The virtual spectrum of invariant differential operators on multiplicity free spaces

Let G be a connected recuctive group acting on a finite dimensional vector space U. The algebra $\mathcal{P}(U)$ decomposes as a G-module into a direct sum of irreducibles and U is called a *multiplicity free space* if if every irreducible occurs at most once.

The algebra $\mathcal{PD}(U)^G$ of G-invariant differential operators is commutative. In fact,

$$\mathcal{PD}(U)^G \cong \mathcal{P}(V)^W$$

where $\mathcal{P}(V)^W$ is the algebra of W-invariant function on some vector space V and $W \subseteq GL(V)$, the little Weyl group of V, is a certain finite reflection group. The identification assigns to any invariant differential operator D its "radial part" p_D. The eigenvalues of D on $\mathcal{P}(U)$ are the values of p_D in certain differential operator D its "radial part" p_D. The eigenvalues of D on $\mathcal{P}(U)$ are the values of p_D in certain points. In other words, p_D encodes the spectrum of D on $\mathcal{P}(U)$.

Multiplicity free spaces have one important feature: there is a simple a priori construction of certain invariant differental operators forming a basis of $\mathcal{PD}(U)^G$. This gives rise to polynomials $p_\lambda \in \mathcal{P}(V)^W$. It is possible to characterize them as elements of $\mathcal{P}(V)^W$ without any reference to their origin from U. This characterization has a certain flexibility which allows to define a continuous family of polynomials $p_\lambda(z;r)$ without sacrificing the good properties of the p_λ's. In other words, $p_\lambda(z;r)$ behaves for any r as if it came from an invariant differential operator.

In the talk we will explain some of the good properties: the p_λ's are eigenfunctions of difference operators. They exhibit a nice behavior under transposition. They are orthogonal with respect to a scalar product and more if time permits.

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