On copulas subject to increasing conditional dependence

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Abstract:
In many real world phenomena one can observe that the dependence between certain data increases when one factor achieves the extreme values. As the first example may serve the so called market contagion. Namely, let X and Y model the returns from the indices of two stock markets. It is a well-acknowledged stylised fact that the dependence of Y and X subject to the condition $X \leq q_X(\alpha)$ is greater than the unconditional dependence of Y and X. As the other one may serve the observed in reliability fact, that in many complex systems the dependence between the remaining time till the failure of the first component and the remaining time till the failure of the whole system is increasing with regards to the time the system has already worked without flaws.
The aim of my talk is to present the tools for construction of models describing such stylized facts. Since copulas are the most general measure of dependence I will state the results in copula based language. First I will give the complete characterization of bivariate copulas that are invariant with respect to univariate conditioning. Next basing on the above I will deal with the family of bivariate copulas $C(u, v)$ such that:
1. C describes the interdependencies between positively quadrant dependent random variables X, Y,

$$\forall u, v \in [0, 1] \;\; C(u, v) \geq \Pi(u, v) = uv.$$

2. The conditional copulas $C_{[\alpha]}(u, v)$ (the copulas of the conditional joint distribution of X and Y with respect to the condition $X \leq q_X(\alpha)$) are monotonic in concordance ordering

$$\alpha_1 \leq \alpha_2 \quad \Longrightarrow \quad \forall u, v \quad C_{[\alpha_1]}(u, v) \geq C_{[\alpha_2]}(u, v)$$

(the smaller quantile the greater dependence).

Keywords:
Dependence, Copulas, Univariate Conditioning, Market Contagion, Concordance Ordering.

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