Stochastic homogenization for porous medium type equations

Given a probability space $(\Omega, \mathcal{F}, \mu)$ endowed with an ergodic dynamical system $\{T(x) : x \in \mathbb{R}^N\}$, we consider the homogenization problem for general porous medium type equations of the form: for any $\omega \in \Omega$, $\epsilon > 0$

$$\begin{cases} \partial_t u^\epsilon = \Delta f\left(T\left(\frac{x}{\epsilon}\right)\omega, u^\epsilon\right) & \text{in} \quad \mathbb{R}^+ \times \mathbb{R}^N \\ u^\epsilon(0,x) = u_0\left(x, T\left(\frac{x}{\epsilon}\right)\omega\right) & \text{on} \quad \mathbb{R}^N \end{cases} \tag{1}$$

where $f(\omega, \cdot)$ is an increasing Lipschitz function. We show that as $\epsilon \to 0$ the weak solution $u^\epsilon(t,x,\omega)$ of (1) converges to $\overline{u}(t,x)$ solution of an homogenized deterministic porous medium type equation.

The case Ω compact was previously studied by Ambrosio, Frid and Silva with different techniques.

This is a joint work with Professor P. E. Souganidis (University of Chicago).