Influence of boundary on the motility of micro-swimmers

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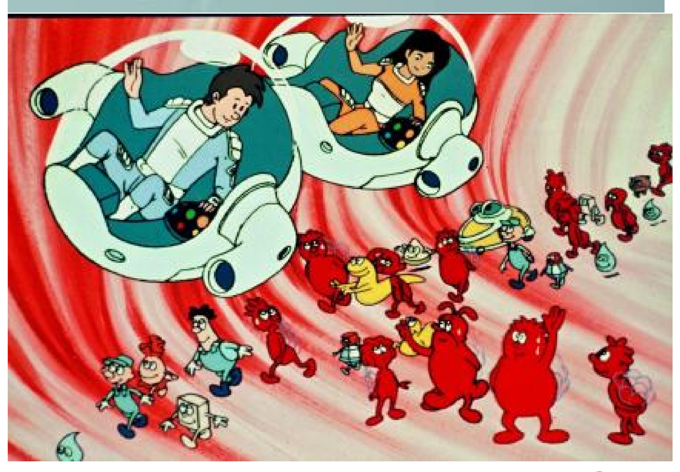
Motivations

- Self-propulsion at micro-scales?
- Applications on fertility, on human diagnosis and therapy...

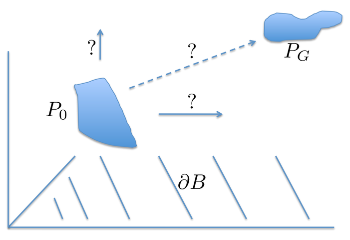
- Physicians and biologists noticed that micro-swimmers as Spermatozoid are attracted by the wall. ([H. Winet et al., Reproduction, 1984]).



- **Does the boundary have an effect on the controllability of the swimmer?**

Controllability issues

- Is it possible to control the state of the system?
- Does the boundary impact the controllability of the swimmer?



Model swimmer/fluid

The swimmer is described by the vector (ξ, p) such as :

- ξ is a function which defines the shape of the swimmer.
- $p = (c, R) \in \mathbb{R}^3 \times SO(3)$ parametrizes the swimmer's position.

The swimmer changes its shape $\Longrightarrow \xi(t)$ pushes the fluid.
The fluid reacts, under the Stokes Equation

$$\begin{cases} -\nu\Delta u + \nabla q = f, \\ \operatorname{div} u = 0. \end{cases}$$

Self-propulsion constraints $\Longrightarrow \begin{cases} \sum \text{Forces} = 0 \\ \text{Torque} \quad = 0 \end{cases}$

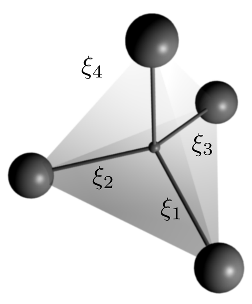
$$\Longleftrightarrow \begin{cases} \int_{\partial\Omega} DN_{p,\xi}\left((\partial_p\Phi)\dot{p} + (\partial_\xi\Phi)\dot{\xi}\right) dx_0 = 0 \\ \int_{\partial\Omega} x_0 \times DN_{p,\xi}\left((\partial_p\Phi)\dot{p} + (\partial_\xi\Phi)\dot{\xi}\right) dx_0 = 0. \end{cases}$$

As a result the swimmer moves, under the ODE,

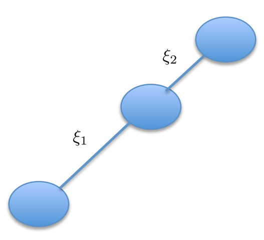
$$\dot{p} = V(p, \xi)\dot{\xi}\,.$$

The swimmers

- The swimmer that we consider consists of n spheres connected by the swimmer's arm.



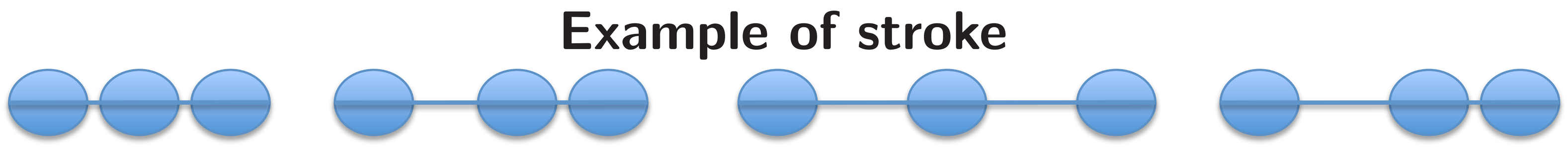
4-sphere swimmer



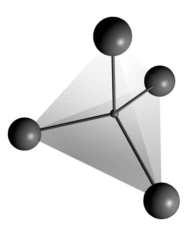
3-sphere swimmer
[Golestanian, Najafi 2004]

- The change of the swimmer's shape consists in changing the length of its arms $(\xi_i)_i$.

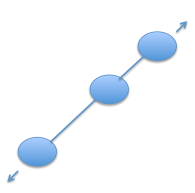
Example of stroke



Controllability's result in $\mathbb{R}^3$ – [Alouges, DeSimone, Heltai, Lefevbre, Merlet]

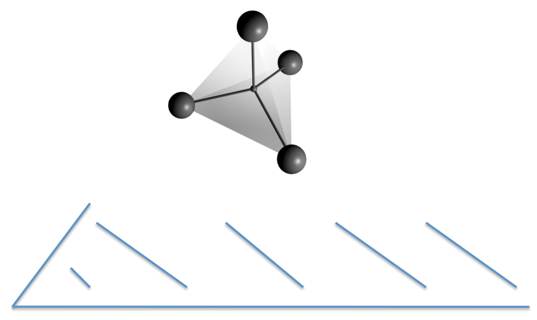


- The 4-sphere swimmer is globally controllable on $\mathbb{R}^3$.

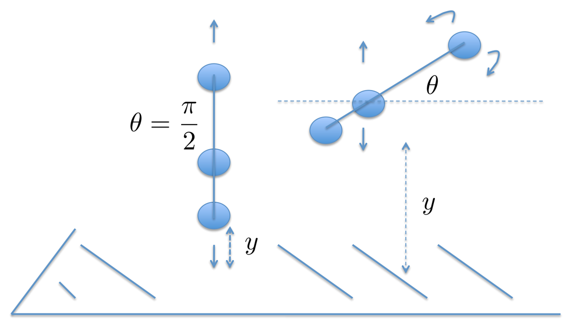


- The 3-sphere swimmer is globally controllable on $\mathbb{R}$.

Influence of a plane wall - Joint work with F. Alouges



- The 4-sphere swimmer is controllable on an dense open set.



- For almost (x_0, y_0, θ_0) such that $\theta_0 \neq \frac{\pi}{2}$, the 3-sphere swimmer is locally controllable on (x_0, y_0, θ_0).
  If $\theta_0 = \frac{\pi}{2}$ then it moves along a vertical line.

Outline of the proofs

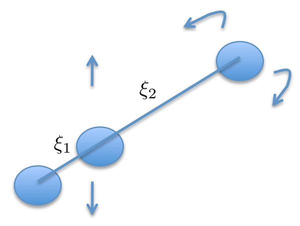
The proofs are based on the study of the subspace $\mathrm{Lie}_{(p,\xi)}((\mathbf{V}_i)_{i=1..M})$, where $(\mathbf{V}_i)_i$ are the vector fields of the motion equation,

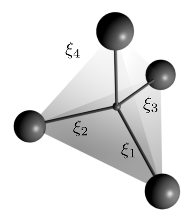
$$\dot{p} = \sum_{i=1}^{M} \mathbf{V}_i(p, \xi)\dot{\xi}\,.$$

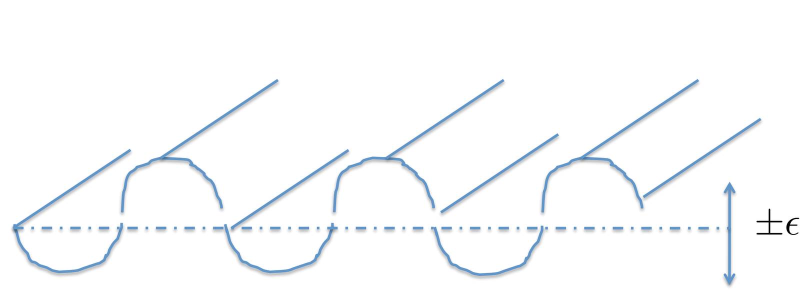
- By using the limit and the case without wall
- The orbit with a 3 dimensional Lie space (if $\theta_0 = \frac{\pi}{2}$).
  - By symmetry.
- The others such that the dimension is equal to 5.
  - By using an integral representation of the solution of the Stokes problem, we get an expansion of the Neumann-To-Dirichlet map for large arm and small spheres.
  - Calculation of the Lie brackets and application of Chow Theorem.
  - Application of the Nagano Theorem.

Rough no slip wall - Work in Progress with D. Gérard-Varet

- The 4-sphere remain controllable on an dense open set.
- The dimension of the reachable set of the 3-sphere is greater than or equal to 5.







Outline of the proof

- The Green function of the Stokes problem is implicitly defined.
- Analyticity of the Green Function.
- Analyticity of the Neumann-to-Dirichlet map.
- Expansion of the Neumann-to-Dirichlet map for small ϵ.
- By using the limit of the family of vector-fields which defines the equation of motion,
  - when the altitude of the swimmer is large
  - when the parameter ϵ is small
- Application of the preceding results.

References

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Optimally swimming stokesian robots, arXiv :1007.4920v1 [math.OC], 2010.

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