The insulating state of matter: A geometrical theory

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In a milestone paper appeared in 1964, W. Kohn showed that the insulating state of matter reflects a peculiar organization of the electrons in their *ground state*: the cause for the insulating behavior is electron localization [1]. Such localization, however, manifests itself in a very subtle way, fully elucidated much later. I will present the modern developments of the theory, which started in 1999. and continue to these days. The many-body wavefunction of an insulator is characterized by means of geometrical concepts.

We consider a many-body system, whose most general Hamiltonian, includes both a "twist" (alias "flux"), and a vector potential $\mathbf{A}$ of magnetic origin:

$$\hat{H}(\mathbf{k}) = \frac{1}{2m}\sum_{i=1}^{N}(\mathbf{p}_i + \frac{e}{c}\mathbf{A}(\mathbf{r}_i) + \hbar\mathbf{k})^2 + \hat{V},$$

where the potential $\hat{V}$ includes a one-body term (the external potential) and a two-body one (the electron-electron interaction). For any $\mathbf{k}$, and for any system size L, the ground wavefunction obeys periodic (toroidal) boundary conditions. The Provost-Vallee quantum metric tensor [2], is in our case:

$$g_{\alpha\beta}(\mathbf{k}) = \mathrm{Re}\ \langle\partial_\alpha\Psi_0(\mathbf{k})|\partial_\beta\Psi_0(\mathbf{k})\rangle - \langle\partial_\alpha\Psi_0(\mathbf{k})|\Psi_0(\mathbf{k})\rangle\langle\Psi_0(\mathbf{k})|\partial_\beta\Psi_0(\mathbf{k})\rangle,$$

where $\partial_\alpha = \partial/\partial k_\alpha$. The corresponding many-body Berry curvature is

$$\Omega_{\alpha\beta}(\mathbf{k}) = i[\,\langle\partial_\alpha\Psi_0(\mathbf{k})|\partial_\beta\Psi_0(\mathbf{k})\rangle - \langle\partial_\beta\Psi_0(\mathbf{k})|\partial_\alpha\Psi_0(\mathbf{k})\rangle\,],$$

and vanishes for time-reversal invariant systems.

The insulating state of matter is characterized by having $g_{\alpha\beta}(0)/N$ finite in the thermodynamic limit, at variance with metals where it diverges [3]. This quantity provides a measure of Kohn's localization in *any kind* of insulator, and has the physical meaning of the ground-state quantum fluctuation of macroscopic polarization. In the simple case of a crystalline system of noninteracting electrons, this same quantity sets a lower bound for the quadratic spread of the Wannier functions (which, again, diverges in the metallic case).

In 2d the Chern number is $C_1 = 4\pi^2\Omega_{12}(0)/L^2$ in the large-L limit. For a quantum Hall fluid, the transverse conductivity is proportional to C_1, ergo to $\Omega_{12}(0)/N$. The system is indeed an insulator: in fact the longitudinal conductivity vanishes, and $g_{11}(0)/N$ is finite [4].

[1] W. Kohn, *Theory of the insulating state*, Phys. Rev. **133**, A171 (1964)

[2] J. P. Provost and G. Vallee, Commun. Math Phys. **76**, 289 (1980).

[3] R. Resta, *Why are insulators insulating and metals conducting?*, J. Phys.: Condens. Matter **14**, R625 (2002).

[4] R. Resta, *Electron Localization in the Quantum-Hall Regime*, Phys. Rev. Lett. **95**, 196805 (2005).