"Mathematics and its applications"
a joint SIMAI-SMAI-SMF-UMI Meeting

ABSTRACTS

Special Session 16: Qualitative Methods for Hamilton-Jacobi Equations and Applications, Torino, July 4, 2006

Organizers
M. Falcone (Roma "La Sapienza")
R. Monneau (CERMICS, Marne la Vallèe)
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Numerical approximation for a stochastic control problem with unbounded controls

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We consider a stochastic control problem with unbounded controls, coming from Finance [1].

We propose a numerical approximation using a generalized finite difference method [2]. We also discuss convergence results and error estimates.

This is a joint work with: B. Bruder, S. Marozo, H. Zidani

References:
[1] B. Bruder, "Super-replication of European options with a derivative asset under constrained finite variation strategies", preprint, 2005.
[2] F. Bonnans - E. Ottenwaelter - H. Zidani, "A fast algorithm for the two dimensional HJB equation of stochastic control" ESAIM:M2AN 38-4 (2004).

Singular perturbations and PDE Aubry-Mather theory

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In this talk I will describe a new approach to singular perturbation problems concerning the asymptotic behavior of random processes, obtained by perturbing a dynamical system through the addition of a small noise. The method exploits the viscosity solution theory and the so-called metric approach to Hamilton-Jacobi equations.

Convergence of a Fast Marching algorithm for the non-convex eikonal equation

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We present a new Fast Marching algorithm for the non-convex eikonal equation modeling front propagation in the normal direction.

The algorithm is an extension of the Fast Marching Method introduced by Sethian. In particular, the new scheme has a new interesting feature: it can deal with a *time-dependent velocity without restriction on its sign.*

We prove convergence in the class of discontinuity viscosity solutions and present some numerical simulations.

Joint work with: M. Falcone, N. Forcadel, R. Monneau.

A metric approach to Hamilton Jacobi equations in bouded domains

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We consider in a bounded open smooth set Ω the Hamilton-Jacobi equation $H(x, Du(x)) = 0$. The Hamiltonian H is assumed to be continuous, convex, coercive. Our analysis concerns the case in which the problem does not possess any strict subsolution in Ω: so it admits viscosity solutions which are in general not unique, not even up to the addition of a constant. We adapt to our setting the so called metric approach proposed by Fathi and Siconolfi to study critical periodic equations. A notion of intrinsic length for curves in $\overline{\Omega}$ is adopted, and a notion of Aubry set, adjusted to the setting, is given. We show that this set plays a crucial role in the analysis of the solutions to $H(x, Du(x)) = 0$ in Ω: it acts as an hidden boundary on which the datum must be fixed in order to obtain a unique solution to the equation coupled with a suitable Neumann boundary condition. We show that a natural Neumann condition for the problem is $H_p(x, Du(x)) \cdot n(x) = 0$. We discuss also the case of state constraints boundary conditions and Dirichlet boundary conditions.
Joint work with F. Camilli and A. Siconolfi.

A variational approach to the macroscopic electrodynamics of hard superconductors

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The coarse-grained electrodynamics in a long cylindrical hard superconductor can be described by the Bean's critical state model. The equations describing the evolution of the magnetic field $\vec{H}$ and of the electric field $\vec{E}$ inside the superconductor are solved using a quasistatic approximation based on a variational approach proposed by Badía and López. I shall show that $\vec{H}$ and $\vec{E}$ admit an explicit representation in terms of geometric quantities related to the cross-section of the superconductor. Our technique can also be applied in the case of an anisotropic behavior of the sample.

Joint work with: A. Malusa

Bolza Problems with discontinuous Lagrangians

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We study the local Lipschitz–continuity of the value function v associated with a Bolza Problem in presence of a Lagrangian $L(x, q)$, convex and uniformly superlinear in q, but only Borel–measurable in x. Under these assumptions, the associated integral functional is not lower semicontinuous with respect to the suitable topology which assures the existence of minimizers, so all results known in literature fail to apply. We propose a new approach, based on suitable reparametrization arguments, to derive suitable *a priori* estimates on the Lipschitz constants of quasi–minimizers. As a consequence of our analysis, we establish the Lipschitz–continuity of v and a compactness result for value functions associated with sequences of locally equi–bounded discontinuous Lagrangians.

Time–step adaptation in a Semi–Lagrangian scheme for Mean Curvature Motion

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The use of large time-step schemes in PDEs often leads to an underresolution of smaller scales. In level–set methods, this causes a lack in accuracy when tracking small structures, like corners. On the other hand, a reduction of the time step would cut the efficiency of the scheme once the solution is smoothed out, which is typically the case in curvature–driven motions.

We present a time–adaptive version of a Semi–Lagrangian (SL) scheme for Mean Curvature Motion, stating general requirements for the construction of an adaptation strategy, and showing some practical examples of such a strategy. We also discuss the efficiency of this approach on classical benchmarks for level–set techniques.

Joint work with M. Falcone and E. Carlini.

A problem of Mather and applications in dynamics

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I will present a recent result obtained in collaboration with A. Fathi and L. Rifford, in which we show that, under some assumption on the form on the Lagrangian and its regularity, the quotient Aubry set is totally disconnected. This gives a partial answer to a problem stated by Mather in 2004. Finally, I will show somme applications of our result in Dynamics.

A numerical model for growing sandpiles on partially open tables.

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We present a finite-difference scheme for the dynamics of a growing sandpile on an open flat table when (infinite) vertical walls are present on part of the boundary. This approach generalizes the one presented in [Falcone-Finzi Vita, '06] for the numerical resolution of the two-layers model proposed by Hadeler and Kuttler for the completely open table problem, where an advection equation for the rolling layer is coupled with an eikonal equation for the standing one. The presence of walls strongly affects the equilibrium solutions for this model, introducing singularities which propagates from the extreme points of the walls.

Joint work with: G. Crasta, M. Falcone

Dislocations dynamics and mean curvature motion

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Plastic deformation is mainly due to the movement of linear defects called dislocations. In the first part of the talk, we will describe a mathematical model for dislocations dynamics which is a non local first order equation. Then, we will consider the convergence at a large scale of dislocations dynamics and we will prove that the limit equation is an anisotropic mean curvature motion of variational type. As a by-product of our analysis, we will show that all mean curvature motion usually obtained in the literature as the limit of general Merriman Bence Osher schemes are also of variational type.

Joint work with F. Da Lio et R. Monneau.

Homogenization of first order equations with (u/ε)-periodic Hamiltonians

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We present a result of homogenization of first order Hamilton-Jacobi equations with (u/ε)-periodic Hamiltonians. On the one hand, under a coercivity assumption on the Hamiltonian (and some natural regularity assumptions), we prove an ergodicity property of this equation and the existence of non periodic approximate correctors. On the other hand, the proof of the convergence of the solution, usually based on the introduction of a perturbed test function in the spirit of Evans' work, uses here a twisted perturbed test function for a higher dimensional problem.

Joint work with R. Monneau.

Maximal solutions of viscous Hamilton-Jacobi equations with degeneracy

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We present some recent results concerning existence and uniqueness of maximal solutions for fully nonlinear stationary elliptic equations, having a possible degenerate principal part and first order terms with polynomial growth. Maximal solutions are viscosity solutions inside the domain and supersolutions up to the boundary. They arise naturally in stochastic control problems with state constraints, and their precence may cause an obstruction to the solvability of the Dirichlet problem. Our results extend to the degenerate case those obtained by J.M. Lasry and P.L. Lions for a Brownian diffusion.
Joint work with I. Capuzzo Dolcetta and A. Porretta

Viscosity solutions and minimizing movements in shape optimization

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We are interested in a model problem related to shape optimization, namely, Bernoulli's problem which can be stated as follows: given a smooth compact subset $S \subset \mathbb{R}^N$, we look for a compact subset $\Omega \supset\supset S$ which minimizes the capacity with a volume constraint. The gradient flow associated with this problem leads to the study of a nonlocal front propagation problem with a prescribed normal velocity of type $V = -1 +$ Hele-Shaw term. We produce long-time geometric viscosity solutions to this evolution and prove they converge to a free boundary problem which is formally equivalent to Bernoulli's problem. Then, we study more precisely the link between these two problems introducing some minimizing movements. It allows us to prove that the energy of the flow is nonincreasing.

Joint work with P. Cardaliaguet (Brest).

Integral formulations of the geometric eikonal equation

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We prove integral formulations of the eikonal equation, equivalent to the notion of viscosity solution in the framework of the set-theoretic approach to front propagation problems. We apply these integral formulations to investigate the regularity of the front: we prove that under regularity assumptions on the velocity c, the front has locally finite perimeter in the region where c does not vanish, and we give a time-integral estimate of its perimeter.

Lower semicontinuous solutions of Hamilton-Jacobi equations with state constraints

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We are interested by optimal control problems in presence of state constraints, specially in the case when no controllability assumption is satisfied. For these problems, the value function is known to be discontinuous.

We discuss the HJB equation related to such problems as well as its numerical approximation.

Joint work with: O. Bokanowski, N. Megdich