Quantum Feedback Control - Lecture 3

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Lecture 1 *Introduction and basic concepts*
Quantum technology, quantum control, postulates of quantum mechanics, quantum probability.

Lecture 2 *Measurement feedback quantum control*
Open quantum systems, quantum stochastic models, quantum filtering, optimal measurement feedback control, risk-sensitive quantum control.

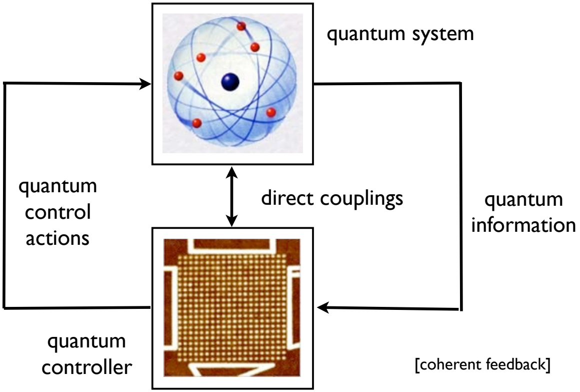
Lecture 3 *Coherent feedback quantum control*
Quantum feedback networks, quantum dissipative systems, control by interconnection, linear quantum systems.

Lecture 3 - Outline

1. Quantum Feedback Networks
2. Quantum Dissipative Systems
3. Quantum Feedback Control by Interconnection
4. Linear Quantum Systems

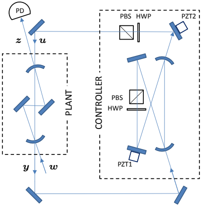
Quantum Feedback Networks

- Quantum information is lost when measurements are made.
- Coherent feedback loops need not involve measurements, and so allow for the flow of quantum information. The controller is another quantum system.

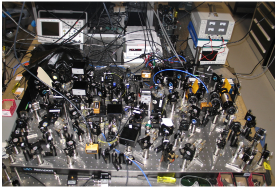


- Exchange of quantum information may occur via
    - direct physical couplings
    - indirect couplings using freely travelling *quantum fields* serving as *'quantum wires'*.
- According to Mabuchi 2008:

> *"...gives rise to a genuinely new category of control-theoretic problems as it encompasses non-commutative signals and quantum-dynamical transformations thereof"* and *"...relatively little is yet known about the systematic control theory of coherent feedback"*.

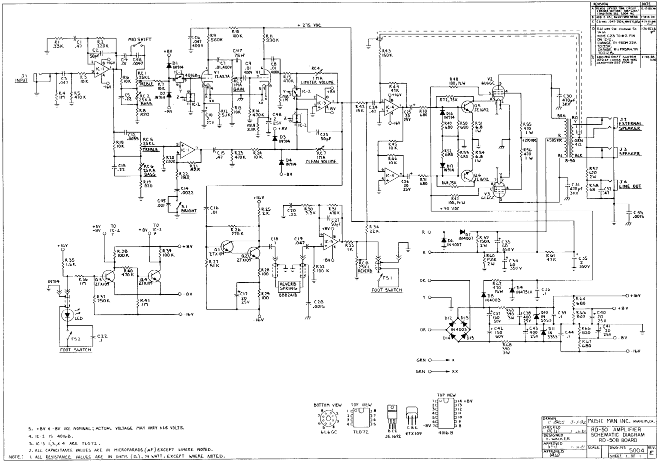


[James-Nurdin-Petersen 2007, Mabuchi 2008]



[Mabuchi Lab, Stanford]

Circuit diagrams are widely used in classical engineering.

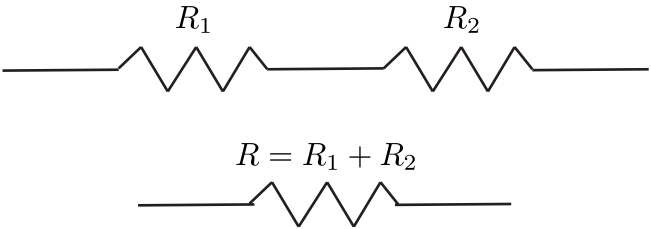


Some basic requirements of quantum network models:

- Capture the quantum physics
- Be capable of representing classical components
- Include dissipative mechanisms - noise, uncertainty, decoherence
- Preserve canonical structure - e.g. commutation relations, energy
- Network of interconnected components should also be a quantum system - recursive
- Efficient methods for representation, interconnection, manipulation, and physical realization
- Seamlessly integrates classical components.
- Efficient methods for analysis, design, and synthesis

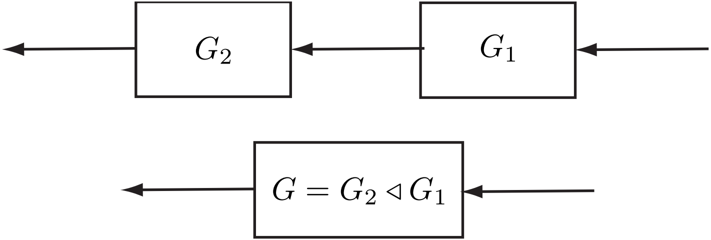
Basic ideas

Classical: series connection of resistors.



The simple algebraic formula $R = R_1 + R_2$ is based on underling physics (electromagnetism).

Quantum: series connection of open systems.



The simple algebraic formula

$$G_2 \triangleleft G_1 = (S_2 S_1, L_2 + S_2 L_1, H_1 + H_2 + \mathrm{Im}[L_2^\dagger S_2 L_1])$$

is based on underlying physics (quantum mechanics).

Where does this come from?

$$dB_{1,out} = L_1 dt + S_1 dB_{1,in}$$

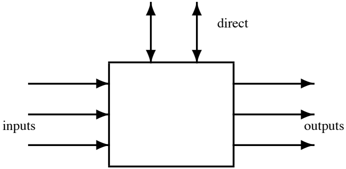
$$dB_{2,out} = L_2 dt + S_2 dB_{2,in}$$

$$B_{2,in}(t) = B_{1,in}(t - \tau), \quad \tau \downarrow 0.$$

$$dB_{2,out} = (L_2 + S_2 L_1) dt + S_2 S_1 dB_{1,in}$$

[Gardiner, 1994; Carmichael, 1994; Gough-James, 2009]

Network components - open quantum systems



Parameters

interface

internal energy

$$\mathbf{G} = (\mathbf{S}, \mathbf{L}, \mathbf{Z}, H)$$

indirect field connections

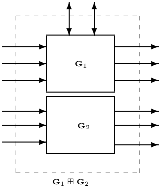
direct connections

Elementary network constructs

[Gough and James, 2008, 2010]

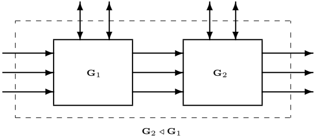
concatenation product

$$\mathbf{G}_1 \boxplus \mathbf{G}_2 = (\begin{pmatrix} \mathbf{S}_1 & 0 \\ 0 & \mathbf{S}_2 \end{pmatrix}, \begin{pmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \end{pmatrix}, \begin{pmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \end{pmatrix}, H_1 + H_2)$$



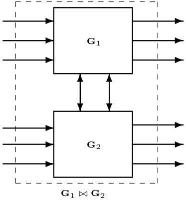
series product

$$\begin{aligned} \mathbf{G}_2 \triangleleft \mathbf{G}_1 &= \mathbf{S}_2\mathbf{S}_1, \mathbf{L}_2 + \mathbf{S}_2\mathbf{L}_1, \begin{pmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \end{pmatrix}, \\ & H_1 + H_2 + \frac{1}{2i}(\mathbf{L}_2^\dagger \mathbf{S}_2 \mathbf{L}_1 - \mathbf{L}_1^\dagger \mathbf{S}_2^\dagger \mathbf{L}_2)) \end{aligned}$$



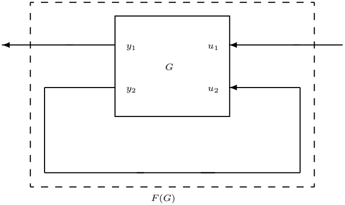
direct connection product

$$\mathbf{G}_1 \bowtie \mathbf{G}_2 = \left(\begin{pmatrix} \mathbf{S}_1 & 0 \\ 0 & \mathbf{S}_2 \end{pmatrix}, \begin{pmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \end{pmatrix}, -, H_1 + H_2 + \mathbf{Z}_2^\dagger \mathbf{Z}_1 + \mathbf{Z}_1^\dagger \mathbf{Z}_2 \right)$$



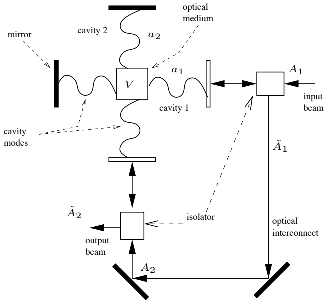
linear fractional transformation (LFT)

$$G = \left(\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}, \begin{pmatrix} L_1 \\ L_2 \end{pmatrix}, H \right)$$

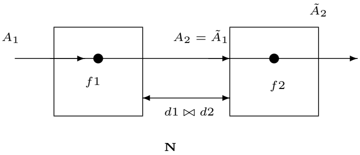


$$F(G) = (S_{11} + S_{12}(I - S_{22})^{-1} S_{21},\ L_1 + S_{12}(I - S_{22})^{-1} L_2,$$
$$H + \mathrm{Im}\{L_1^\dagger S_{12}(I - S_{22})^{-1} L_2\} + \mathrm{Im}\{L_2^\dagger S_{22}(I - S_{22})^{-1} L_1\})$$

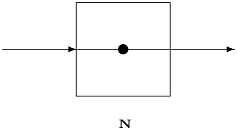
Reducible networks in quantum optics



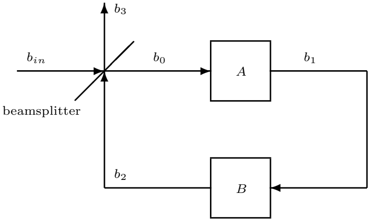
[Wiseman-Milburn, 1994]



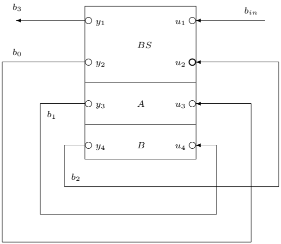
$$
\begin{aligned}
\mathbf{N} &= \mathbf{G}_1 \wedge \mathbf{G}_2 = (\mathbf{G}_{f2} \lhd \mathbf{G}_{f1}) \boxplus (\mathbf{G}_{d1} \bowtie \mathbf{G}_{d2}) \\
&= (1, \sqrt{\gamma_2}\, a_2 + \sqrt{\gamma_1}\, a_1, \_, \\
&\quad \Delta_1 a_1^* a_1 + \Delta_2 a_2^* a_2 - ig(a_2 a_1^* - a_2^* a_1))
\end{aligned}
$$



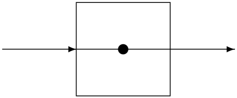
Non-reducible networks in quantum optics



[Yanagisawa-Kimura, 2003]



$$N = F\left((BS \boxplus A \boxplus B) \triangleleft T)\right)$$



N

Direct measurement feedback

[Wiseman, 1994]

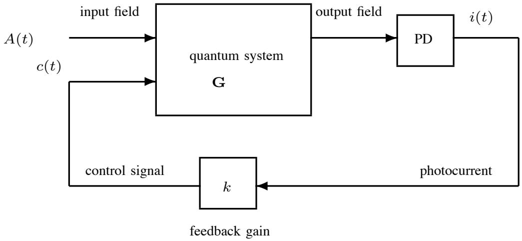
Controlled Hamiltonian

$$H_0 + Fc$$

Before feedback, the quantum system is described by

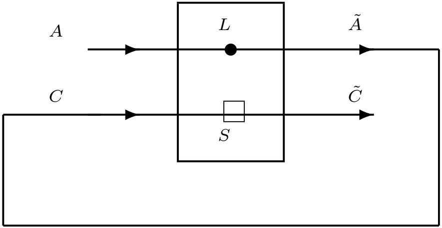
$$\mathbf{G} = (1, L, H_0) \boxplus (S, 0, 0)$$

where $S = e^{-iF}$ is unitary (describes the classical input as an equivalent field input that models photodetection).



After feedback, we have

$$\mathbf{G}_{cl} = (S, 0, 0) \triangleleft (1, L, H_0) = (S, SL, H_0)$$



Realistic detection

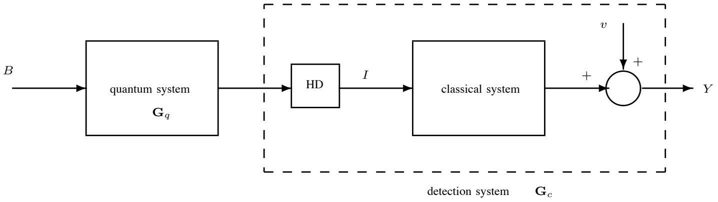
[Warszawski-Wiseman-Mabuchi, 2002]

The quantum system is given by

$$\mathbf{G}_q = (1, L_q, H_q),$$

and the classical detection system is given by the classical stochastic equations

$$\begin{aligned} dx(t) &= \tilde{f}(x(t))dt + g(x(t))dw(t), \\ dY(t) &= h(x(t))dt + dv(t), \end{aligned}$$



The classical system is equivalent to

$$\mathbf{G}_c = (1, L_{c1}, H_c) \boxplus (1, L_{c2}, 0)$$

where $L_{c1} = -ig^T p - \frac{1}{2}\nabla^T g$, $L_{c2} = \frac{1}{2}h$ and $H_c = \frac{1}{2}(f^T p + p^T f)$.

The complete cascade system is

$$\begin{aligned} \mathbf{G} &= ((1, L_{c1}, H_c) \triangleleft (1, L_q, H_q)) \boxplus (1, L_{c2}, 0) \\ &= (\mathbf{I}, \begin{pmatrix} L_1 + L_{c1} \\ L_{c2} \end{pmatrix}, H_q + H_c + \frac{1}{2i}(L_{c1}^* L_q - L_q^* L_{c1})) \end{aligned}$$

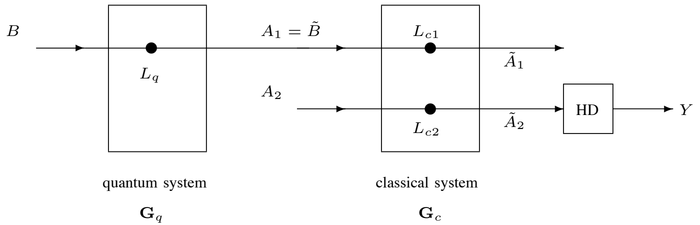
The unnormalized quantum filter for the cascade system is

$$d\sigma_t(X) = \sigma_t(-i[X, H_q + H_c + \frac{1}{2i}(L_{c1}^* L_q - L_q^* L_{c1})] + \mathcal{L}_{\begin{pmatrix} L_1 + L_{c1} \\ L_{c2} \end{pmatrix}}(X))dt + \sigma_t(L_{c2}^* X + X L_{c2})dy.$$

For instance, $X = X_q \otimes \phi$, where ϕ is a smooth real valued function on $\mathbb{R}^n$.

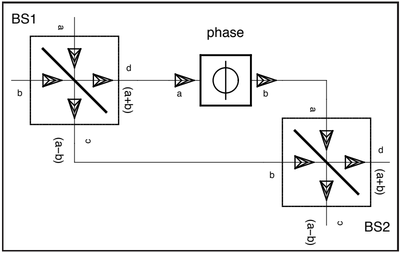
Filtered estimate of quantum variables:

$$\pi_t(X_q) = \sigma_t(X_q)/\sigma_t(1)$$



Quantum Hardware Description Language (QHDL)

[Mabuchi and colleagues, 2011]



$$MZ = BS_2 \triangleleft (phase \boxplus I) \triangleleft BS_1$$

```
entity Mach_Zehnder is
    generic (phi_mz: real := 0);
    port (In1, VacIn: in fieldmode; Out1, Out2: out fieldmode);
end Mach_Zehnder;
```

```
architecture structure_MZ of Mach_Zehnder is
    component beamsplitter
        port (a, b: in fieldmode; c, d: out fieldmode);
    end component beamsplitter;

    component phase
        generic (phi: real);
        port (a: in fieldmode; b: out fieldmode);
    end component phase;

    signal bs1_phase, bs1_bs2, phase_bs2: fieldmode;

begin
    BS1: beamsplitter
        port map (a => In1, b => VacIn, c => bs1_bs2, d => bs1_phase);
    phase: phase
        generic map (phi => phi_mz);
        port map (a => bs1_phase, b => phase_bs2);
    BS2: beamsplitter
        port map (a => phase_bs2, b => bs1_bs2, c => Out1, d => Out2);
end structure_MZ;
```

[Mabuchi and colleagues, 2011]

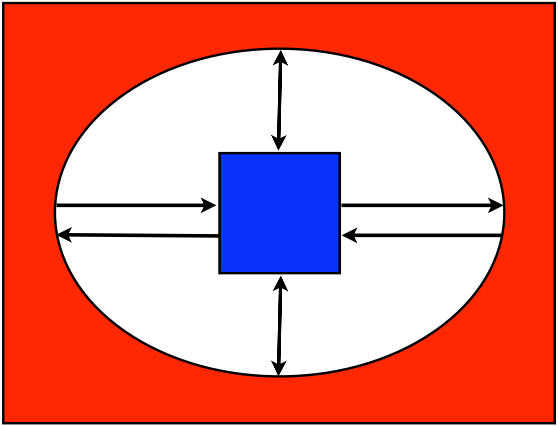
Quantum Dissipative Systems

Combine perspectives from

- quantum physics - damping, commutation relations
  - quantum noise (e.g. Gardiner-Collett, 1985, etc)
- control engineering - signals, disturbances, uncertainty
  - passivity, gain (e.g. Zames, 1965, Willems, 1972, 1997, etc)

in order to develop analysis and design tools.

The plant is the system of interest, interacting with its environment.



Environment can include infinite heat baths, as well as other systems - a network.

Given two reducible systems $\mathbf{P} = \boxplus_j \mathbf{P}_j$ and $\mathbf{W} = \boxplus_{j'} \mathbf{W}_{j'}$, an interaction Hamiltonian

$$K = -i \sum_k (N_k^* M_k - M_k^* N_k),$$

where $N_k \in \mathscr{A}_{\mathbf{P}}$, $M_k \in \mathscr{A}_{\mathbf{G}}$, and a list of series connections

$$\mathscr{S} = \{\mathbf{W}_k \triangleleft \mathbf{P}_j, \, \mathbf{P}_{k'} \triangleleft \mathbf{W}_{j'}\},$$

one can form a network

$$\mathbf{N} = \mathbf{P} \wedge \mathbf{W}.$$

We call $\mathbf{W}$ an *exosystem*, and keeping the interconnection structure fixed, we let $\mathbf{W}$ vary in a *class* $\mathscr{W}$ of exosystems.

Lindblad generator for a system $\mathbf{G} = (\mathbf{S}, \mathbf{L}, H)$:

$$\mathcal{G}_{\mathbf{G}}(X) = \mathcal{L}_{\mathbf{L}}(X) - i[X, H]$$

where

$$\mathcal{L}_{\mathbf{L}}(X) = \frac{1}{2}\mathbf{L}^\dagger[X, \mathbf{L}] + \frac{1}{2}[\mathbf{L}^\dagger, X]\mathbf{L}.$$

Then

$$\mathbb{E}_s[X(t)] = X(s) + \int_s^t \mathbb{E}_s\left[\mathcal{G}_{\mathbf{G}}(X(r))\right] dr$$

for all $t \geq s$.

Plant

$$\mathbf{P} = (\mathbf{S}, \mathbf{L}, H)$$

Exosystem

$$\mathbf{W} = (\mathbf{R}, \mathbf{w}, D) \in \mathscr{W}$$

Supply rate

$$r_{\mathbf{P}}(\mathbf{W}) \in \mathscr{A}_{\mathbf{P}} \otimes \mathscr{A}_{ex}$$

a self-adjoint symmetrically ordered function of the exosystem parameters, depending on the plant parameters.

We say that the plant **P** is *dissipative* with supply rate r with respect to a class $\mathscr{W}$ of exosystems if there exists a non-negative system observable $V \in \mathscr{A}_{\mathbf{P}}$ such that

$$\mathbb{E}_0 \left[V(t) - V - \int_0^t r(\mathbf{W})(s) ds \right] \leq 0$$

for all exosystems $\mathbf{W} \in \mathscr{W}$ and all $t \geq 0$.

Infinitesimal characterization

The plant $\mathbf{P}$ is *dissipative* with supply rate r with respect to a class $\mathscr{W}$ of exosystems if and only if there exists a non-negative system observable $V \in \mathscr{A}_{\mathbf{P}}$ such that

$$\mathcal{G}_{\mathbf{P} \wedge \mathbf{W}}(V) - r(\mathbf{W}) \leq 0$$

for all exosystem parameters $\mathbf{W} \in \mathscr{W}$.

Special case from now on:

$$\mathbf{P} \wedge \mathbf{W} = \mathbf{P} \triangleleft \mathbf{W}$$

and

$$\mathbf{W} = (\mathbf{I}, \mathbf{L}, H)$$

Consider

$$\mathbf{W} = (\mathbf{R}, \mathbf{w}, -i(\mathbf{v}^\dagger \mathbf{K} - \mathbf{K}^\dagger \mathbf{v}))$$

where $\mathbf{v}$ commutes with plant variables and $K_P \in \mathscr{A}_{\mathbf{P}}$.
Open quantum systems are dissipative with respect to the "natural" supply rate

$$\begin{aligned} r_0(\mathbf{W}) &= \mathcal{G}_{\mathbf{P} \triangleleft \mathbf{W}}(V_0) \\ &= \mathcal{L}_{\mathbf{w}}(V_0) + \mathcal{L}_{\mathbf{L}}(V_0) + \begin{pmatrix} \mathbf{w}^\dagger & \mathbf{v}^\dagger \end{pmatrix} \mathbf{Z} + \mathbf{Z}^\dagger \begin{pmatrix} \mathbf{w} \\ \mathbf{v} \end{pmatrix}, \end{aligned}$$

where $V_0 \geq 0$ commutes with H.

$$\mathbf{Z} = [V_0, \begin{pmatrix} \mathbf{L} \\ \mathbf{K} \end{pmatrix}]$$

Transformation under series product

Let $\mathbf{P}_1$ and $\mathbf{P}_2$ be dissipative with respect to supply rates $r_{\mathbf{P}_1}(\mathbf{W})$ and $r_{\mathbf{P}_2}(\mathbf{W})$, storage functions V_1 and V_2, and exosystem classes $\mathscr{W}_1$ and $\mathscr{W}_2$ respectively.

The series system $\mathbf{P}_2 \triangleleft \mathbf{P}_1$ is dissipative with storage function $V_1 + V_2$ and supply rate

$$r_{\mathbf{P}_2 \triangleleft \mathbf{P}_1}(\mathbf{W}) = r_{\mathbf{P}_1}(\mathbf{P}_2' \triangleleft \mathbf{W}) + r_{\mathbf{P}_2}(\mathbf{P}_1 \triangleleft \mathbf{W}),$$

with respect to the exosystem class

$$\mathscr{W} = \{\mathbf{W} \ : \ \mathbf{P}_2' \triangleleft \mathbf{W} \in \mathscr{W}_1 \text{ and } \mathbf{P}_1 \triangleleft \mathbf{W} \in \mathscr{W}_2\},$$

where

$$\mathbf{P}_2' = (\mathbf{S}_1^\dagger \mathbf{S}_2 \mathbf{S}_1, \ \mathbf{S}_1^\dagger (\mathbf{S}_2 - \mathbf{1}) \mathbf{L}_1 + \mathbf{S}_1^\dagger \mathbf{L}_2, \ H_2 + \mathrm{Im} \left\{ \mathbf{L}_2^\dagger (\mathbf{S}_2 + \mathbf{1}) \mathbf{L}_1 - \mathbf{L}_1^\dagger \mathbf{S}_2 \mathbf{L}_1 \right\}).$$

Example

Open harmonic oscillator (e.g optical cavity)

$$\mathbf{P} = (1, \sqrt{\gamma} a, \omega a^* a)$$

Let $V_0 = H/\omega = a^* a$.

$$r_0(\mathbf{W}) = \mathcal{G}_{\mathbf{P} \triangleleft \mathbf{W}}(V_0) \;\; = \;\; -\gamma a^* a - \sqrt{\gamma}(w^* a + a^* w) + \mathcal{L}_w(V_0) - i[V_0, D]$$

By completion of squares the supply rate can be re-written

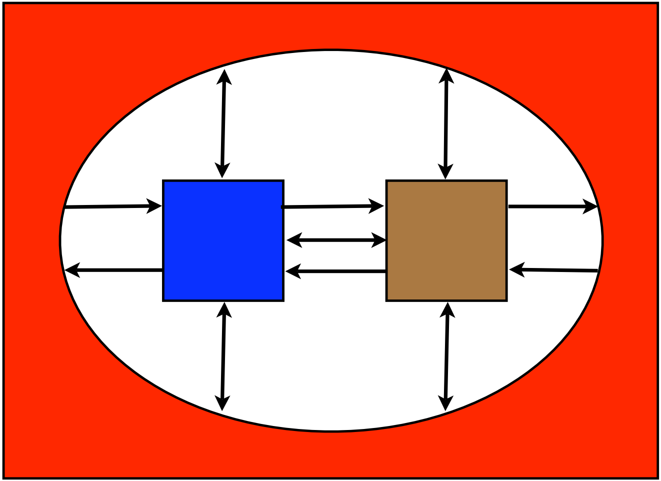
$$r_0(\mathbf{W}) = -(\sqrt{\gamma}\, a + w)^*(\sqrt{\gamma}\, a + w) + w^* w + \mathcal{L}_w(V_0) - i[V_0, D]$$

and hence the system has gain 1 relative to the output quantity $\sqrt{\gamma}\, a + w$ and commuting inputs w.
Note that if we include ground state energy and write $V = a^* a + \frac{1}{2} = q^2 + p^2$ (here $q = a + a^*$, $p = -i(a - a^*)$), then passivity and gain holds but with $\lambda = \gamma > 0$.

Quantum Feedback Control by Interconnection

- Inspired by energy-based design methods for classical mechanical systems (e.g. robotics) (e.g. Ortega and Spong, 1989, etc)
- Control design as network design
- Controller may be classical, quantum, or a mixture of the two
- Design focusses on the physical structure
- Interconnections can be field-mediated and/or direct interactions
- Covers standard problems of stabilization, regulation, robustness

The plant and the controller may interact with their environment.



Methodology

Specify the control objectives by encoding them in

- a non-negative observable $V_d \in \mathscr{A}_{\mathbf{P}} \otimes \mathscr{A}_{\mathbf{C}}$,
- a supply rate $r_d(\mathbf{W})$,
- and a class of exosystems $\mathscr{W}_d$ for which a network $(\mathbf{P} \wedge \mathbf{C}) \wedge \mathbf{W}$ is well defined.

One then seeks to find, if possible, a controller $\mathbf{C}$ such that

$$\mathcal{G}_{(\mathbf{P} \wedge \mathbf{C}) \wedge \mathbf{W}}(V_d) - r_d(\mathbf{W}) \leq 0$$

for all exosystem parameters $\mathbf{W} \in \mathscr{W}_d$.

Example

Cavity $\mathbf{P} = (1, a, 0)$ with vacuum input.
Wish to maintain steady-state photon number $\alpha^*\alpha$.
Consider simple direct plant-controller interaction

$$\mathbf{C} = (1, 0, -i(K_P^*\nu - \nu^* K_P)),$$

where K_p is a plant operator and ν is a complex number, both to be chosen.
Closed loop system

$$\mathbf{P} \wedge \mathbf{C} = \mathbf{P} \boxplus \mathbf{C}.$$

We set

$$V_d = (a-\alpha)^*(a-\alpha) = a^*a - \alpha^*a - a^*\alpha + \alpha^*\alpha,$$

and for a positive real number c,

$$r_d(\mathbf{W}) = -cV_d,$$

with $\mathscr{W}_d = \{(\_, \_, 0)\}$, which consists only of the trivial exosystem.

The design problem is to select, if possible, K_P, a plant operator, and ν, a complex number, such that

$$\mathcal{G}_{\mathbf{P} \boxplus \mathbf{C}}(V_d) + cV_d \leq 0$$

for suitable $c > 0$. We choose $K_P = a$.
Evaluate LHS, and set $c = 1/2$, $\nu = -\alpha/2$.

Physically, this control design corresponds to a classical energy source connected to the cavity, such as when the vacuum field is replaced by a coherent field (signal plus noise), i.e. a laser beam.

Linear Quantum Systems

We consider noncommutative stochastic systems of the form

$$\begin{aligned} dx(t) &= Ax(t)dt + Bdw(t) \\ dz(t) &= Cx(t)dt + Ddw(t) \end{aligned}$$

where A, B, C and D are real matrices, and

$$x(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix}$$

is a vector of possibly noncommutative plant variables.

[James, Nurdin and Petersen, 2008]

[General Ito algebra theory: Belavkin]

The initial system variables $x(0)$ are Gaussian with state ρ, and satisfy the commutation relations

$$[x_j(0), x_k(0)] = C^{xx}_{jk} = 2i\Theta_{jk}, \quad j, k = 1, \ldots, n,$$

where Θ is a real antisymmetric matrix.
For example, a system with one classical variable and two conjugate quantum variables is characterized by

$$\Theta = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}.$$

The vector quantity w describes input channels and is assumed to admit the decomposition

$$dw(t) = \beta_w(t)dt + d\tilde{w}(t)$$

where $\beta_w(t)$ is the self-adjoint finite variation part, and $\tilde{w}(t)$ is the (Gaussian) noise part of $w(t)$ with Ito table

$$d\tilde{w}(t)d\tilde{w}^T(t) = F_{\tilde{w}}dt,$$

where $F_{\tilde{w}}$ is a non-negative Hermitian matrix.

$$F_{\tilde{w}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & i \\ 0 & -i & 1 \end{bmatrix}$$

describes a noise vector with one classical component and a pair of conjugate quantum Gaussian noises.

Physical systems

Physical systems impose constraints on the matrices A, B, C, D, e.g.:

$$\begin{aligned} JA + A^\dagger J + C^\dagger JC &= 0 \\ B = -JC^\dagger J \end{aligned}$$

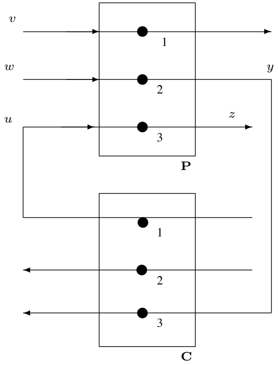
for the case where $\Theta = J$ and $F = I + iJ$ (non-degenerate case).

We say that A, B, C, D are physically realizable if the corresponding equations correspond to those arising from a system of quantum harmonic oscillators coupled to boson fields in the vacuum state.

The degenerate extension of this allows for mixed quantum-classical situations.

H^∞ robust control

The control objective is to reduce the gain from input w to output z by an appropriate choice of controller **C**.



Plant

$$\begin{aligned}\mathbf{P} &= \mathbf{P}_1 \boxplus \mathbf{P}_2 \boxplus \mathbf{P}_3 \\ &= (1, \sqrt{\kappa_1}\, a,\, 0) \boxplus (1, \sqrt{\kappa_2}\, a,\, 0) \boxplus (1, \sqrt{\kappa_3}\, a,\, 0),\end{aligned}$$

Controller

$$\mathbf{C} = \mathbf{C}_1 \boxplus \mathbf{C}_2 \boxplus \mathbf{C}_3.$$

The plant-controller network is

$$\mathbf{P} \wedge \mathbf{C} = \mathbf{P}_1 \boxplus (\mathbf{C}_3 \triangleleft \mathbf{P}_2) \boxplus (\mathbf{P}_3 \triangleleft \mathbf{C}_1) \boxplus \mathbf{C}_2.$$

The supply rate is

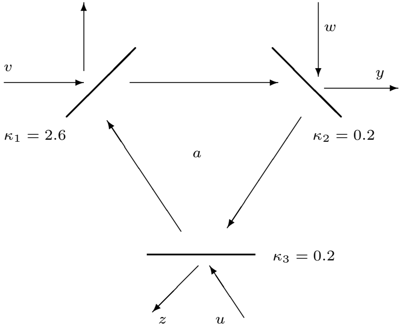
$$r(\mathbf{W}) = g^2 w^* w - (\sqrt{\kappa_3}\, a + w)^* (\sqrt{\kappa_3}\, a + w)$$

for exosystems $\mathbf{W} \in \mathscr{W}_d$, where

$$\mathscr{W}_d = \{\mathbf{W} = (1, 0, 0) \boxplus (1, w, 0) \boxplus (1, 0, 0) \boxplus (1, 0, 0) \ : \ \ w \text{ commutes with } .$$

Example

The plant is an optical cavity:



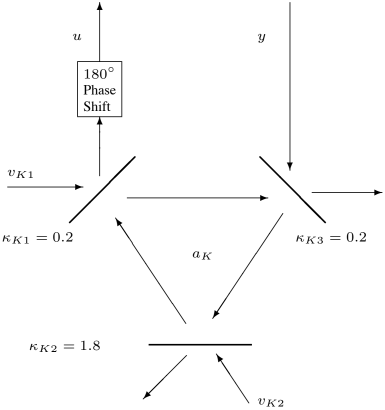
For the plant parameters $\kappa_1 = 2.6$, $\kappa_2 = 0.2$, $\kappa_3 = 0.2$, a controller was realized as a cavity with annihilation operator b:

$$\mathbf{C} = (-1, -\sqrt{0.2}\, b,\ 0) \boxplus (1, \sqrt{1.8}\, b,\ 0) \boxplus (1, \sqrt{0.2}\, b,\ 0).$$

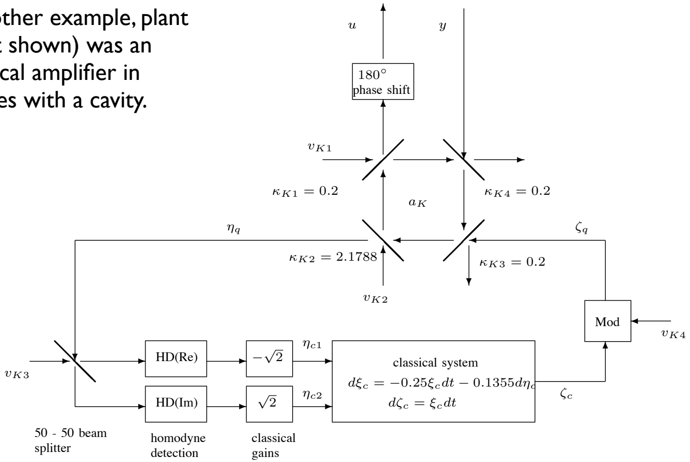
This construction had two steps:

1. Evaluation of quadratic forms with respect to Gaussian states, and using some classical results. This gives *part, not all*, of the solution.
2. Completing the design by adding field couplings to ensure commutation relations preserved. This is algebraic.

Controller, specified to be quantum, realized as a cavity:



Another example, plant (not shown) was an optical amplifier in series with a cavity.



Here, the controller was specified to have quantum and classical degrees of freedom.

'...the most fruitful areas for growth of sciences were those ... between various established fields.'

'It is these boundary regions of science which offer the richest opportunities to the qualified investigator.'

Norbert Wiener, *Cybernetics*, 1948

Thanks for your attention!