Workshop
MEAN FIELD GAMES AND RELATED TOPICS
Dipartimento di Matematica
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ABSTRACTS

Y. Achdou (Université Paris 7)

Mean Field Games: numerical methods for finite horizon problems

We consider the system of two PDEs arising from the theory of MFG [3,4,5], consisting of a forward in time Hamilton-Jacobi-Bellman coupled with a backward in time Fokker-Planck equation. Finite difference schemes for this system have been proposed [1,2]. They lead to large systems of nonlinear equations. In this talk, we discuss different strategies for solving these systems by Newton methods, in particular recent attempts with multigrid methods (work in progress with V. Perez, Ph.D student at Université Paris Diderot)).

References

[1] Y. Achdou, F. Camilli, I. Capuzzo Dolcetta. Mean field games: numerical methods for the planning problem, hal-00465404.
[2] Y. Achdou and I. Capuzzo Dolcetta. Mean field games: Numerical methods. SIAM J. Numer. Anal., 48(3), 1136-1162, 2010.
[3] J.-M. Lasry and P.-L. Lions. Jeux à champ moyen. I. Le cas stationnaire. C. R. Math. Acad. Sci. Paris, 343(9), 619-625, 2006.
[4] J.-M. Lasry and P.-L. Lions. Jeux à champ moyen. II. Horizon fini et controle optimal. C. R. Math. Acad. Sci. Paris, 343(10), 679-684, 2006.
[5] J.-M. Lasry and P.-L. Lions. Mean field games. Jpn. J. Math., 2(1), 229-260, 2007.

M. Bardi (Università di Padova)

Explicit solutions of some Linear-Quadratic Mean Field Games

I consider N-person differential games where the state of each player is 1-dimensional and governed by a stochastic differential equation linear in the state and in the control. I take an ergodic cost criterion with running cost quadratic with respect to state and control and consider the system of Hamilton-Jacobi-Bellman and Kolmogorov equations introduced by Lasry and Lions in the theory of Mean Field Games. Under assumptions of symmetric and almost identical players I solve explicitly the system.

Then I compute the limit as the number N of players goes to infinity, assuming they are identical and with suitable scalings of the parameters. This provides an explicit solution of the resulting Mean Field Game differential equations, unique under a natural normalization (different from the one of the Lasry-Lions theory for the space periodic setting). A full discussion of the dependence of the solution on the parameters (such as the coefficients of self, primary and secondary interaction among the players) can be performed.

J.D. Benamou (INRIA)

Numerical resolution of time extended Mass Transport Problems

we will give an overview of

1. the time formulation of the Monge Kantorovitch (MK2) Mass Transport problem and several proposed extensions;

2. some gradient descent attempts for its numerical simulation.

G. Buttazzo (Università di Pisa)

Evolution Models for Mass Transportation Problems

The presentation aims to provide a dynamical formulation for a class of mass transportation problems using a Benamou-Brenier approach consisting in the minimization of a suitable functional depending on the density and on the velocity of the flow, coupled with the continuity equation.

The cases of congestion and concentration effects are considered, the first ones occurring for instance in traffic flows problems and in movement of crowds under panic effects, the second ones in several models of branching transportation as roots of trees, roads, communication networks, delta of rivers, blood vessels.

P.E. Caines (McGill University, Montreal)

A Continuum Mean Field Stochastic Control Based Consensus Problem

A continuum Mean Field (MF) (or Nash Certainty Equivalence (NCE)) stochastic control approach to the consensus problem is introduced. In this problem formulation: (i) each agent has simple stochastic dynamics with inputs directly controlling its state's rate of change, and (ii) each agent seeks to minimize its individual cost function involving a mean field coupling to the states of all other agents. For this dynamic game problem, a set of coupled deterministic (backward in time) Hamilton-Jacobi-Bellman and (forward in time) Fokker-Planck-Kolmogorov equations which is coupled to a (spatially averaged) cost coupling function is derived approximating the stochastic system of agents in the continuum (i.e., as the population size N goes to infinity).

In a finite population system (analogous to the individual based approach): (i) the resulting MF control strategies possess an ϵ_N-Nash equilibrium property where ϵ_N goes to zero as the population size N approaches infinity, and (ii) these MF control strategies steer each individual's state toward the initial state population mean which is reached asymptotically as time goes to infinity. Hence, the system with decentralized MF control strategies reaches mean-consensus on the initial state population mean asymptotically (as time goes to infinity).

In this talk an evolution (i.e., forward in time) Mean Field (MF) consensus equation system consisting of two coupled (forward in time) deterministic PDEs coupled to the (spatially averaged) cost coupling function will also be presented; then the reconciliation between the stochastic MF consensus problem with standard consensus problems (i.e., consensus algorithms with online feedback) via Q-learning techniques will be discussed. Finally, the extensions of the obtained results to the analysis of the MF synthesized Cucker-Smale flocking model will be addressed.

In collaboration with Mojtaba Nourian (CIM, Montreal) and Roland P. Malhamé (GERAD, Montreal).

P. Cardaliaguet (CEREMADE, Paris)

Long time average for the system of mean field games

We consider the time dependent system of mean field game and investigate the average as time tends to infinity. The interesting point is that the original system has an initial and a terminal condition.

G. Carlier (CEREMADE, Paris)

From discrete to continuous equilibrium models of congested transport

In the first part of the talk, I will introduce a standard model of congestion on a finite network an will explain how equilibrium configurations can be found by some convex minimization problem. In the second part of the talk, a continuous model will be obtained by Gamma-convergence. Finally, I will emphasize some links between the continuous model and some degenerate elliptic PDE's and explain how to solve the continuous problem by a flow argument à la Moser.

D. Gomes (Istituto Superior Tecnico, Lisboa)

Finite space, continuous time mean field games: ODE and PDE methods

Mean field games is a recent area of study introduced by Lions and Lasry in a series of seminal papers. Mean field games model situations of competition between large number of rational agents that play non-cooperative dynamic games under certain symmetry assumptions. A key step is to develop a mean field model, in a similar way to what is done in statistical physics in order to construct a mathematically tractable model. A main question that arises in the study of such mean field problems is the rigorous justification of the mean field models by a limiting procedure. We consider the mean field limit of two-state Markov decision problem as the number of players $N \to \infty$. First we establish the existence and uniqueness of a symmetric partial information Markov perfect equilibrium. Then we derive a mean field model and characterize its main properties. This mean field limit is a system of coupled ordinary differential equations with initial-terminal data. Our main result is the convergence as $N \to \infty$ of the N player game to the mean field model and an estimate of the rate of convergence.

O. Gueant (Université Paris 7)

Change of variable for mean field games with quadratic Hamiltonian

In this talk, we will focus on mean field games with quadratic hamiltonian function and show that a change of variable reduces the MFG equations to forward/backward heat equations with non-linear source terms. Monotonic algorithms linked to this change of variable will then be presented with numerical examples.

J.-M. Lasry (CEREMADE)

Modelizations MFG de dynamiques d'investissement

P.-L. Lions (College de France)

TBA

P. Markowich (Cambridge University)

On some PDE models in the social sciences

We present three PDE models describing important phenomena in the social sciences: a Boltzmann type equation for the spread of opinions in human societies, a hyperbolic conservation law for human crowd motion and a free boundary parabolic equation for price formation in commodity markets.

F. Santambrogio (Université de Paris-Sud Orsay)

Density constraints and pressures in gradient flows and mean-field games

I will start from a PDE formulation for crowd movement given by B. Maury as a continuous counterpart of a discrete model where individual are small disks subject to non-overlapping constraints. The key point is the constraint $\rho \leq 1$ and the fact that the velocity field advecting ρ in the continuity equation is the projection of the desired one on the set of admissible fields (where admissible fields are those that have negative divergence on the set where $\rho = 1$). The corresponding PDE is quite hard and existence has been proven via optimal transport techniques, in particular in a gradient-flow framework.

After that, I'll try to base on the same idea a mean-field game, where costs depending on the $\rho(x)$ are replaced by constraints. The main difficulty is making it meaningful, since we can always think that every agent, facing an already admissible density ρ, may move with no special constraints, since one only extra particle will never violate the condition $\rho \leq 1$. In order to give a proper definition of the equilibrium and of the equations, we will use the pressure field.

H. Tembine (SUPELEC, Gif sur Yvette)

Risk-sensitive mean field stochastic games

The talk will discuss the mean field equilibrium in risk-sensitive Markov games in a large-population regime, with connections to the propagation of chaos. Risk-sensitivity is captured through exponential of long-term payoff for each player. Under specific structure of transition kernels, mean-field limit arises because of the large number of players and the presence of some small parameters in the risk-sensitivity index. We derive a risk-sensitive version of Bellman-Shapley equation via multiplicative dynamic programming and a Kolmogorov forward equation. As a special case results, irreducible states will be presented.

G.Turinici (CEREMADE, Paris)

Numerical approaches for MFG

Motivated by a MFG model for the choice of technologies we consider the associated optimization problem and introduce a specific algorithm to find the MFG equilibria. We close with numerical results, including the multiplicity of equilibria and further considerations on numerical approaches.

M.T. Wolfram (Wien University)

On a mean field game approach modeling pedestrian motion

Crowd motion modeling has become an active area of research within the last decades. In this talk we focus on a mean field games approach modeling pedestrian dynamics, in particularly on the interactions between two pedestrian groups. A well known phenomena in crowd motion is the formation of lanes with uniform walking direction in streets. Here people tend to stay within their own group (walking in one direction) and avoid congestion with the other group (walking in the opposite direction).

The presented mean field game model includes these congestion effects. We discuss the mathematical modeling and present a numerical solver for the stationary problem. Furthermore we illustrate the behavior of the model with various numerical simulations.