

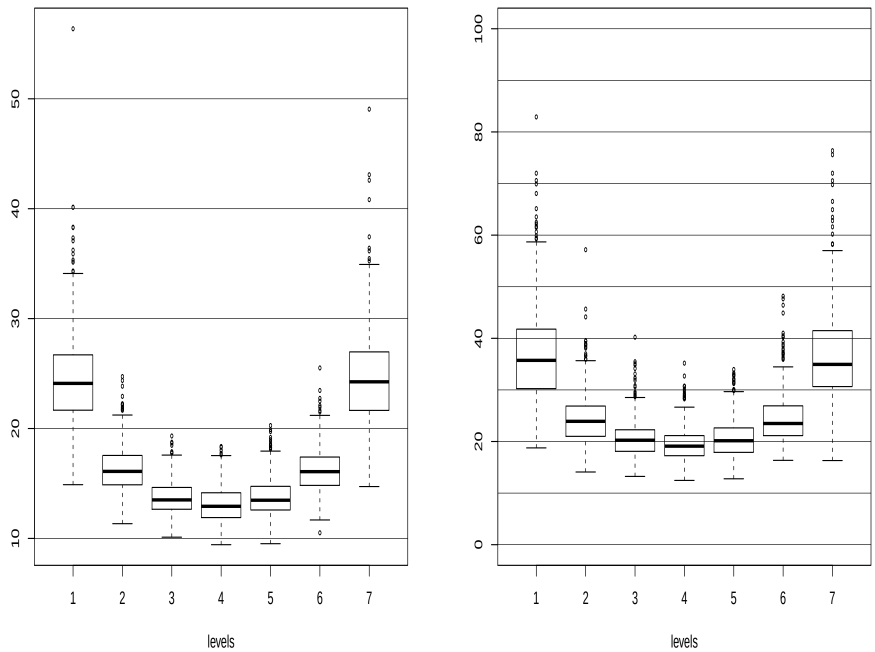
The NCEP/NCAR Reanalysis 1 project[27] is an analysis/forecast system to perform data assimilation using past data from 1948 to the present. Data are freely available at the web page
`https://psl.noaa.gov/data/gridded /data.ncep.reanalysis.pressure.html`.
The database contains a global grid (144x73) with spatial coverage 2.5 degree latitude x 2.5 degree longitude of U-wind and V-wind components measured 4-times daily. We have computed the speed from the U-wind and V-wind components data in year 2019 consisting in series of 1460 values. We have randomly selected a sample of 200 series from the database and estimated the inter-event profile both of the V-wind series and of the speed value. The box plots of profiles, each containing 200 values, are represented in fig. 6 (A) and (B). In replicated samplings the profiles of the speed show significant asymmetry, according to the Wilcox test of comparison of location parameters for the inter-event times at levels 1 and 7. Conversely the opposite asymmetry with minor evidence or no asymmetry is observed for the wind component, a behavior that deserves further investigation.

B. Lsi-Lastem

The second database contains data collected at an Italian waste to energy plant available at the web page `http://hera.meteo.lsi-lastem.it /pages/download.aspx`. We have used wind speed series sampled every 10 minutes in the year 2018. The series consisting in more than 52000 values was split into segments of length 1000. The level-crossing profile for 7 levels is estimated in each segment and the box plot is represented in fig.7 (A). The Wilcox test of comparison reveals that the inter-event times at levels 1 and 7 have significantly different location parameter.

C. Enel Green Power

The third database provided by Enel Green Power contains measures at a wind farm with 32 turbine generators located in



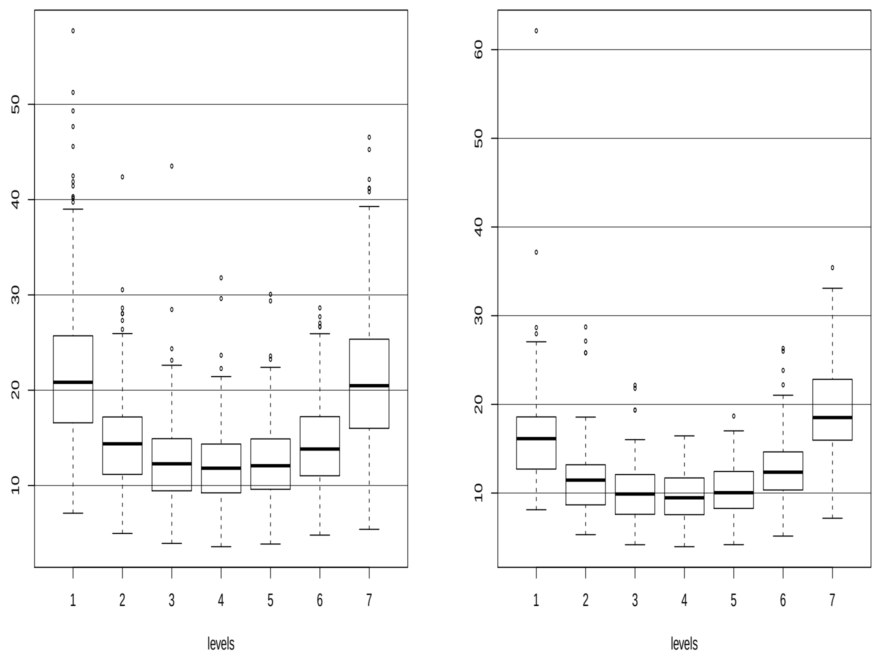
(a) (b)

FIG. 5: Box plots of the level-crossing profile for 7 probability levels in 500 realizations of simulated series. (A) AR(2) process; (B) GARCH(1,1) process.

Sicily. The wind speed was recorded in year 2015 every 10 minutes so that each series has 52560 values. Since some series are affected by artifacts and missing values occurring at different times, we have extracted 168 segments of 5200 values having less than 150 artifacts. For each segment we have estimated the level-crossing profile for 7 levels. Box plots are in fig. 7 (B). The Wilcox test of comparison reveals that the inter-event times at levels 1 and 7 have significantly different location parameter.

V. DISCUSSION AND CONCLUSIONS

In our study we have considered the profile of the inter-event times of the up-crossing of a threshold in function of the probability. Our main result is a representation of the profile for Gaussian processes from which the symmetry of the profile is obtained. One of the consequences concerns the applicability of Gaussian and non Gaussian ARMA processes in modeling of wind speed. Gaussian ARMA models were used in[4],[9] and non Gaus-

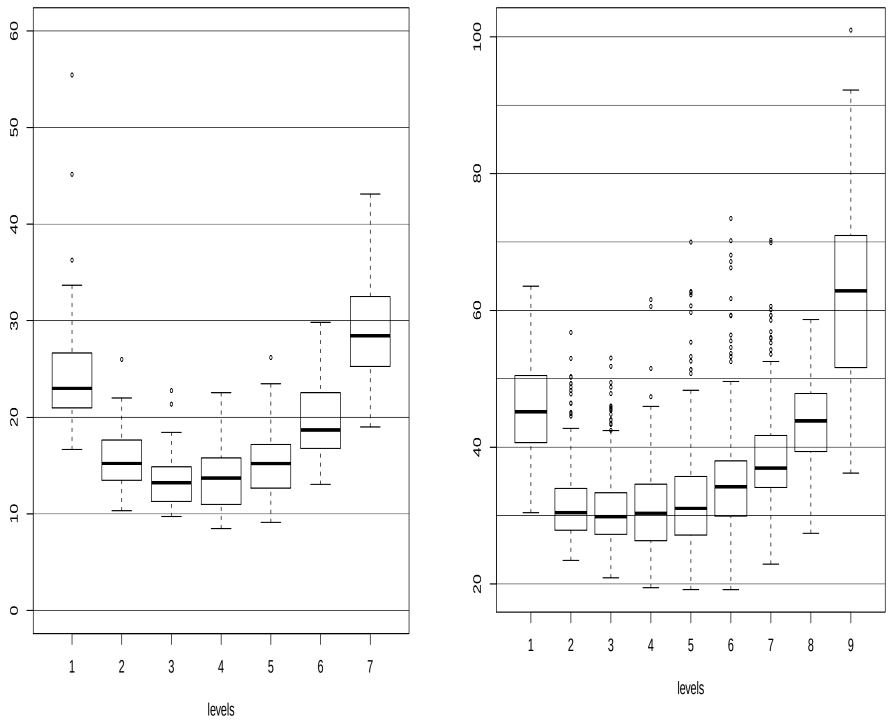


(a) (b)

FIG. 6: Box plots of the level-crossing profile for 7 probability levels in data series from NCEP database. (A) Component of wind velocity; (B) Wind speed.

sian models obtained applying a monotonic transformation to a Gaussian series were used in[12]. Our result shows that both these models have a symmetric profile. The application of a monotonic transformation produces a skewed univariate distribution of the series, that is used to fit the observed one, but does not changes the symmetry of the level-crossing profile. Numerical simulations have provided evidence that ARMA-GARCH models, used for instance in[7], share the same symmetric profile of ARMA models. On the other side, data analysis has shown that the profile estimated in wind speed is not symmetric, suggesting that the above models are not capable to reproduce all the features of wind speed series. The level-crossing profile allows to capture some aspects of the dependence among the variables not previously detected.

We have also provided numerical evidence that the application of a non monotonic transformation to an ARMA process produces an asymmetry in the profile similar to the one observed in wind speed data. The observed asymmetry could be related to the



(a) (b)

FIG. 7: Box plots of the level-crossing profile for 7 levels in data series. (A) Wind speed data from LASTEM; (B) Wind speed data from Enel Green Power.

peak - throat asymmetry that is typical of many environmental data series, previously detected by means of the visibility graph[21],[20], not yet detected for wind data series.

An advantage of our approach is the interpretability of the result, since it concerns a feature, the up-crossing of a threshold, that is relevant in any prediction task. A qualitative argument is the following. Fixed a probability p corresponding to a threshold x, the profile gives the average of the time that one has to expect to get the next up-crossing of the threshold x. This provides a prediction time that depends on x, differently from other methods where the prediction time is a parameter independent on the threshold. In Gaussian series and non Gaussian ones obtained applying a monotonic transformation the prediction time has a minimum at $p = 1/2$ corresponding to the median. The profiles estimated in wind series have an asymmetric U-shape where the minimum is attained at a value less than $1/2$. At the left and right extremes of the profile (p close to 0 and 1 respectively) the profile is high, meaning that one has to expect a long time to get a new up-crossing. During this time the system is first above the threshold and then below, and the sojourn time above the threshold for instance is an important variable to be considered. The value at the minimum, say m, represents a time scale, in units of sampling, that can be used to quantify the predictability of the series. In a time interval greater that m, in the average, the system has performed a new up-crossing and a new renewal cycle has started. If one assumes independence of events occurring in two consecutive renewal cycles, no prediction can be performed for time horizon greater than m, based on the informations of the previous cycle. In this case the only informations that can be used concern the distribution of the inter-event times, for instance the second moment, and the distribution of the sojourn times above the threshold. A final remark concerns the time scale of the profile. It depends on the aggregation period of the data, that in our examples varies from 10 minutes to 6 hours. This has to be taken into account when estimating the real time scale of the forecasting horizon.

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