Economic model predictive control: closed-loop optimality and distributed implementation

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NUMOC 2017, June 20

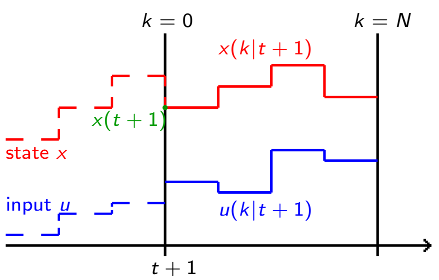
Outline

- Model predictive control (MPC) and economic MPC
- The role of dissipativity
  - Classifying optimal operating behaviors
  - Closed-loop convergence
- Distributed implementation for large-scale systems
- Application: Economic dispatch in power systems
- Conclusions

Introduction - Model predictive control

Model predictive control (MPC)

- Modern, optimization-based control technique
- Successful applications in many industrial fields



Basic MPC scheme

At each time t,

- solve finite horizon optimal control problem
- apply first part of optimal solution

Main advantages of MPC

- Can handle hard **constraints** on states and inputs
- Optimization of some **performance criterion**
- Applicable to **nonlinear, MIMO** systems

Model predictive control

- Nonlinear discrete time system $x(t+1) = f(x(t), u(t))$
- State and input constraints $x(t) \in \mathbb{X}$, $u(t) \in \mathbb{U}$

Standard MPC problem formulation

$$J_N^*(x(t)) = \min_{u(\cdot|t)} \sum_{k=0}^{N-1} \ell(x(k|t), u(k|t)) + V^f(x(N|t))$$

$$\text{s.t.} \quad x(0|t) = x(t), \quad x(k+1|t) = f(x(k|t), u(k|t)), \quad k = 0, \dots, N-1$$
$$x(k|t) \in \mathbb{X}, \; u(k|t) \in \mathbb{U}, \quad k = 0, \dots, N-1$$
$$x(N|t) \in \mathbb{X}^f$$

- Most results in MPC literature: classical control objective of **setpoint stabilization** is considered
- MPC controller design: determine V^f and $\mathbb{X}^f$ s.t. closed loop is stable [Chen & Allgöwer '98, Mayne et al. '00, Grüne '09, ...]
- Basic assumption: stage cost ℓ is **positive definite** w.r.t. setpoint to be stabilized
- However: **different** control objective is of interest in many applications

Economic model predictive control

- Maximization of product in process industry
- Minimization of energy consumption in building climate control
- Manufacturing industry: cost efficient scheduling of production process

$\Rightarrow$ Setpoint stabilization is **not** primary control objective

Economic MPC

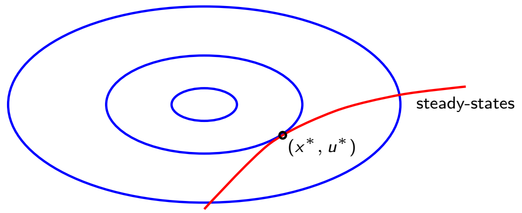
- Stage cost ℓ can be **general cost function**, need not be positive definite
  $\Rightarrow$ Closed-loop system does not necessarily converge to steady-state
- Resulting questions:
  - What is the **optimal operating regime** (steady-state, periodic, ...)?
  - Does the **closed-loop system** "find" optimal operating behavior?

Dissipativity in economic MPC

Definition - optimal operation at steady-state

- Optimal steady-state: $(x^*, u^*) = \underset{x\in\mathbb{X}, u\in\mathbb{U}, x=f(x,u)}{\arg\min} \ \ell(x,u)$
- A system is **optimally operated at steady-state** if for each feasible state and input sequences $x(\cdot)$ and $u(\cdot)$ the following holds:

$$\liminf_{T\to\infty} \sum_{t=0}^{T-1} \frac{\ell(x(t),u(t))}{T} \geq \ell(x^*,u^*).$$



Dissipativity in economic MPC

Definition - Dissipativity [Willems '72, Byrnes & Lin '94]

A system is strictly **dissipative** with respect to the **supply rate** s if there exists a **storage function** λ such that for all $x \in \mathbb{X}$ and $u \in \mathbb{U}$ it holds that

$$\lambda(f(x,u)) - \lambda(x) \leq s(x,u) - \alpha(\|x - x^*\|), \qquad \alpha \in \mathcal{K}_\infty.$$

Dissipativity and optimal steady-state operation

additional controllability condition [Müller et al. '13,15]

Optimal operation at steady-state $\Longrightarrow$ Dissipativity w.r.t. supply rate $s(x,u) = \ell(x,u) - \ell(x^*,u^*)$

Dissipativity w.r.t. supply rate $s(x,u) = \ell(x,u) - \ell(x^*,u^*)$ $\Longrightarrow$ Optimal operation at steady-state

[Angeli et al. '12]

Dissipativity in economic MPC

If steady-state operation is optimal, does closed-loop converge to x^*?

Strict dissipativity and optimal steady-state operation

local controllability at x^* [Müller et al. 15] ⟹

[Diehl et al. '11, Angeli et al. '12, Grüne '13, Müller et al. '14, Faulwasser&Bonvin '15] ⟹

optimal operation at steady-state (strict form)

Strict dissipativity w.r.t. supply rate $s(x,u) = \ell(x,u) - \ell(x^*,u^*)$

Closed-loop convergence to x^*

⟸

Discussion

- Stability analysis different than in stabilizing MPC
- Closed-loop system "does the right thing", i.e., "finds" optimal operating behavior
- Can be concluded **without** having to compute storage function λ

Dissipativity in economic MPC

Results can be extended to optimal periodic behavior:

Dissipativity and optimal periodic operation

local controllability at Π^* [Müller et al. 15] ⟹

[Müller&Grüne'16, Zanon et al. '17] ⟹

Optimal periodic operation

Strict dissipativity of P-step system w.r.t. $s(\tilde{x},\tilde{u}) = \tilde{\ell}(\tilde{x},\tilde{u}) - \tilde{\ell}(\tilde{x}^*,\tilde{u}^*)$

Closed-loop convergence to Π^*

⟸

Discussion

- Dissipativity plays central role in economic MPC
- Closed-loop system "does the right thing", i.e., "finds" optimal operating behavior
- Can be concluded **without** having to compute storage function
- Results hold for both optimal steady-state and periodic behavior

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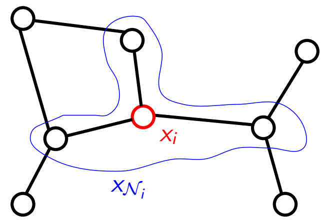
Distributed implementation for large-scale systems

- Large-scale system, composed of M **interconnected subsystems**



- **Coupled linear dynamics**
  $x_i^+ = A_{\mathcal{N}_i} x_{\mathcal{N}_i} + B_i u_i$
- **Coupled constraints**
  $\mathbb{X}_{\mathcal{N}_i} = \{x_{\mathcal{N}_i} | C_{\mathcal{N}_i} x_{\mathcal{N}_i} \le c_i\}, \quad \mathbb{U}_i = \{u_i | D_i u_i \le d_i\}$
- **Convex economic stage cost**: $\ell_i(x_i, u_i)$
  ⇝ also coupled cost functions are possible

Distributed implementation for large-scale systems

Centralized (overall) optimization problem

$$\min \sum_{i=1}^{M} \sum_{k=0}^{N-1} \ell_i(x_i(k|t), u_i(k|t)) + V_i^f(x_i(N|t))$$

$$\text{s.t. } x_i(k+1|t) = A_{\mathcal{N}_i} x_{\mathcal{N}_i}(k|t) + B_i u_i(k|t), \quad k = 0,..,N-1$$

$$x_{\mathcal{N}_i}(k|t) \in \mathbb{X}_{\mathcal{N}_i}, \quad u_i(k|t) \in \mathbb{U}_i, \quad k = 0,..,N-1$$

$$x_i(0|t) = x_i(t), \quad x(N|t) \in \mathbb{X}^f.$$

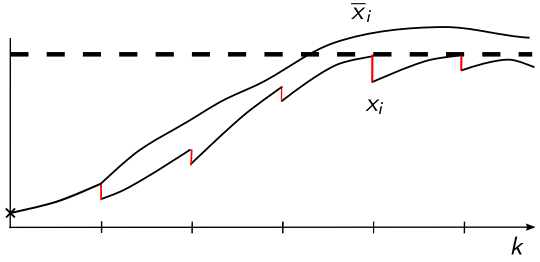
- Many different **distributed** MPC implementation approaches available
  [Stewart et al. '10, Grüne & Worthmann '12, Müller et al. '12, Conte et al. '12, . . .]
- Methods for computation of distributed terminal region available
  [Müller et al. '12, Conte et al. '12, Köhler et al. '17]
- In case of coupled dynamics: dual decomposition methods well suited, e.g., ADMM, dual gradient methods
- Problem: **Inexact minimization** due to real-time requirements
  ⇝ Closed-loop constraint satisfaction? Recursive feasibility?

Distributed implementation for large-scale systems

- Define **consolidated trajectory** $\overline{x}$ satisfying dynamic constraints, i.e.

$$\overline{x}_i(k+1|t) = A_{\mathcal{N}_i} \overline{x}_{\mathcal{N}_i}(k|t) + B_i u_i(k|t)$$

- **Problem:** Consolidated trajectory might **not** satisfy constraints!



Distributed implementation for large-scale systems

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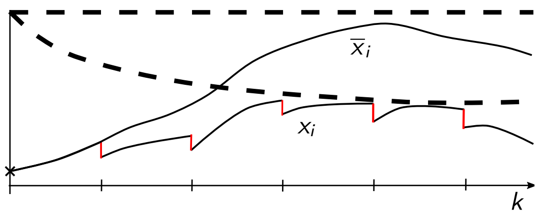
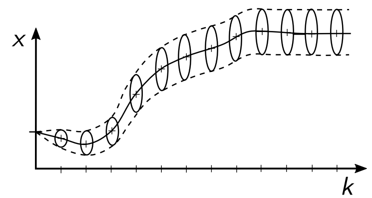
- **Problem:** Consolidated trajectory might **not** satisfy constraints!
- **Remedy**: tighten constraints s.t. $\overline{x}$ satisfies constraints
- Employ ideas from **robust tube MPC**

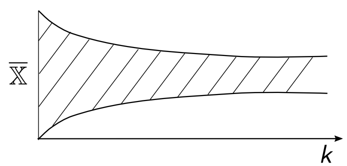


Distributed implementation for large-scale systems

Robust tube MPC

- System dynamics: $x^+ = Ax + Bu + Ew$, $w \in \mathcal{W}$
- Use **parameterized feedback** $u = Kx + v$, where v is MPC input
  ⇝ Uncertainty in predicted state trajectory can be bounded





Theorem [Chisci et al. '01]

Suppose

- suitable constraint tightening,
- suitable terminal region / terminal cost.

Then the closed-loop converges to the minimal robust positively invariant set.

Idea to handle inexact distributed optimization:

- define relaxed optimization problem
- assume maximum constraint violation ε through inexact minimization
- use suitable constraint tightening

$$\min \sum_{i=1}^{M} \sum_{k=0}^{N-1} \ell_i(x_i(k|t), u_i(k|t)) + V_i^f(x_i(N|t)) \quad (1a)$$

$$\text{st. } x_i(0|t) = x_i(t) \quad (1b)$$

$$\|(A_{\mathcal{N}_i} + B_i K_{\mathcal{N}_i}) x_{\mathcal{N}_i}(k|t) + B_i u_i(k|t) - x_i(k+1|t)\|_\infty \leq \varepsilon_{z_i,k} \quad (1c)$$

$$x_{\mathcal{N}_i}(k|t) \in \overline{\mathbb{X}}_{\mathcal{N}_i,k}, \quad (1d)$$

$$v_i(k) + K_{\mathcal{N}_i} x_{\mathcal{N}_i}(k|t) \in \overline{\mathbb{U}}_{i,k} \quad (1e)$$

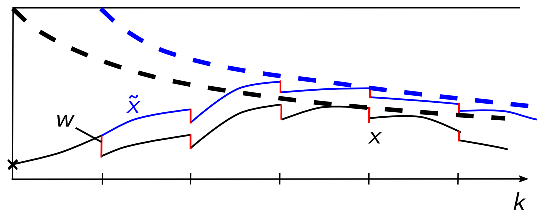
$$x(N|t) \in \overline{\mathbb{X}}^f. \quad (1f)$$

Inexact Solution:
Assume maximum constraint violation $\varepsilon_i, \varepsilon_{u_i}, \varepsilon_{x_i}, \varepsilon_f$ in (1c)-(1f)

Candidate solution at time $t+1$:

$$\tilde{v}(k|t+1) = \begin{cases} v(k+1|t), & k = 0, ..., N-2 \\ 0 & k = N-1 \end{cases}$$

$$\tilde{x}(k|t+1) = \begin{cases} x(t+1) =: x(1|t) + w & k = 0 \\ x(k+1|t) + (A+BK)^k w & k = 1, ..., N-1 \\ (A+BK)\tilde{x}(N-1|t+1) & k = N \end{cases}$$



Tightened Constraints

- Dynamic constraints: $\varepsilon_{z_i,k+1} = \varepsilon_{z_i,k} - \varepsilon_i$.
- State constraints: $\overline{\mathbb{X}}_{\mathcal{N}_i,k} = \{x_{\mathcal{N}_i} | C_{\mathcal{N}_i} x_{\mathcal{N}_i} \leq \overline{c}_{i,k}\}$ with $\overline{c}_{i,k,j} = c_{i,j} - \sigma_{\mathcal{W}}(\overline{C}^\top_{\mathcal{N}_i,j}, k) - (k+1)\varepsilon_{x_i}$
- Similar for input and terminal constraints

Theorem [Köhler et al. '17]

Suppose

- distributed optimization with maximum constraint violation $\varepsilon_i, \varepsilon_{u_i}, \varepsilon_{x_i}, \varepsilon_f$,
- suitable constraint tightening,
- suitable terminal region / terminal cost.

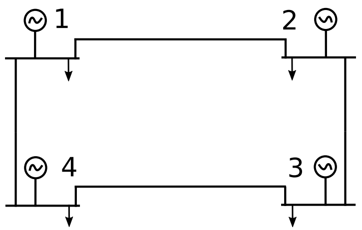
Then the MPC optimization problem is recursively feasible and the closed-loop satisfies state and input constraints.

Remarks:

- Closed-loop performance guarantees can be obtained by assuming a certain degree of suboptimality for the distributed optimization.
- Maximum constraint violation needs to be guaranteed $\rightsquigarrow$ can be verified online, but in general no a priori bound on number of iterations.

Outline

Application: Economic dispatch in power systems



Power system dynamics:
$x_i = [P_i^M, \omega_i, \delta_i]$, $u_i = P_i^C$, $d_i = P_i^L$
$x_i^+ = A_{\mathcal{N}_i} x_{\mathcal{N}_i} + B_i u_i + E_i d_i$

Economic dispatch: Power load P_i^L changes
$\Rightarrow$ drive system to (new) economic steady state

$$\min \sum_{i=1}^{M} \left(P_i^M\right)^2 \quad \text{st.} \quad \sum_{i=1}^{M} P_i^M = \sum_{i=1}^{M} P_i^L$$

Application: Economic dispatch in power systems

State of the art solution:
Economic Automatic Generation Control (EAGC) [Li et al. '14]
$\Rightarrow$ converges to (new) economic optimal steady state

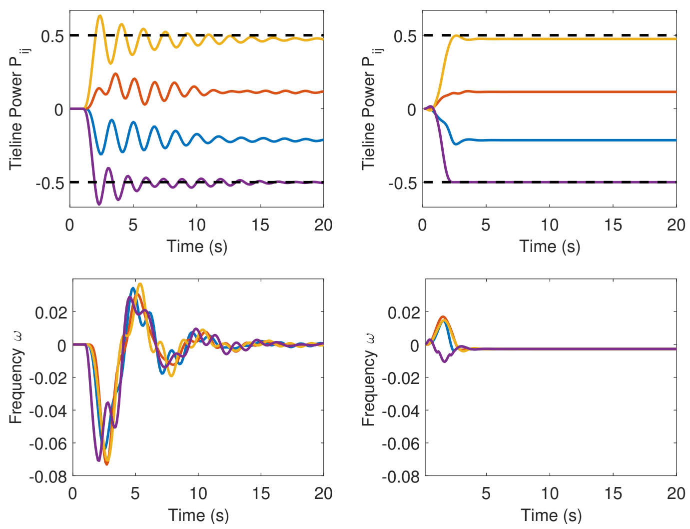
Distributed Economic MPC solution:

- Economic stage cost: $l_i = (P_i^M)^2 + c\omega_i^2$
- Constraints: $P_i^M \in [P_M^{\min}, P_M^{\max}]$, $P_{ij} \in [P_{ij}^{\min}, P_{ij}^{\max}]$
- Average constraints: $Av[\omega_i] = 0$
- Strict dissipativity: system is optimally operated at steady-state

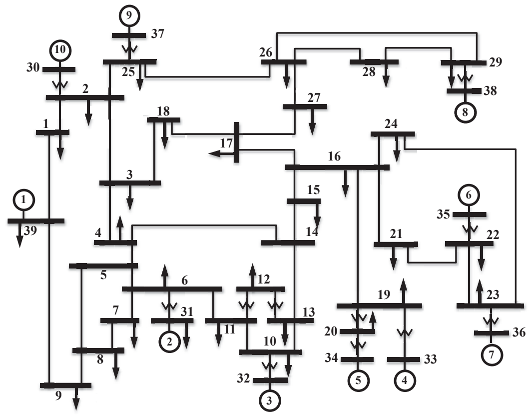
Application: Economic dispatch in power systems

Simulation Result

EAGC DEMPC

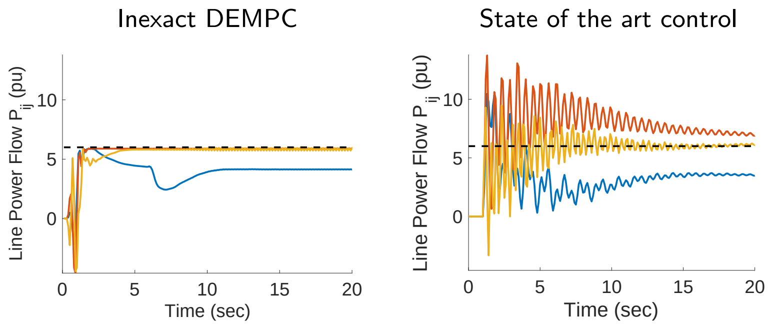


Application: Economic dispatch in power systems



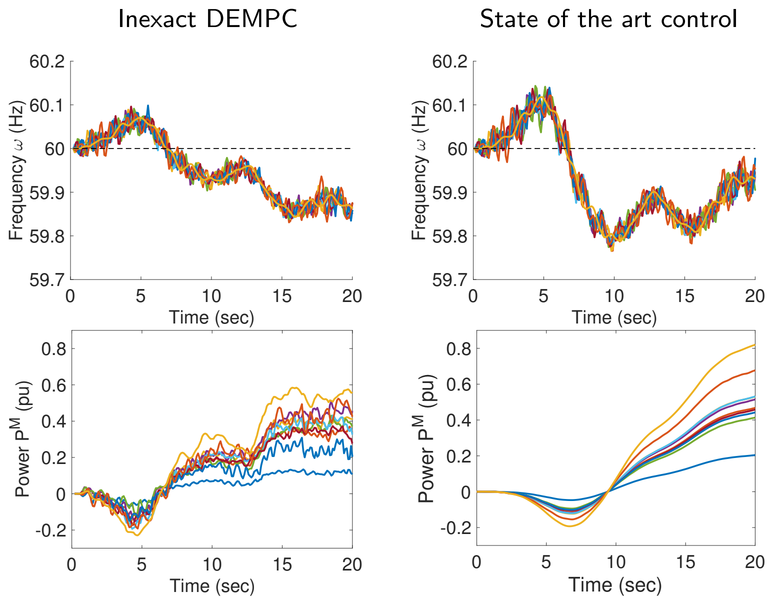
IEEE 39 bus system: New England power grid

Application: Economic dispatch in power systems



Case study: large load change

Application: Economic dispatch in power systems



Case study - random load fluctuations

Conclusions

Conclusions

- **Economic model predictive control**
  - Consideration of general control objectives
  - Closed-loop system not necessarily convergent
- **Dissipativity** plays a crucial role in economic MPC:
  - Classify optimal operating behavior
  - Convergence analysis of closed-loop system
- **Distributed implementation**
  - Important to consider inexact optimization due to real-time constraints
  - Recursive feasibility and constraint satisfaction can be achieved using suitable constraint tightening

Acknowledgements

- Johannes Köhler, Frank Allgöwer, University of Stuttgart
- David Angeli, Imperial College London and University of Florence
- Lars Grüne, University of Bayreuth
- Na Li, Harvard University