Summer School and Workshop
Harmonic Analysis, Spectral Theory and PDE's
Rome-SAPIENZA, 12-15 September 2017

Titles and Abstracts

September 12, 2017

1. **Valeria Banica.**

   Title: *1-D cubic NLS with several Diracs as initial data and consequences*

   Abstract: We solve the cubic nonlinear Schrödinger equation on $\mathbb{R}$ with initial data a sum of Diracs. Then we describe some consequences for a class of singular solutions of the binormal flow, that is used as a model for the vortex filaments dynamics in 3-D fluids and superfluids

2. **Roland Donninger.**

   Title: *Self-similar blowup for supercritical wave maps*

   Abstract: I will report on recent progress in the study of self-similar blowup for supercritical wave maps. I will introduce the "hyperboloidal similarity coordinates" which cover a large portion of spacetime up to the Cauchy horizon of the singularity. This coordinate system makes it possible to obtain a stability result for the blowup profile in the whole space and even beyond the blowup time. This is joint work with Pawel Biernat (Bonn) and Birgit Schörkhuber (Vienna).

3. **Sebastian Herr.**

   Title: *Nonlinear Dirac equations and related systems*

   Abstract: Results concerning the longtime behavior of solutions of cubic Dirac equations (the Soler model) and of the Dirac-Klein-Gordon system will be presented and the connection to Euclidean harmonic analysis will be outlined.

4. **Carlos Kenig.**

   Title: *The energy critical nonlinear wave equation*

   Abstract: Lecture 1: An overview on theenergy critical wave equation. Abstract: In this lecture we will give an overview of the work in the last 12 years on the long-time behavior of large solutions of the energy critical wave equation. The emphasis will be on the asymptotic behavior near the final time of existence. We hope that the results described will be a model of what to strive for in the study of the asymptotic behavior of large solutions of general critical nonlinear dispersive equations.

   Lecture 2: The radial case. Abstract: In this lecture we will describe work of Duyckaerts-Kenig-Merle (2012-2013) on soliton resolution for radial solutions of the energy critical wave equation in 3 space dimensions.

Lecture 3: The nonradial case. Abstract: In this lecture we will describe recent works of Duyckaerts-Kenig-Merle (2016) and of Duyckaerts-Jia-Kenig-Merle (2016) on soliton resolution along well-chosen sequences of time for bounded (in the energy space) solution of the energy critical wave equation, in all (low) dimensions, in the non-radial case. Further work and open problems will also be discussed, time permitting.

5. **David Krejčiřík.**

   Title: *Absence of eigenvalues of Schrödinger operators with complex potentials*

   Abstract: We prove that the spectrum of Schrödinger operators in three dimensions is purely continuous and coincides with the non-negative semiaxis for all potentials satisfying a form-subordinate smallness condition. By developing the method of multipliers, we also establish the absence of point spectrum for electromagnetic Schrödinger operators in all dimensions under various alternative hypotheses, still allowing complex-valued potentials with critical singularities. This is joint work with Luca Fanelli and Luis Vega.

6. **Ari Laptev**

   Title: *Spectral and Functional inequalities*

   Abstract: We shall cover a number of functional inequalities which have application to Spectral Theory of Partial Differential Operators. In particular, we shall briefly describe the theory of coherent state transformations and its applications to the spectral theory. As a corollary we obtain a new result related to Weyl's type asymptotics and spectral estimates of functional-difference operators associated to mirror curves of special del Pezzo Calabi-Yau threefolds.

7. **Loic Letreust.**

   Title: *Asymptotic expansion of eigenvalues for the MIT bag model*

   Abstract: In this talk we present some spectral asymptotic results of the MIT bag model. This model is the Dirac operator, defined on a smooth and bounded domain of $\mathbb{R}^3$, with certain boundary conditions. This model was developed to get a better understanding of the phenomenons involved in the confinement of quarks. We study the self-adjointness of the operator and describe the limiting behavior of the eigenvalues of the MIT bag Dirac operator as the mass m tends to 8. This is a joint work with N. Arrizabalaga and N. Raymond.

8. **Albert Mas.**

   Title: *Shell interactions for Dirac operators and approximation by short range potentials*

   Abstract: In this talk we will discuss the approximation of a relativistic δ-shell interaction in $\mathbb{R}^3$ by interactions with short range potentials. The relativistic δ-shell interaction is described by the coupling of the free Dirac operator with a singular potential supported on the boundary of a smooth bounded domain Ω, and the short range potentials are given by cutoff functions in a neighbourhood of $\partial\Omega$. During the approximation procedure, we will see how singular integral operators on $\partial\Omega$ come into play and how standard techniques in Calderón-Zygmund theory allow us to develop the approximation mentioned before. In particular, we will talk about the use of maximal operators and pointwise limits almost everywhere to deal with the limit of short range relativistic interactions. This is a joint work with F. Pizzichillo (BCAM - Basque Center for Applied Mathematics, Bilbao).

9. **Carlos Perez.**

   Ttitle: Rough singular integrals and rough Hardy Inequalities via maximal functions

   Abstract: In this expository lecture we will discuss some new results for rough singular singular integral operators improving some weighted estimates by J. Duoandikoetxea and J. L. Rubio

de Francia and E. Sawyer from the 80's. We also show that these results have a connection with some of the classical Hardy inequalities. An important theme is the central role played by appropriate maximal functions. Extrapolation type theorems and sparse domination bounds will be tools from modern Harmonic Analysis that will play a central role in the proofs.

10. **Julien Sabin.**

    Title: *Maximizers for the Stein-Tomas inequality*

    Abstract: We give a necessary and sufficient condition for the precompactness of all optimizing sequences for the Stein-Tomas inequality. In particular, if a well-known conjecture about the optimal constant in the Strichartz inequality is true, we obtain the existence of an optimizer in the Stein-Tomas inequality. Our result is valid in any dimension. This is a joint work with R. Frank (Munich) and E. Lieb (Princeton).

11. **Nicola Visciglia.**

    Title: *Scattering theory for the generalized kdv equation in the energy space*

    Abstract: We study the long time behavior of solutions to gKdV. Our approach is based on the Kenig-Merle concentration-compactness/rigidity Theory. This is joint work with Farah-Linares-Pastor.

12. **James Wright.**

    Title: *Connections between Classical Fourier series and Calderón-Zygmund theory*

    Abstract: The 1950s was an exciting era in analysis for many reasons. It was a golden age for Fourier series where several interesting problems arose, generalised and found deep, definitive solutions. It was also the origin of the real variable method and Calderón-Zygmund theory. In this summer school we will examine two classical Fourier series problems with roots from the 1950s and establish fascinating connections with certain problems in Calderón-Zygmund theory. Viewing a problem from more than one perspective is a fortuitous situation and gives one great advantage to solve it.

    The first problem has its root in the famous Beurling-Helson theorem from the early 1950s where mappings of the circle which preserve absolutely convergent Fourier series are characterised. This was generalised to mappings between any two tori and a definitive solution was eventually given by Paul Cohen in the early 1960s. In the later 1950s Kahane asked what happens when absolute convergence is relaxed to uniform convergence. Through the 1960s, 70s and 80s much progress was made on Kahane's problem in one dimension but the question in higher dimensions seemed formidible at the time. It turns out there is a close connection between Kahane's problem and Singular Radon Transforms from Calderón-Zygmund theory whose recent generalisation to the multi-parameter setting by Stein and Street allows us to consider seriously Kahane's problem in higher dimensions.

    The second problem has its roots in the theory of Sidon sets from the 1950s. These are spectral sets with the property that every continuous function which is Fourier supported in the set automatically has an absolutely convergent Fourier series. Such spectral sets are necessarily very sparse. In 1953, Il'yanov asked what happens when one relaxs absolute convergence to uniform convergence. He called such sets Spectral Sets of Uniform Convergence and the basic problem was to decide how sparse these sets are – can they be polynomial? The eventual solution to this problem by Arkhipov and Oskolkov from the late 1980s is intimately connected to multipliers arising from Discrete Analogues of Harmonic Analysis whose theory in the late 1980s was being developed by Stein and Wainger and is the singular integral variant of discete maximal functions considered by Bourgain in his generalisation of Birkhoff's pointwise ergodic theorem to sparse arithmetic sequences. These Discrete Analogues in Harmonic Analysis have now been sufficiently developed by numerous people and we are in a position to address Il'yanov's original problem in higher dimension.