Quantum Feedback Control - Lecture 1

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CENTRE FOR QUANTUM COMPUTATION & COMMUNICATION TECHNOLOGY
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Lecture 1 *Introduction and basic concepts*
Quantum technology, quantum control, postulates of quantum mechanics, quantum probability.

Lecture 2 *Measurement feedback quantum control*
Open quantum systems, quantum stochastic models, quantum filtering, optimal measurement feedback control, risk-sensitive quantum control, linear quantum systems.

Lecture 3 *Coherent feedback quantum control*
Quantum feedback networks, quantum dissipative systems, control by interconnection, linear quantum systems.

'Technology seems to advance in waves. Small advances in science and technology accumulate slowly ... until a critical level...

'Woven into the rich fabric of technological history is an invisible thread that has a profound effect on each of these waves...

'This thread is the idea of feedback control.

Dennis Bernstein, *History of Control*, 2002

Lecture 1 - Outline

1. Quantum Technology
2. Quantum Control
3. Quantum Mechanics
4. Quantum Probability

Quantum Technology

Quantum technology is the application of quantum science to develop new technologies. This was foreshadowed in a famous lecture:

1959: Richard Feynman, Plenty of Room at the Bottom

> *"What I want to talk about is the problem of manipulating and controlling things on a small scale."*

Key drivers for quantum technology:

- Miniaturization - quantum effects can dominate
  - Microelectronics - feature sizes of 10s nm (Moore's Law)
  - Nanotechnology - nano electromechanical devices have been made sizing 10s nm
- Exploitation of quantum resources
  - Quantum Information - (ideally) perfectly secure communications
  - Quantum Computing - algorithms with exponential speed-ups
  - Metrology - ultra-high precision measurements

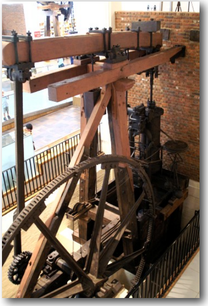
[Dowling-Milburn, 2003]

Quantum technology revolutions

[Dowling-Milburn, 2003]

- First: [QM used to understand what exists]
  - wave-particle duality
  - semiconductors
  - information age
- Second: [QM used to engineer new things]
  - artificial atoms
  - man-made quantum states
  - quantum engineering

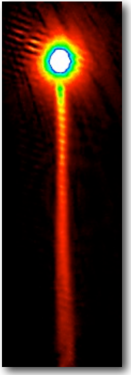
Quantum Control



[Boulton and Watt, 1788, London Science Museum]

Watt used a governor to control steam engines - very macroscopic.

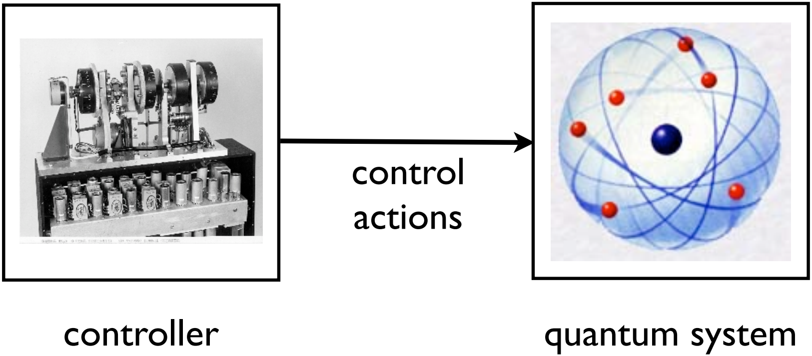
Now we want to control things at the quantum level - e.g. atoms



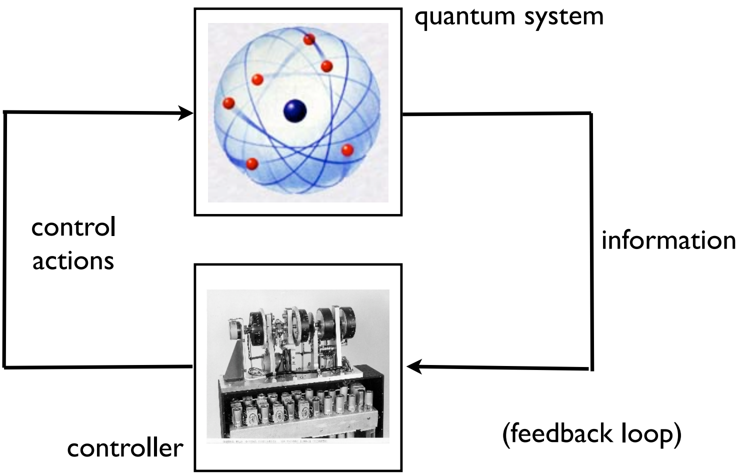
[ANU atom laser, 2007, Canberra]

Types of Quantum Control:

Open loop - control actions are predetermined, no feedback is involved.



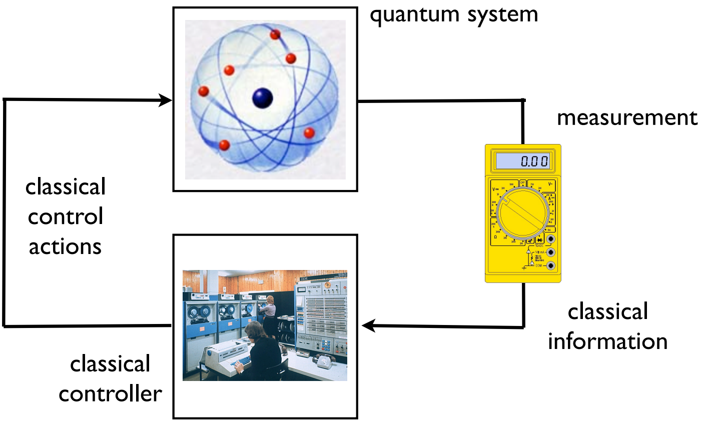
Closed loop - control actions depend on information gained as the system is operating.



Types of Quantum Feedback:

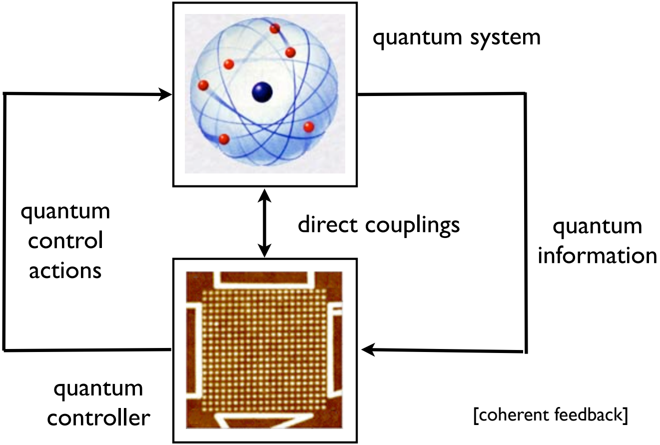
Using measurement

The classical measurement results are used by the controller (e.g. classical electronics) to provide a classical control signal.



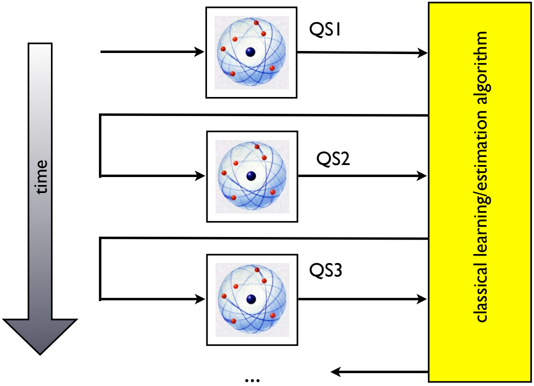
Not using measurement

The controller is also a quantum system, and feedback may involve a flow of quantum information, as well as direct couplings.



Iterative learning control

Same scheme for estimation from repeated identical experiments.
Fresh quantum system in each iteration.

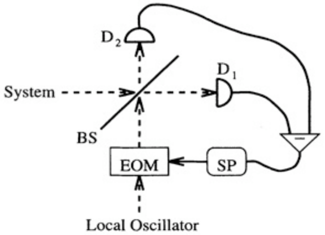


Examples of quantum feedback control

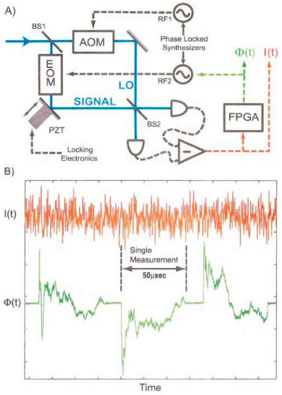
Adaptive phase measurement

[Wiseman 1995]

- the first quantum measurement feedback control experiment (a very important experimental test)

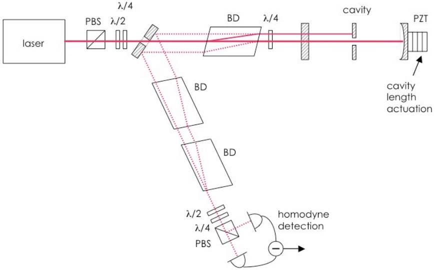


[Armen, Au, Stockton, Doherty, Mabuchi 2002]

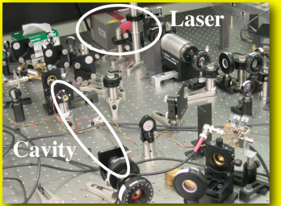


Laser-cavity locking

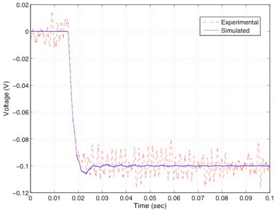
- quantum LQG measurement feedback control experiment



[Huntington, James, Petersen, Sayed Hassen, Heurs, 2009]



[Huntington Lab, ADFA@UNSW]

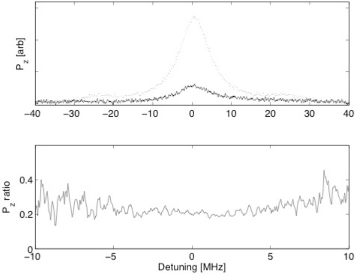


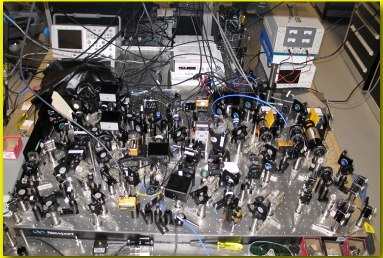
Coherent quantum feedback control

- quantum coherent feedback control experiment

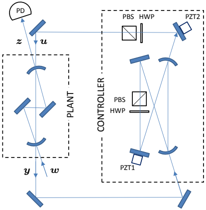
[Mabuchi, 2008]

[James, Nurdin, Petersen, 2008]

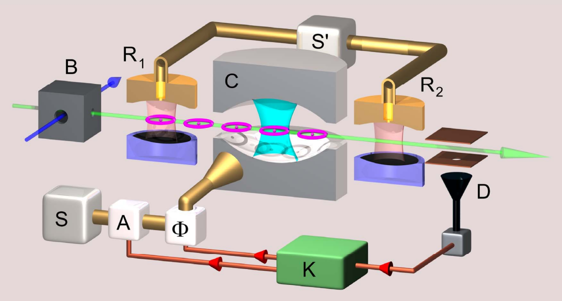




[Mabuchi Lab, Stanford]



Closed-loop QED experiment



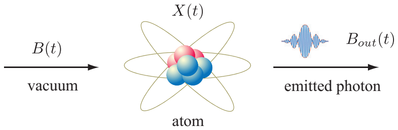
C. Sayrin et al., Nature, 1-September 2011

[Image courtesy of Hadis Amini, Igor Dotsenko]

Quantum Mechanics

A little history

- Black body radiation (Plank)
- Photoelectric effect (Einstein)
- Atomic quantization (Bohr)
- Quantum probability (Born)
- Spontaneous and stimulated emission of light (Einstein)
- Matter waves (De Broglie)
- Matrix mechanics, uncertainty relation (Heisenberg)
- Wave functions (Schrodinger)
- Entanglement (EPR)
- Axiomatization, quantum probability (von Neumann)



Non-commuting observables

$$[Q, P] = QP - PQ = i\hbar\, I$$

Expectation

$$\langle Q \rangle = \int q |\psi(q,t)|^2 dq$$

Heisenberg uncertainty

$$\Delta Q \Delta P \geq \frac{1}{2} |\langle i[Q,P] \rangle| = \frac{\hbar}{2}$$

Schrodinger equation

$$i\hbar \frac{\partial \psi(q,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(q,t)}{\partial q^2} + V(q)\psi(q,t)$$

Some mathematical preliminaries

Hilbert space with inner product $\langle \cdot, \cdot \rangle$

We take $\mathfrak{H} = \mathbf{C}^n$, n-dimensional complex vectors, $\langle \psi, \phi \rangle = \sum_{k=1}^{n} \psi_k^* \phi_k$.

Vectors are written (Dirac's *kets*)

$$\phi = |\phi\rangle \in \mathfrak{H}$$

Dual vectors are called *bras*, $\psi = \langle \psi | \in \mathfrak{H}^* \equiv \mathfrak{H}$, so that

$$\langle \psi, \phi \rangle = \langle \psi || \phi \rangle = \langle \psi | \phi \rangle$$

Let $\mathscr{B}(\mathfrak{H})$ be the Banach space of *bounded operators* $A : \mathfrak{H} \to \mathfrak{H}$.

For any $A \in \mathscr{B}(\mathfrak{H})$ its *adjoint* $A^* \in \mathscr{B}(\mathfrak{H})$ is an operator defined by

$$\langle A^* \psi, \phi \rangle = \langle \psi, A\phi \rangle \quad \text{for all } \psi, \phi \in \mathfrak{H}.$$

Also define

$$\langle A, B \rangle = \mathrm{Tr}[A^* B], \quad A, B \in \mathscr{B}(\mathfrak{H})$$

An operator $A \in \mathscr{B}(\mathfrak{H})$ is called *normal* if $AA^* = A^*A$. Two important types of normal operators are *self-adjoint* ($A = A^*$), and *unitary* ($A^* = A^{-1}$).

The *spectral theorem* for a self-adjoint operator A says that (finite dimensional case) it has a finite number of real eigenvalues and that A can be written as

$$A = \sum_{a \in \mathrm{spec}(A)} a P_a$$

where P_a is the projection onto the eigenspace corresponding to the eigenvalue a (diagonal representation).

The Postulates of Quantum Mechanics

The basic quantum mechanical model is specified in terms of the following:

Observables

Physical quantities like position, momentum, spin, etc., are represented by self-adjoint operators on the Hilbert space $\mathfrak{H}$ and are called *observables*. These are the noncommutative counterparts of random variables.

States

A state is meant to provide a summary of the status of a physical system that enables the calculation of statistical quantities associated with observables. A generic state is specified by a *density matrix* ρ, which is a self-adjoint operator on $\mathfrak{H}$ that is positive $\rho \geq 0$ and normalized $\mathrm{Tr}[\rho] = 1$. This is the noncommutative counterpart of a probability density.

The expectation of an observable A is given by

$$\langle A \rangle = \langle \rho, A \rangle = \mathrm{Tr}[\rho A]$$

Pure states: $\rho = |\psi\rangle\langle\psi|, \quad \psi \in \mathbf{H}$ so that

$$\langle A \rangle = \mathrm{Tr}\left[|\psi\rangle\langle\psi|A\right] = \langle \psi, A\psi \rangle = \langle \psi|A|\psi \rangle$$

Measurement

A *measurement* is a physical procedure or experiment that produces numerical results related to observables. In any given measurement, the allowable results take values in the spectrum $\mathrm{spec}(A)$ of a chosen observable A.

Given the state ρ, the value $a \in \mathrm{spec}(A)$ is observed with probability $\mathrm{Tr}[\rho P_a]$.

Conditioning

Suppose that a measurement of A gives rise to the observation $a \in \mathrm{spec}(A)$. Then we must condition the state in order to predict the outcomes of subsequent measurements, by updating the density matrix ρ using

$$\rho \mapsto \rho'[a] = \frac{P_a \rho P_a}{\mathrm{Tr}[\rho P_a]}.$$

This is known as the "projection postulate".

Evolution

A *closed* (i.e. isolated) quantum system evolves in a *unitary* fashion: a physical quantity that is described at time $t = 0$ by an observable A is described at time $t > 0$ by [Heisenberg picture]

$$A(t) = U(t)^* A U(t),$$

where $U(t)$ is a unitary operator for each time t. The unitary is generated by the *Schrödinger equation*

$$i\hbar \frac{d}{dt} U(t) = H(t) U(t),$$

where the (time dependent) Hamiltonian $H(t)$ is a self-adjoint operator for each t.

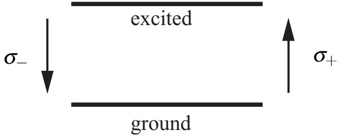
States evolve according to [Schrodinger picture]

$$\rho(t) = U(t) \rho U^*(t)$$

The two pictures are equivalent (dual): ($\langle A, B \rangle = \mathrm{Tr}[A^* B]$)

$$\langle \rho(t), A \rangle = \langle \rho, A(t) \rangle$$

The two-level system (qubit).



$H = \mathbf{C}^2$, ground $|g\rangle$ and excited $|e\rangle$ states.
Raising σ_+ and lowering σ_- operators:

$$\sigma_+|g\rangle = |e\rangle, \quad \sigma_-|e\rangle = |g\rangle.$$

Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The quantum harmonic oscillator.

$H = L^2(\mathbf{R})$,

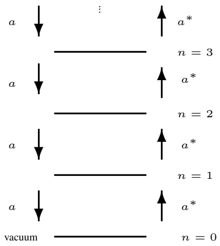
$$(Q\psi)(q) = q\psi(q), \quad (P\psi)(q) = -i\frac{d}{dq}\psi(q)$$

Annihilation and creation operators (up to constants)

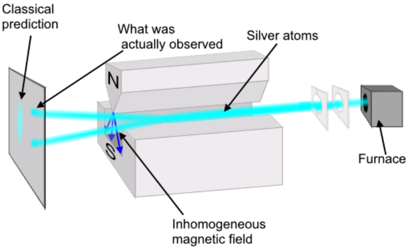
$$a = Q + iP, \quad a^* = Q - iP$$

Commutation relations

$$[a, a^*] = 1$$



Example - Stern-Gerlach experiment



Let $\mathfrak{H} = \mathbf{C}^2$, and consider the observable

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

representing spin in the z-direction.
Measurements of this quantity take values in

$$\mathrm{spec}(\sigma_z) = \{-1, 1\}$$

which correspond to spin down and spin up, respectively.

We can write

$$\sigma_z = P_{z,1} - P_{z,-1}$$

[spectral representation]

where

$$P_{z,1} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad P_{z,-1} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},$$

Consider a pure state, given by the vector

$$\psi = \begin{pmatrix} c_1 \\ c_{-1} \end{pmatrix}$$

with $|c_1|^2 + |c_{-1}|^2 = 1$

If we observe σ_z, we obtain

- the outcome 1 (spin up) with probability $\langle \psi, P_{z,1}\psi \rangle = |c_1|^2$, or
- the outcome -1 with probability $\langle \psi, P_{z,-1}\psi \rangle = |c_{-1}|^2$.

Compatible and incompatible observables

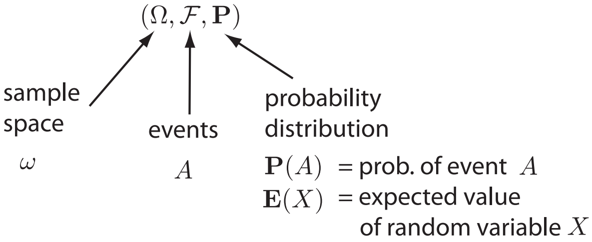
One of the key differences between classical and quantum mechanics concerns the ability or otherwise to simultaneously measure several physical quantities. In general it is not possible to exactly measure two or more physical quantities with perfect precision if the corresponding observables do not commute, and hence they are *incompatible*.

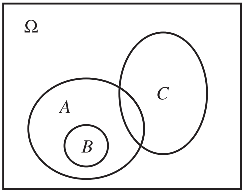
A consequence of this is lack of commutativity is the famous *Heisenberg uncertainty principle*.

Quantum Probability

Classical probability

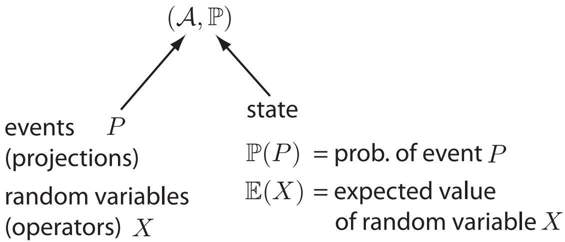
Classical physics is built on foundations of classical logic, which is closely related to classical probability.





Quantum probability

We may think of quantum mechanics as the description of physical systems using a *non-commutative probability theory*.



States may be defined using pure states $|\psi\rangle$ or density operators ρ:

$$\mathbb{E}[X] = \langle\psi|X|\psi\rangle, \text{ or } \mathbb{E}[X] = \mathrm{Tr}[\rho X].$$

Algebras $\mathcal{A}$ of events describe information in both classical and quantum probability.

The *spectral theorem* tells us that a commutative quantum probability space is equivalent to a classical probability space.

$$\underset{\text{commutative}}{(\mathscr{C}, \mathbb{P})} \longleftrightarrow (\Omega, \mathscr{F}, \mathbf{P})$$

This is the mathematics corresponding to the measurement postulate.

Example (spin)
Set $\mathfrak{H} = \mathbf{C}^2$ and choose $\mathscr{A} = M_2$ (2×2 complex matrices).

The pure state is defined by $\mathbb{P}(A) = \langle \psi | A | \psi \rangle$
(recall that $|\psi\rangle = (c_1 \ c_{-1})^T$ with $|c_1|^2 + |c_{-1}|^2 = 1$).

The observable σ_z, used to represent spin measurement in the z direction, generates a commutative $*$-subalgebra

$$\mathscr{C}_z \subset \mathscr{A}.$$

Now $\mathscr{C}_z$ is the linear span of the events (projections) $P_{z,1}$ and $P_{z,-1}$.
Spectral theorem: gives the probability space $(\Omega_z, \mathcal{F}_z, \mathbf{P}_z)$ where

$$\Omega_z = \{1, 2\},$$

$$\mathcal{F}_z = \{\varnothing, \{1\}, \{2\}, \Omega_z\}.$$

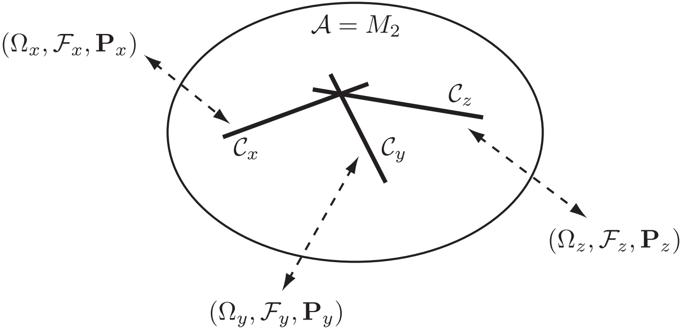
The observables

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

correspond to spin in the x and y directions, and they do not commute with σ_z, and so are incompatible with σ_z.

Their joint statistics are undefined; hence they cannot both be observed in the same realization.

This leads to distinct commutative subspaces:

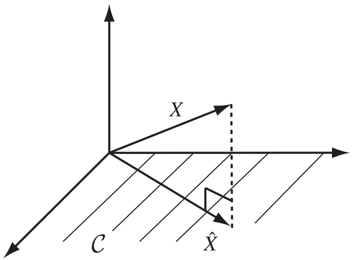


Quantum conditional expectation

Let X commute with a commutative subspace $\mathcal{C}$. The conditional expectation

$$\hat{X} = \pi(X) = \mathbb{E}[X|\mathcal{C}]$$

is the orthogonal projection of $X \in \mathcal{A}$ onto $\mathcal{C}$.



$\hat{X}$ is the minimum mean square estimate of X given $\mathcal{C}$.

By the spectral theorem, $\hat{X}$ is equivalent to a classical random variable.

Example

Consider $\mathfrak{H} = \mathbf{C}^3$, $\mathscr{A} = M_3$ (3 $\times$ 3 matrices), and $\mathbb{E}(X) = \langle \psi | X | \psi \rangle$ with $\psi = (1\ 1\ 1)^T / \sqrt{3}$.

Let

$$\mathcal{C} = \left\{ a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + b \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \ : \ a, b \in \mathbf{C} \right\}$$

and

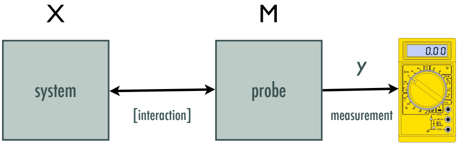
$$X = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

Then X commutes with $\mathcal{C}$ and

$$\mathbb{E}(X|\mathcal{C}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} = 1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + 2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \in \mathcal{C}.$$

[orthogonal projection]

Probe model for quantum measurement



The conditional expectation (least squares best estimate)

$$\pi(X) = \mathbb{E}[U^*(X \otimes I)U | U^*(I \otimes M)U]$$

is well defined because $X \otimes I$ commutes with $I \otimes M$.

This allows statistical estimation for system observables given measurement data.

The von Neumann "projection postulate" is a special case.

In continuous time, this leads to *quantum filtering* (Lecture 3).

Entanglement is a resource unique to the quantum world.

Particle A	EPR pair	Particle B
$Q = \pm 1$ $R = \pm 1$		$S = \pm 1$ $T = \pm 1$

Classical correlations:
For all joint classical probability distributions **P** on
$\Omega = \{-1, +1\} \times \{-1, +1\}$ we have *Bell inequality*

$$\mathbf{E}[QS + RS + RT - QT] \leq 2$$

Quantum correlations:
There exists a quantum state $\mathbb{E}$ on $M_2 \otimes M_2$ and choice of Q, R, S, T such that

$$\mathbb{E}[QS + RS + RT - QT] > 2$$