The first Dirac eigenvalue in a conformal class

B. Ammann (Nancy, France)

Let (M, g_0) be a compact n-diemnsional Riemannian manifold equiped with a fixed spin structure. Let $[g_0]$ be the set of all metrics conformal to g_0 having volume 1. We study the first positive eigenvalue of the Dirac operator as a function on $[g_0]$. At first, we sketch the proof that the first positive Dirac eigenvalue is not bounded from above. Then we turn our attention to the infimum, denoted by $\mu(M, [g_0])$. We will show that $\mu(M, [g_0])$ is always positive. In order to discuss whether this infimum is attained, we reformulate the problem as a variational problem. The infimum is attained if

$$\mu(M, [g_0]) < \mu(\mathbb{S}^n)$$

where $\mathbb{S}^n$ denotes the round sphere. Roughly speaking, this inequality avoids concentration of minimizing sequences for our functional. We discuss the Euler-Lagrange equation of the variational problem. In dimension 2 the spinorial Weierstrass representation can be used to transform the Euler-Lagrange equation into an evenly branched conformal immersion into R^3 such that the image has constant mean curvature. The existence of certain periodic constant mean curvature surfaces is obtained as a corollary. In the remaining part we discuss several conditions implying the inequality

$$\mu(M, [g_0]) < \mu(\mathbb{S}^n).$$

With an Aubin type construction of a test spinor, one sees that this inequality holds when M is not conformally flat and $\dim M > 6$. Other conditions are known if M is conformally flat and for lower dimension.