Recent Advances on Theory and Applications of Semi-Lagrangian Methods Rome

Abstracts

5-6 December 2011

Roldolfo Bermejo
Universidad Politécnica de Madrid

A semi-Lagrangian particle level set finite element method for interface problems

We present a high order quasi-monotone semi-Lagrangian particle level set (QMSL-PLS) method for moving interfaces. The quasi-monotone semi-Lagrangian method is a blend of a low order monotone and a high order semi-Lagrangian methods. The QMSL-PSL method is easy to implement, efficient and well adapted for unstructured, either simplicial or hexahedral, meshes. We prove that it is unconditionally stable in the maximum discrete norm, $\| \cdot \|_{h,\infty}$, and the error analysis shows that when the level set solution $u(t)$ is in the Sobolev space $W^{r+1,\infty}(D)$, $r \geq 0$, the convergence in the maximum norm is of the form $(KT\Delta t)\min(1, \Delta t \| v \|_{h,\infty}/h)((1-\alpha)h^p + h^q)$, $p = \min(2, r+1)$, and $q = \min(3, r+1)$, where v is a velocity. This means that at high CFL numbers, that is, when $\Delta t > h$, the error is $O(\frac{(1-\alpha)h^p+h^q)}{\Delta t})$, whereas at CFL numbers less than 1, the error is $O((1-\alpha)h^{p-1} + h^{q-1})$. The parameter $\alpha \in [0,1]$ controls the oscillatory behavior of the contribution to the numerical solution by the high order semi-Lagrangian method, in regions where the solution is smooth enough, α is very close to 1, so that in these regions the method will behave as a high order method. We have tested our method with satisfactory results in benchmark problems such as the Zalesak´s slotted disc, the single vortex flow and the rising bubble.

Elena Celledoni
Norwegian University of Science and Technology

Semi-Lagrangian methods: high order discretizations in space and time

In this talk I will review two classed of high order time integration methods for convection diffusion problems amenable to semi-Lagrangian implementations. I will show the performance of the methods when applied to a range of test problems. I will discuss briefly their stability properties and their performance in convection dominated problems.

Holger Heumann
University of Nice Sophia-Antipolis
Semi-Lagrangian Methods for Advection-Diffusion for Differential Forms

We consider generalized linear transient advection-diffusion problems for differential forms on a bounded domain in $\mathbb{R}^n$. We provide comprehensive a priori convergence estimates for their spatio-temporal discretization by means of a semi-Lagrangian approach combined with a discontinuous Galerkin method. Under rather weak assumptions on the velocity underlying the advection we establish an asymptotic L^2-estimate $O(\tau + \epsilon h^r + h^{r+1}\tau^{-\frac{1}{2}} + \tau^{\frac{1}{2}})$, where h is the spatial meshwidth, τ denotes the timestep, r is the polynomial degree of the forms used as trial functions, and ϵ stands for a lower bound on the diffusion parameter. This estimate can even be improved considerably in a variety of special settings.
In collaboration with Ralf Hiptmair

Roberto Mecca
Sapienza Università di Roma
A Semi-Lagrangian scheme for Shape-from-Shading via Photometric Stereo

In this talk we introduce a new model for the Shape from Shading problem using Photometric Stereo techniques (SfS-PS). Under appropriate hypotheses, using two or more digital image of an object, the solution to the problem can be characterized via a system of non-linear equations of the Hamilton-Jacobi type. Moreover, the system can be reduced to a single advection equation where the vector field depend on the gray-level of the images. In order to guarantee stability and accuracy, we use a semi-Lagrangian scheme for that associated linear advection problem. Some tests on synthetic and real images will be presented.

Michel Mehrenberger
Université de Strasbourg
Enhanced convergence estimates for semi-Lagrangian schemes with application to the Vlasov-Poisson system

In the framework of the convergence of semi-Lagrangian methods for the Vlasov Poisson system, based on a Strang splitting, we are interessed in the behavior of the error of the advection equation with high order discretizations. We will particularly study the case where the CFL number tends to zero. Usual estimates diverge, what raises question.

Giovanni Russo
Università di Catania
Level-set finite difference ghost-fluid methods for free boundary problems in incompressible fluid dynamics

Free boundary problems for the incompressible Euler and Navier-Stokes equations in primitive variables in two space dimensions are considered, with possible application to deep water flow, and lava flow. The computational domain is a rectangular box, discretized by a regular Cartesian grid. The boundary of the domain containing the fluid is identified by a level set function, which evolves with the fluid itself. The equations are discretized by finite difference on the grid. A variant of the projection methods is used to evolve the velocity field. The evolution equation for the velocity is written in a semi-lagrangian form, thus achieving second order accuracy in space and time, and avoiding CFL restrictions on the time step. The pressure is computed by solving a suitable Poisson equation. Boundary conditions are used to determine the additional equations for the pressure at ghost points. The main novelty of the approach is in a very effective Poisson solver for the pressure equation, based on a relaxation the boundary conditions by a fictitious time method, and a subsequent multigrid solver, and new extension technique for the velocity on ghost points, which guarantees divergence-free field extension up to second order. An implicit approach for the diffusive term in the Navier-Stokes equations is considered. The new velocity and the pressure are computed by solving a single implicit system, which guarantees preservation of stationary solutions at discrete level. Preliminary results of the numerical solution of Navier-Stokes equations with large jump in the viscosity parameter are also presented.

Giovanni Tumolo
ICTP
A semi-implicit, semi-Lagrangian, p-adaptive Discontinuous Galerkin method for the rotating shallow water equations

As a first step in the context of a broader work concerning the construction and analysis of a new generation DG based dynamical core for atmospheric modeling, a semi-implicit and semi-Lagrangian Discontinuous Galerkin method for the shallow water equations with rotation is proposed and analyzed in two dimensions.

The shallow water equations actually contain all of the horizontal operators required in an atmospheric model and thus represent a good first test for newly proposed schemes for atmospheric simulations.

The method is equipped with a simple p-adaptivity criterion, that allows to adjust dynamically the number of local degrees of freedom employed to the local structure of the solution.

Numerical results in the framework of one dimensional and two dimensional test cases prove that the method captures accurately and effectively the main features of linear gravity and inertial gravity waves, as well as reproduces correct solutions in nonlinear open channel flow tests. The effectiveness of the method is also demonstrated by numerical results

obtained at high Courant numbers and with automatic choice of the local approximation degree.

The present research has been carried out during the PhD of the autor (supervisors F. Giorgi and L. Bonaventura) with financial support from the *Abdus Salam International Center for Theoretical Physics* and in collaboration with L.Bonaventura and M. Restelli.

Alexander Vladimirsky
Cornell University

Semi-Lagrangian methods for multiobjective (and integral-constrained) optimal control

Many realistic control problems involve multiple criteria for optimality and/or integral constraints on allowable controls. This can be conveniently modeled by introducing a budget for each secondary criterion/constraint. An augmented HJB equation is then solved on an expanded state space, and its discontinuous viscosity solution yields the value function for the primary criterion/cost. For the case where the resources/budgets are monotone decreasing, we have developed a fast (non-iterative) algorithm based on an explicitly causal semi-Lagrangian discretization. The computational issues are similar to those encountered in traditional (single criterion) time-dependent optimal control, but with weak controllability and special (implicitly defined) state restrictions.
Our recent work concerns a more challenging case, where the resources can be instantaneously renewed (& budgets can be reset) upon entering a pre-specified subset of the state space. This leads to a hybrid control problem with more subtle causal properties of the value function and additional challenges in constructing efficient & accurate semi-Lagrangian discretizations.
The first project is joint work with A.Kumar. The second project is joint work with R.Takei, W.Chen, Z.Clawson, and S.Kirov.

Nigel Wood
Met Office

A weather and climate modelling perspective on recent developments of the semi-Lagrangian method

A brief overview of the current, semi-implicit semi-Lagrangian, dynamical core of the UK Met Office's Unified Model will be given (this is the so-called New Dynamics). The talk will then focus on various aspects of the semi-Lagrangian scheme that have been developed as part of the planned replacement of the dynamical core (ENDGame).
Depending on the time available a selection of the following topics will be included:
- a globally uniform rotation matrix approach to evaluating the departure points on the sphere with extension to non-spherical geometries;
- an issue with, and solution to, the stability of this method;
- a proposal for the consistent handling of trajectories that approach the vertical boundaries

of the numerical domain;
- an inherently conservative semi-Lagrangian scheme (SLICE);
- the role of the departure point calculation in the overall stability of the model;
- the role of the departure point calculation on dynamical consistency.