Unexpected Effects of Endogenous Discount Rates on Global Warming Policies

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Problem

Our aim is to study, in the framework of the standard externality problem 'global warming', the consequences of endogenous discounting on the qualitative behavior and compare this approach with alternatives such as exogenous discounting. The Stern report and the ensuing public and academic debate has made clear how crucial discounting is for evaluating anti-global warming measures. A frequently proposed solution, concerning the issue of not weighting the future consequences of today's decisions enough, is hyperbolic discounting, which in turn leads to the problem of time inconsistency. Our basic assumption is that **discounting decreases as the damage increases**, becoming thereby endogenous. This means that decision makers become **more patient when facing the environmental damages** following their high consumption levels.

The Model

An infinitely-lived decision maker benefits from consumption c and suffers damages D from pollution T. Pollution is a stock externality that accumulates with current 'emissions' that are linked to consumption (say of fossil fuels), for simplicity, linearly. Consumption is normalized in units of emissions and the stock of pollution depreciates at a constant rate (δ).

$$\max_{\{c(t)\geq 0\}} \int_0^\infty \left[e^{-\Theta(t)}(u(c(t)) - D(T(t)))\right] \mathrm{d}t,$$

subject to

$$\dot{T}(t) = c(t) - \delta T(t), \forall t, \qquad T(0) = T_0 = 0,$$

and

$$\dot{\Theta}(t) = f(T(t)),$$

where Θ replaces the usual exogenous discount term rt. Thus, Θ is endogenously determined by the discount function $f(T)$, which fulfills that

$$f(0) = r > 0, f > 0 \text{ and } \boldsymbol{f'(T) < 0}, \forall T,$$
$$\text{and } \lim_{T\to\infty} f(T) = 0.$$

$f' < 0$ reflects our assumption, that the discount rate decreases with respect to the pollution stock.

Further Assumptions: The felicity function $u : \mathbb{R}_+ \to \mathbb{R}$ is at least twice differentiable, $u'(c) > 0, u''(c) < 0, \forall c$. Inada conditions: $\lim_{c\to 0} u'(c) = \infty$, and $\lim_{c\to\infty} u'(c) = 0$. The damage function $D : \mathbb{R}_+ \to \mathbb{R}_+$ is at least twice differentiable, $D'(T) > 0, D''(T) > 0$. Inada-type conditions: $D'(0) = 0$, and $\lim_{T\to\infty} D'(T) = \infty$.

Optimality Conditions

The present value Hamiltonian is defined as follows

$$\mathcal{H} = (u(c) - D(T))e^{-\Theta} + \lambda[c - \delta T] - \mu f(T),$$

leading to the necessary optimality conditions

$$\mathcal{H}_c = \lambda + u' e^{-\Theta} = 0,$$
$$\dot{\lambda} = D'(T)e^{-\Theta} + \lambda\delta + \mu f'(T),$$
$$\dot{\mu} = -(u(c) - D(T))\, e^{-\Theta},$$
$$\lim_{t\to\infty} \mathcal{H}(t) = 0.$$

Solving the Model

The dynamics of $\dot{c}$ are found by differentiating $\mathcal{H}_c$ with respect to time and replacing the costate $\lambda = -u'(c)e^{-\Theta}$. Trough making use of the autonomy of $\mathcal{H}$, or equivalently $\dot{\mathcal{H}} = \mathcal{H}_t = 0$, we can eliminate the costate μ. We arrive at the following system of two equations,

$$\dot{c} = \frac{u'(c)}{u''(c)}\left(f(T) + \delta - \frac{D'(T)}{u'(c)}\right) - \frac{1}{u''(c)}\frac{f'(T)}{f(T)}\left(u(c) - D(T) - \dot{T}u'(c)\right), \qquad (1)$$
$$\dot{T} = c - \delta T.$$

In the counterpart of exogenous discounting, $f(T)$ is replaced by the constant discount rate r, and the colored term does not exist. This raises the following questions: existence of steady states, their properties compared to conventional discounting and the dynamic properties.

Steady States

Existence: (proof not shown here)
The assumptions about $u(c)$ and $D(T)$ guarantee the existence of at least one steady state.
Properties:
Due to the colored term in (1), changing the intercept of the utility function $u(c)$, or the intercept of the damage function $D(T)$ – completely irrelevant in the exogenous setup – can render counter-intuitive qualitative behavior. Using a discount function $f(T) < r, \forall T$, we would expect a smaller endogenous damage level $\bar{T}^n$, and consequently a smaller endogenous consumption level $\bar{c}^n = \delta\bar{T}^n$ in the steady state, since less discounting is equivalent to more patience. In contrast, as shown in Figure 1, we can – but do not have to – end up with **a steady state $\bar{T}^n$ higher than the exogenous steady state $\bar{T}^x$.**

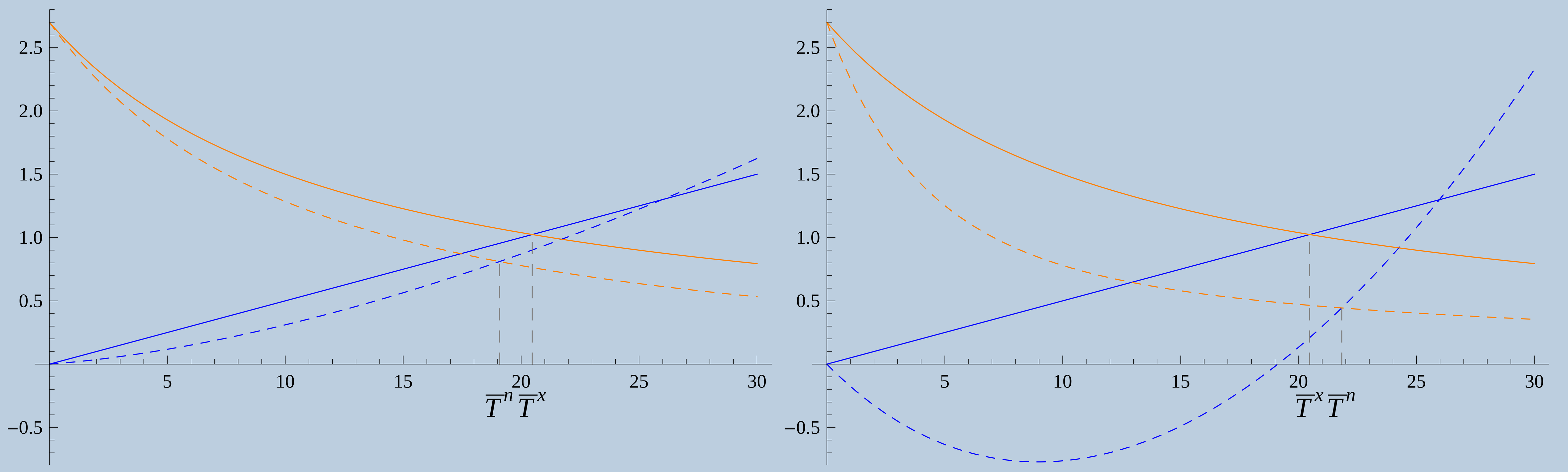

Figure 1: Making use of $\dot{T} = 0$, we replace c by δT in $\dot{c}$. Further, we rearrange $\dot{c} = 0$, so that we arrive at the the endogenous steady state condition $u'(\delta\bar{T}^n)(f(\bar{T}^n) + \delta) = D'(\bar{T}^n) + \frac{f'(\bar{T}^n)}{f(\bar{T}^n)}\left(u(\delta\bar{T}^n) - D(\bar{T}^n)\right)$, or the exogenous steady state condition $u'(\delta\bar{T}^x)(r+\delta) = D'(\bar{T}^x)$ respectively. We print the right and left hand sides of these conditions separately, an intersection characterizes a steady state. The orange graphs illustrate the marginal utility (left hand side of the condition), the blue graphs the marginal damages (right hand side), the solid lines represent the exogenous case, the dashed lines the endogenous one.

In case of exogenous discounting, there exists a unique steady state. Whereas the existence of the colored term in (1) in case of endogenous discounting can trigger **multiple steady states**, as illustrated in Figure 2. Further, we showed that only an uneven number of steady states is possible.

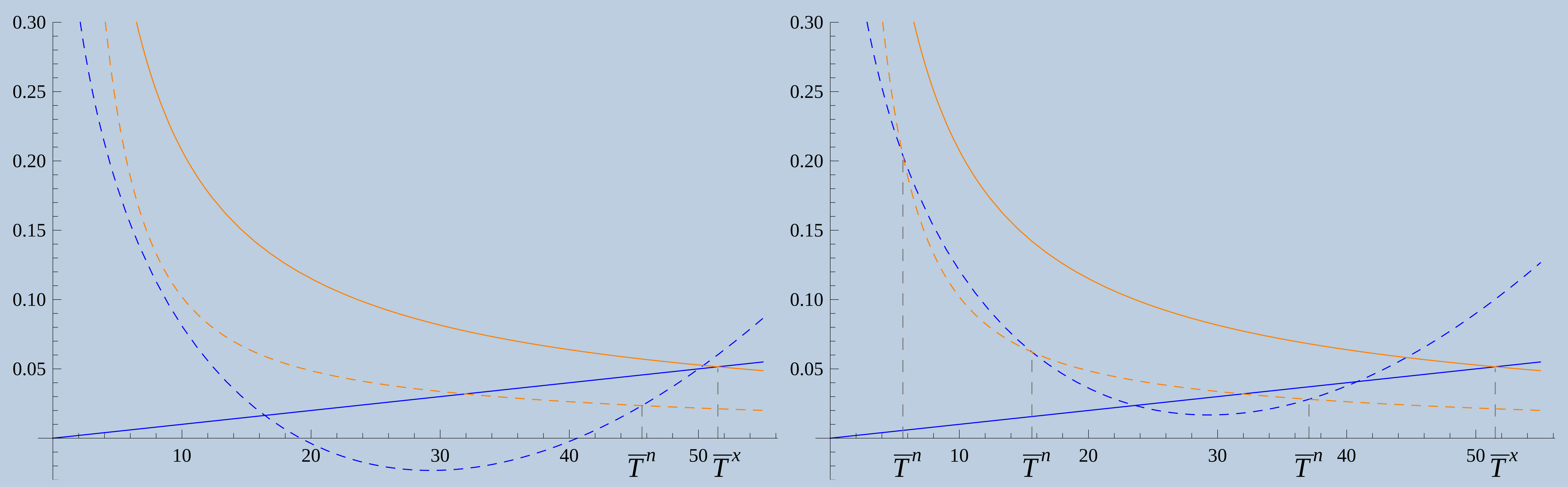

Figure 2: same procedure and same labelling as in Figure 1!

Dynamics

Linearizing the dynamical system around the steady state(s), we use the following Jacobi matrix,

$$J = \begin{bmatrix} \frac{\partial\dot{c}}{\partial c} & \frac{\partial\dot{c}}{\partial T} \\ 1 & -\delta \end{bmatrix} \Rightarrow Det\,(J) = -\left(\delta\frac{\partial\dot{c}}{\partial c} + \frac{\partial\dot{c}}{\partial T}\right) = \frac{\partial\dot{c}}{\partial c}\left(-\frac{\frac{\partial\dot{c}}{\partial T}}{\frac{\partial\dot{c}}{\partial c}} - \delta\right).$$

Thereby, it results that endogenous steady states with an uneven number (and thus also a unique steady state) are always saddlepoint stable. The stability property of (a) steady state(s) with an even number is not that clearly determined, since the sign of the trace is ambiguous. Although it presumably holds that the "middle" steady state(s) are unstable, it can not be arithmetically shown.