POLITECNICO
MILANO 1863

DIPARTIMENTO DI MATEMATICA

Anisotropic mesh adaptation for 3D printing-oriented structural design

Nicola Ferro

February 11, 2021

Mathematical Methods for Objects Reconstruction: from 3D Vision to 3D Printing

Introduction

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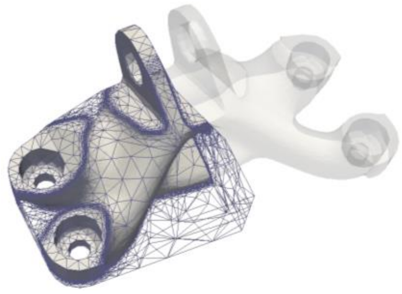

Topology optimization

Find the optimal material distribution in a design domain that guarantees specific constraints, by modifying the topology.

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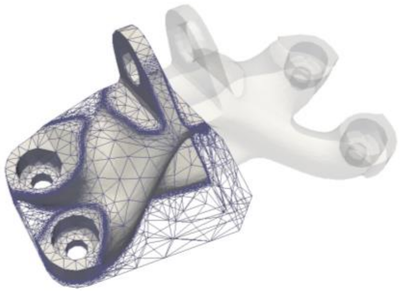
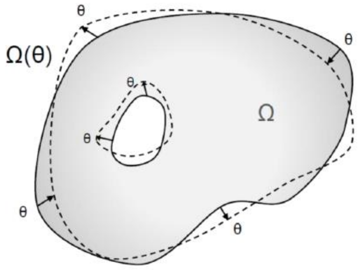


Topology optimization

Find the optimal material distribution in a design domain that guarantees specific constraints, by modifying the topology.



Shape optimization

Find the optimal shape of a structure that guarantees specific constraints, by modifying the boundary of the domain.

Sigmund, Bendsoe, Allaire

Outline

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The methods

Topology optimization

Anisotropic mesh adaptation

The applications

Robust structural optimization

Metamaterial design

Performance-constrained design

AM-ready topology optimization

Ongoing and future projects

Topology optimization

The goal

Find the optimal material distribution in a design domain that guarantees specific constraints, by modifying the topology.

In the literature

- ESO/BESO methods
- Level-set methods
- Phase-field models
- **SIMP method**

$$\rho \in L^\infty(\Omega, [0,1])$$

Density variable

Bendsøe (1995)

Topology optimization

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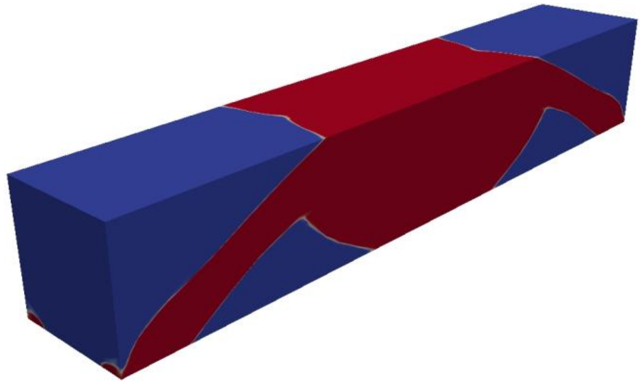
Find the optimal material distribution in a design domain that guarantees specific constraints, by modifying the topology.



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Density variable

identifies how the void and the material are arranged in the domain

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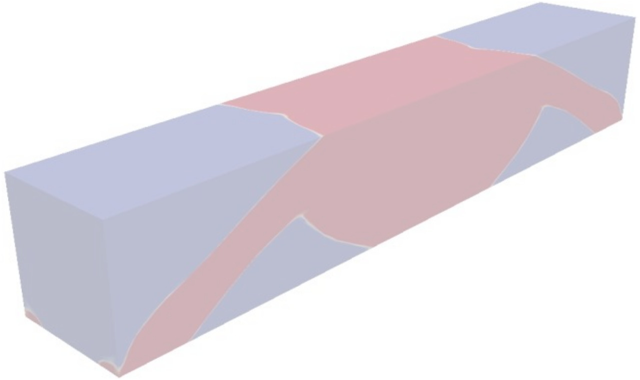

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$$\min_{\rho \in L^\infty(\Omega)} G(\boldsymbol{u}(\rho)): \begin{cases} a_\rho(\boldsymbol{u}(\rho), \boldsymbol{v}) = G(\boldsymbol{v}) \quad \forall \boldsymbol{v} \in U \\ \int_\Omega \rho d\Omega \le \alpha|\Omega| \\ \rho_{min} \le \rho \le 1 \end{cases}$$

Objective

$$G(\boldsymbol{u}) = \int_{\Gamma_N} \boldsymbol{f} \cdot \boldsymbol{u} \, d\gamma$$

$$a_\rho(\boldsymbol{u}, \boldsymbol{v}) = \int_\Omega \sigma_\rho(\boldsymbol{u}) : \epsilon(\boldsymbol{v}) d\Omega$$

$$\sigma_\rho(\boldsymbol{u}) = \rho^p [2\mu\epsilon(\boldsymbol{u}) + \lambda I : \epsilon(\boldsymbol{u})]$$

$$p \ge \max\left\{\frac{2}{1-\nu}, \frac{4}{1+\nu}\right\}$$

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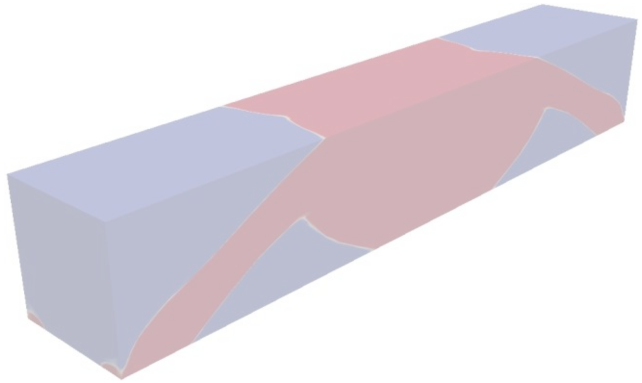

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$$G(\boldsymbol{u}) = \int_{\Gamma_N} \boldsymbol{f} \cdot \boldsymbol{u} \, d\gamma$$

ρ-modified bilinear form

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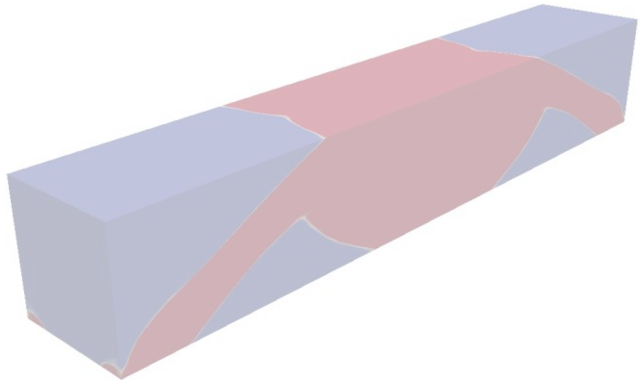

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ρ-modified constitutive law

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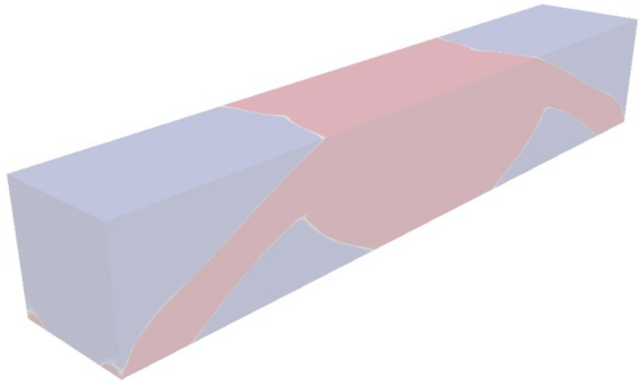

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Penalization exponent

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Topology optimization

The goal

Find
in a
speci
topo

WARNING

The model – without further expedients – does not ensure uniqueness of the solution.

Not convex;
Mesh dependent;

Additional filtering and/or regularization;
Ad hoc discretization for the specific problem.

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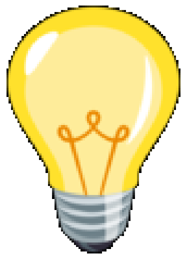

THE IDEA

We couple topology optimization with anisotropic mesh adaptation to provide an automatic enriched framework for structural optimization.

Penalization exponent $p \geq \max\left\{\frac{2}{1-\nu}, \frac{4}{1+\nu}\right\}$

Micheletti, Perotto, Soli (2018) – patent application

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Modify the computational grid according to the physics of the problem to limit the computational burden and guarantee accuracy.

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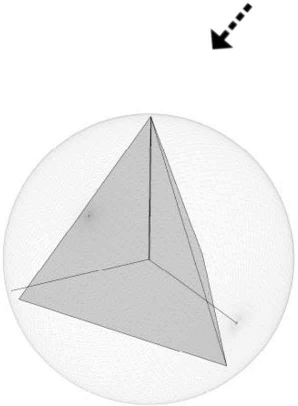
(ρ, u)



Isotropic

Mesh adaptation

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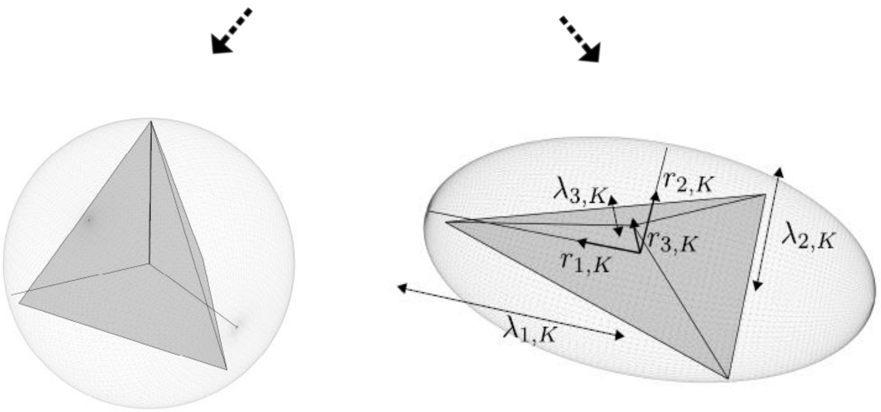

Isotropic

Anisotropic

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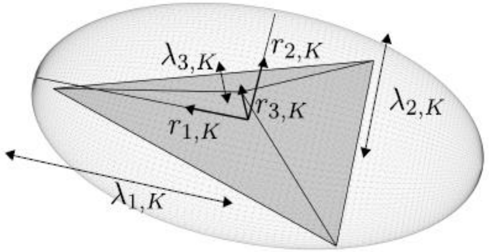
(ρ, u)

Size



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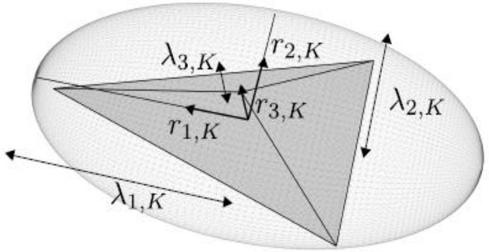
(ρ, u)

h_K



Isotropic

Anisotropic

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(ρ, u)

h_K

Size
Shape
Orientation

Isotropic

Anisotropic

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(ρ, u)

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$$\begin{cases} \lambda_{1,K}, \lambda_{2,K}, \lambda_{3,K} \\ \boldsymbol{r}_{1,K}, \boldsymbol{r}_{2,K}, \boldsymbol{r}_{3,K} \end{cases}$$

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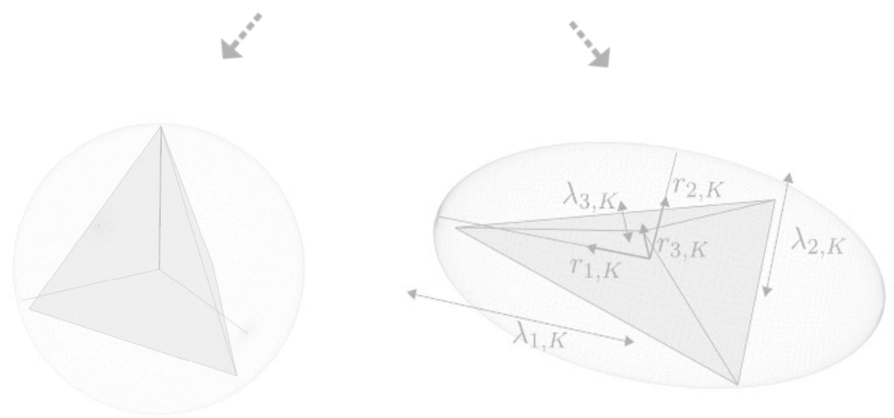

Isotropic

Anisotropic

The isotropic setting

In a standard finite element setting, we exploit the Zienkiewicz-Zhu estimator for the H^1-norm estimation.

$$\eta \simeq |\rho - \rho_h|_{H^1(\Omega)}$$

Size h_K

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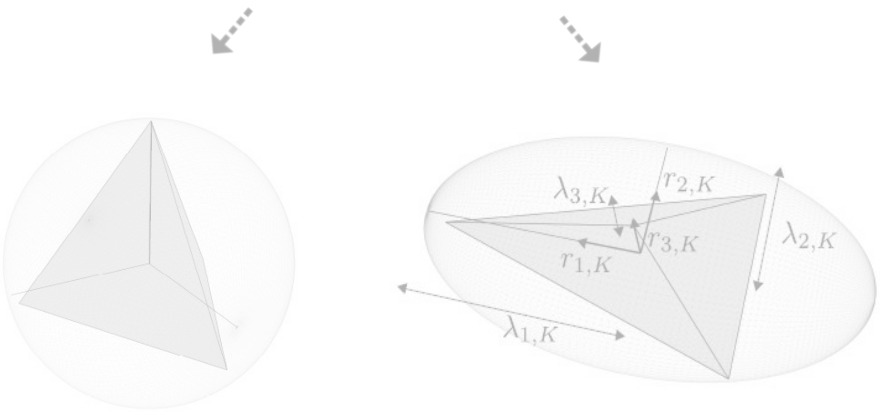

Isotropic

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↓

Size h_K

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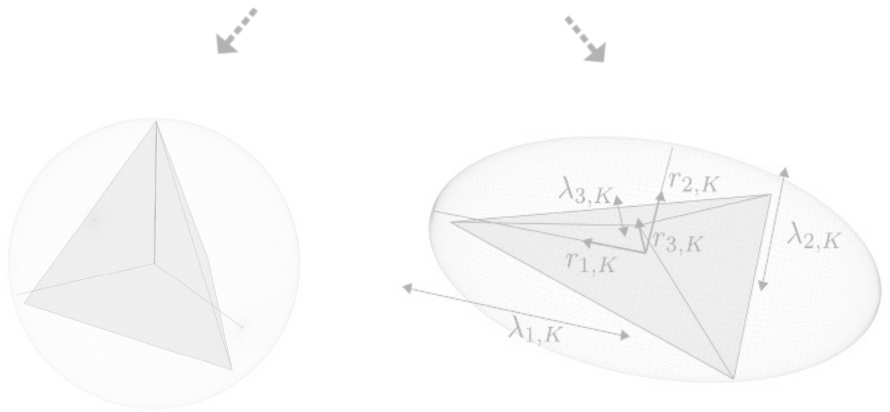

Isotropic Anisotropic

Zienkiewicz, Zhu (1987)

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Size h_K

Computable

Mesh adaptation

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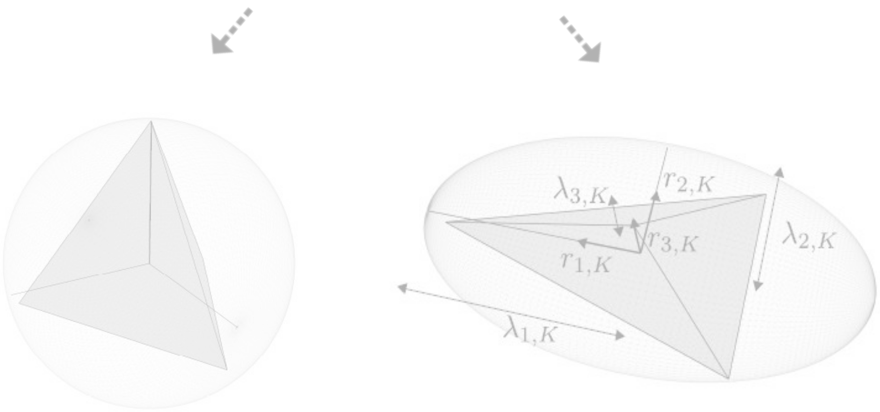

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Size h_K

Not computable

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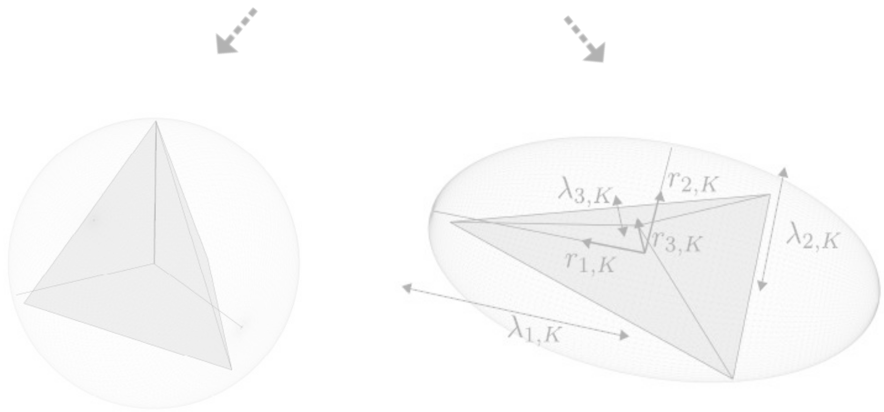

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We introduce a recovery procedure in order to estimate the exact gradient.

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Size h_K

Mesh adaptation

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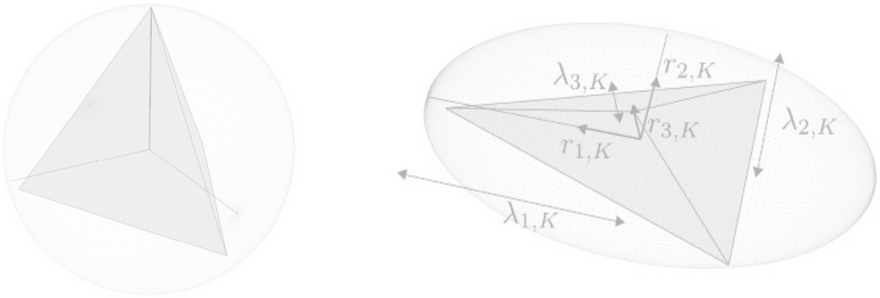

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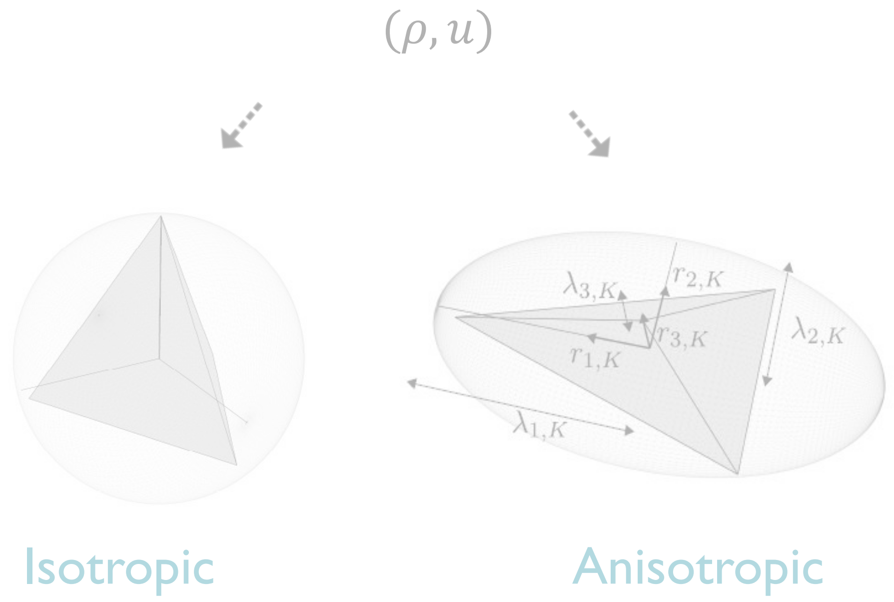

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$\eta \simeq |\rho - \rho_h|_{H^1(\Omega)}$

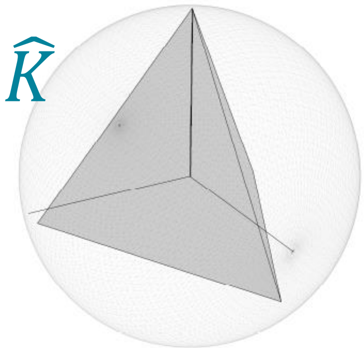
↓

Size h_K

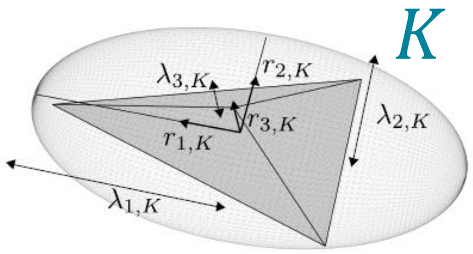
Recovery operator

Mesh adaptation

ANISOTROPIC SETTING



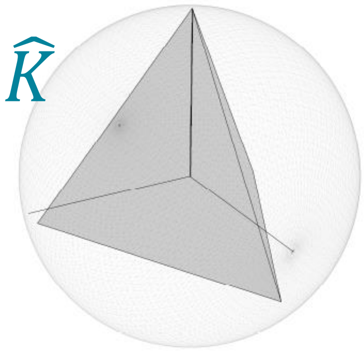
$$M_K \hat{x} + t_K$$



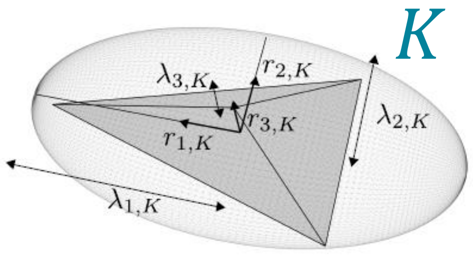
Formaggia, Perotto (2001)

Mesh adaptation

ANISOTROPIC SETTING



$$\left(R_K^T \Lambda_K R_K\right) Z_K \hat{x} + t_K$$



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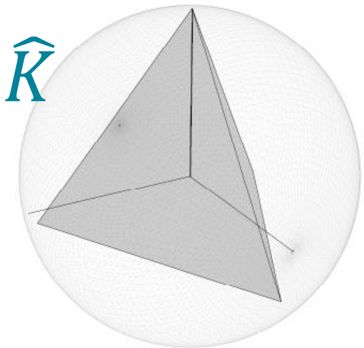

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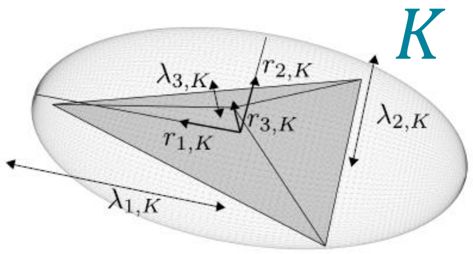

Geometric features
of the generic element K

Formaggia, Perotto (2001)

Mesh adaptation

The anisotropic setting

We resort to a modification of the isotropic setting to determine the size, shape and orientation of the elements.

$$\eta \simeq |\rho - \rho_h|_{H^1(\Omega)} \dashrightarrow \begin{cases} \lambda_{1,K}, \lambda_{2,K}, \lambda_{3,K} \\ \boldsymbol{r}_{1,K}, \boldsymbol{r}_{2,K}, \boldsymbol{r}_{3,K} \end{cases}$$

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$$\eta^2 = \sum_{K \in T_h} \eta_K^2$$

$$\eta_K^2 = \frac{1}{\left(\lambda_{1,K}\lambda_{2,K}\lambda_{3,K}\right)^{2/3}} \sum_{i=1}^{3} \lambda_{i,K}^2 \left(\boldsymbol{r}_{i,K}^T H_{\Delta_K}(E_\nabla)\boldsymbol{r}_{i,K}\right)$$

$$E_\nabla = [P(\nabla\rho_h) - \nabla\rho_h]_{\Delta_K}$$

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Micheletti, Perotto (2010)

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Micheletti, Perotto (2010)

Definition of an anisotropic error estimator

Mesh adaptation

The anisotropic setting

We resort to a modification of the isotropic setting to determine the size, shape and orientation of the elements.

$$\eta \simeq |\rho - \rho_h|_{H^1(\Omega)} \dashrightarrow \begin{cases} \lambda_{1,K}, \lambda_{2,K}, \lambda_{3,K} \\ \boldsymbol{r}_{1,K}, \boldsymbol{r}_{2,K}, \boldsymbol{r}_{3,K} \end{cases}$$

$$\eta^2 = \sum_{K \in T_h} \eta_K^2$$

$$\eta_K^2 = \frac{1}{\left(\lambda_{1,K}\lambda_{2,K}\lambda_{3,K}\right)^{2/3}} \sum_{i=1}^{3} \lambda_{i,K}^2 \left(\boldsymbol{r}_{i,K}^T \, H_{\Delta_K}(E_\nabla)\boldsymbol{r}_{i,K}\right)$$

$$E_\nabla = [P(\nabla \rho_h) - \nabla \rho_h]_{\Delta_K}$$

$$P(\nabla \rho_h)\Big|_{\Delta_K} = \frac{1}{|\Delta_K|} \sum_{T \in \Delta_K} |T| \nabla \rho_h \Big|_T$$

$$[H_{\Delta_K}(w)]_{i,j} = \sum_{T \in \Delta_K} \int_T w_i w_j \, dT$$

Recovered error

Micheletti, Perotto (2010)

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Micheletti, Perotto (2010)

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<u>AIM:</u>

Control and equidistribute the error by imposing the accuracy $MTOL$ and minimizing the total number of elements.

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Mesh adaptation

Local optimization problem

The minimization of the cardinality of T_h is equivalent to maximize the measure of the element K. Hence, we resort to a constrained local minimization problem

$$\min_{\{s_{i,K},\boldsymbol{r}_{i,K}\}_{i=1}^{3}} \sum_{i=1}^{3} s_{i,K}\left(\boldsymbol{r}_{i,K}^{T}\,\widehat{H}_{\Delta_K}(E_\nabla)\boldsymbol{r}_{i,K}\right)$$

such that

$$\boldsymbol{r}_{i,K}\cdot\boldsymbol{r}_{j,K}=\delta_{ij}$$
$$s_{1,K}\geq s_{2,K}\geq s_{3,K}$$
$$s_{1,K}s_{2,K}s_{3,K}=1$$

and

$$\eta_K^2=\frac{MTOL^2}{\#T_h}$$

Micheletti, Perotto, Farrell (2010)

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and

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Micheletti, Perotto, Farrell (2010)

Proposition

Let $\{\boldsymbol{g}_i, g_i\}_{i=1,2,3}$ be the eigeinpairs associated with $\widehat{H}_{\Delta_K}(E_\nabla)$, with $g_1\geq g_2\geq g_3>0$ and $\{\boldsymbol{g}_i\}_{i=1,2,3}$ orthonormal. Then, the optimal geometric values for the element K are

$$\boldsymbol{r}_{1,K}^{OPT}=\boldsymbol{g}_3,\qquad \boldsymbol{r}_{2,K}^{OPT}=\boldsymbol{g}_2,\qquad \boldsymbol{r}_{3,K}^{OPT}=\boldsymbol{g}_1$$

$$\lambda_{1,K}^{OPT}=g_3^{-\frac{1}{2}}\left(\frac{MTOL^2}{3\,\#T_h|\widehat{\Delta_K}|}\right)^{1/3}\left(\prod_{i=1}^{3}g_i\right)^{1/18}$$

$$\lambda_{2,K}^{OPT}=g_2^{-\frac{1}{2}}\left(\frac{MTOL^2}{3\,\#T_h|\widehat{\Delta_K}|}\right)^{1/3}\left(\prod_{i=1}^{3}g_i\right)^{1/18}$$

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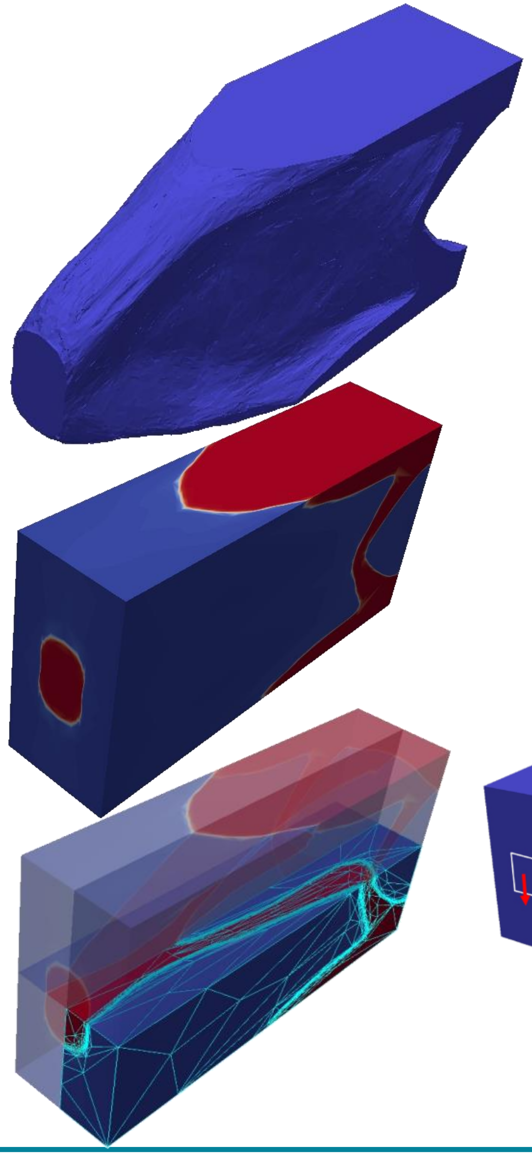
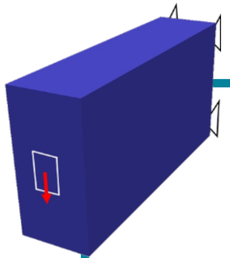
Numerical results



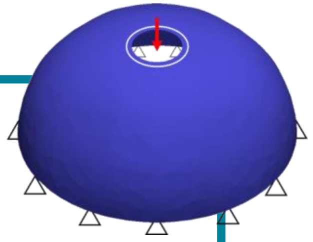


CANTILEVER

$\alpha = 0.5,$

$\#T_h \simeq 96038,$

CPU time = 1.5h



DOME

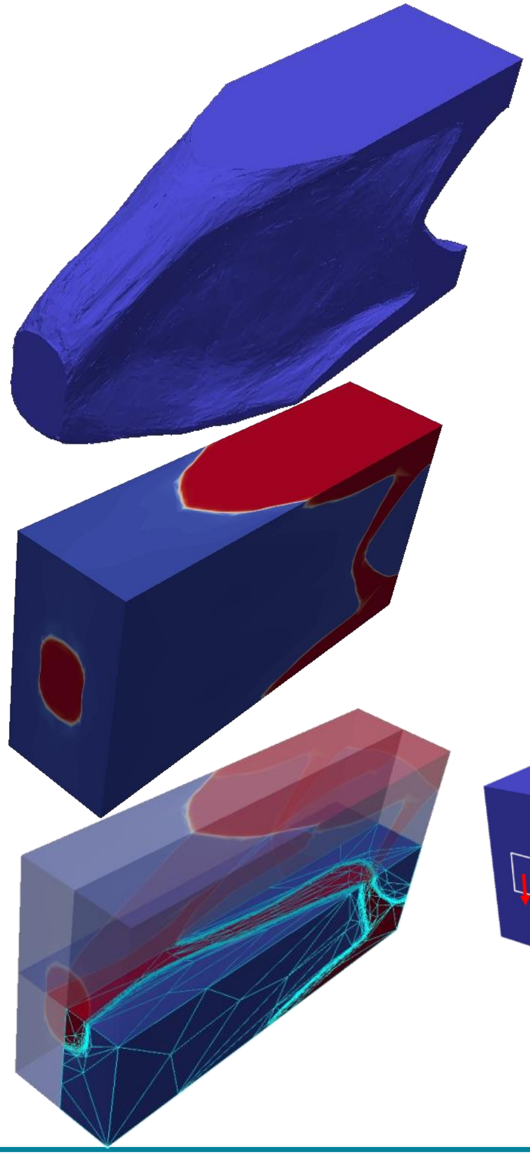
$\alpha = 0.2,$

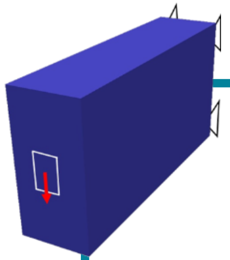
$\#T_h \simeq 118435,$

CPU time = 0.65h

Numerical results

Mesh adaptation amounts to 3-5% of the overall CPU time, adding no considerable overhead.





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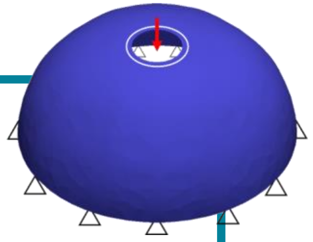

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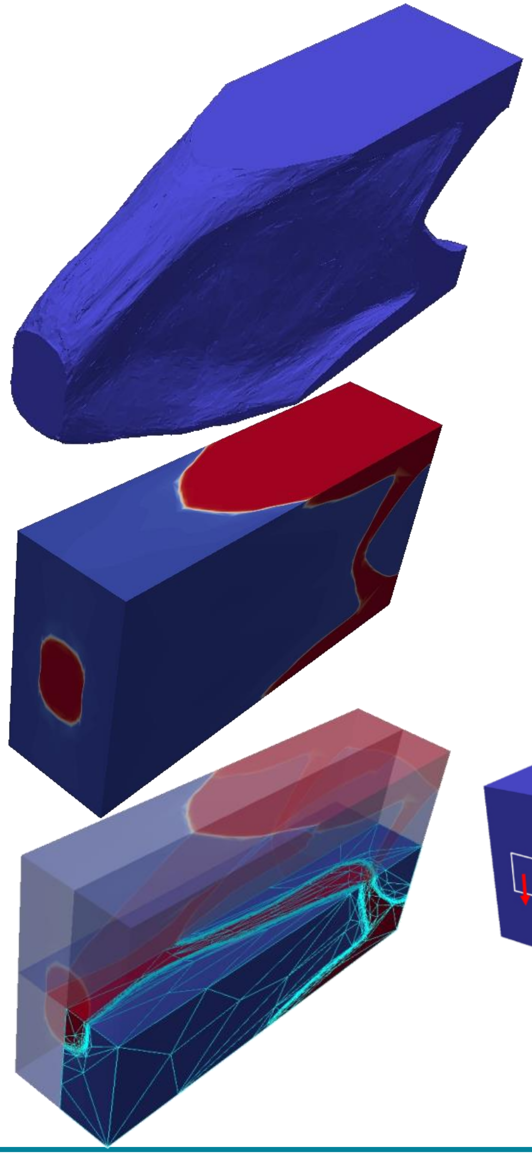
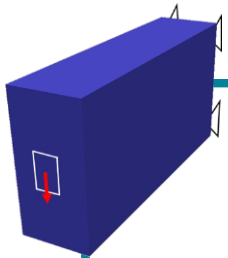


Mesh adaptation amounts to 3-5% of the overall CPU time, adding no considerable overhead.

The algorithm delivers smooth structures with a reduced employment of filters/regularization.



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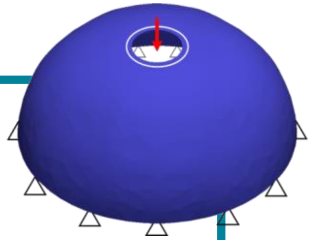

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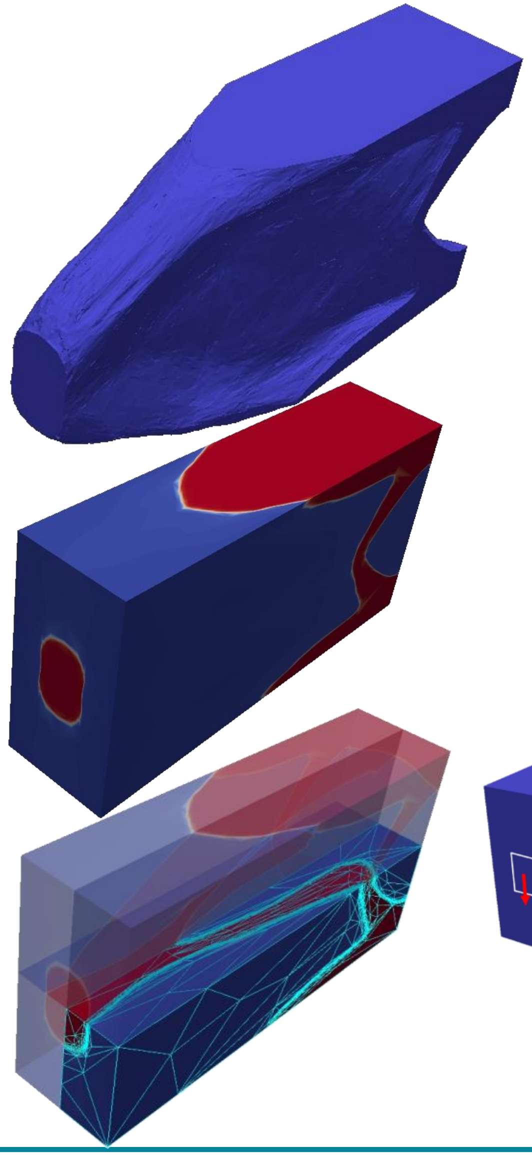

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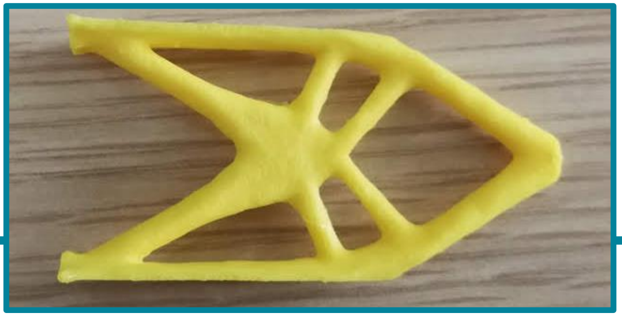

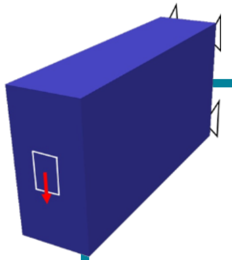

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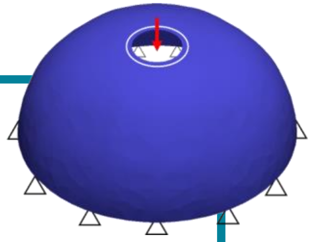

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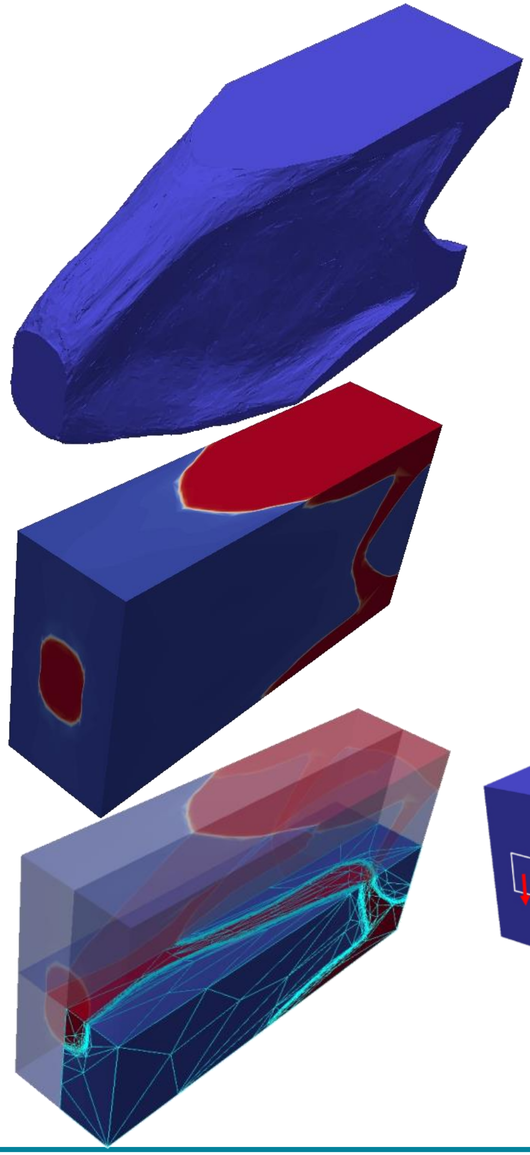

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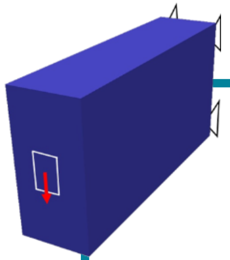
The algorithm delivers smooth structures with a reduced employment of filters/regularization.

The procedure is application-independent, as it relies on the design variable ρ.



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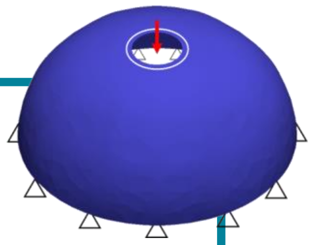

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Outline

The design of structures can be assisted with a mathematically robust optimization framework, involving different goals and design constraints of interest.

The mathematical modeling and the numerical discretization of this class of problem can be addressed in terms of different methods and numerical schemes.

Goal of this presentation is to provide an adaptive versatile numerical discretization procedure, which can assist the structural design problem.

The methods

Topology optimization

Anisotropic mesh adaptation

The applications

Robust structural optimization

Metamaterial design

Performance-constrained design

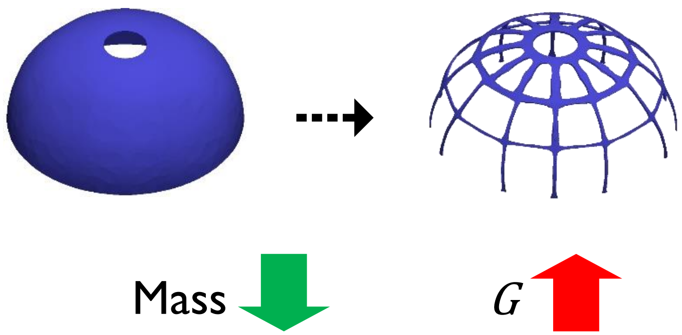
AM-ready topology optimization

Ongoing and future projects

Shape optimization

Topology optimization

Find the optimal material distribution in a design domain that guarantees specific constraints, by modifying the topology.



Shape optimization

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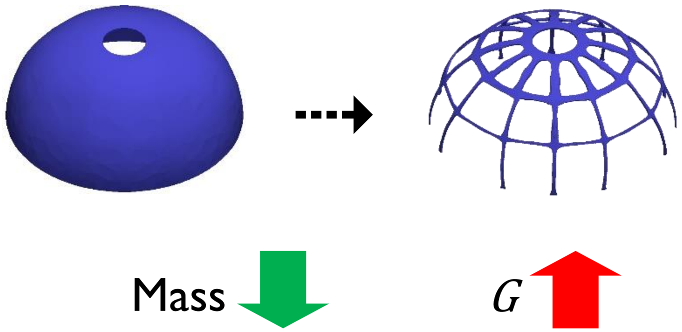
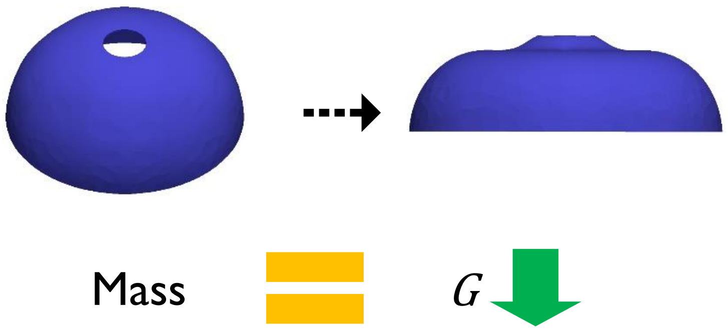


Shape optimization

Find the optimal shape of a structure that guarantees specific constraints, by modifying the boundary of the domain.



Shape optimization

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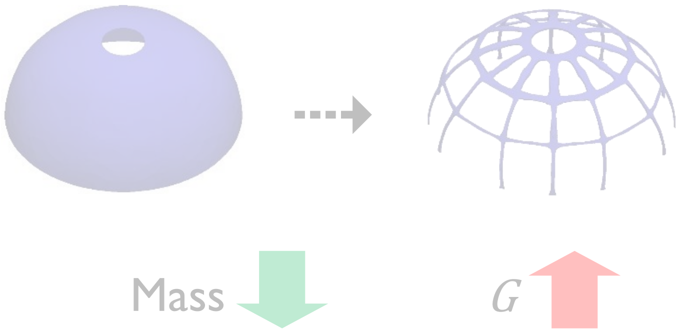
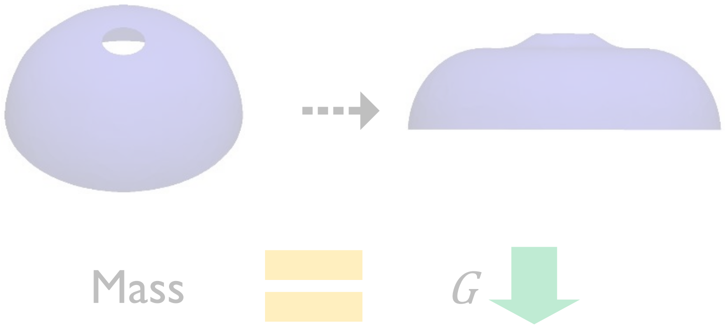
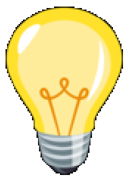
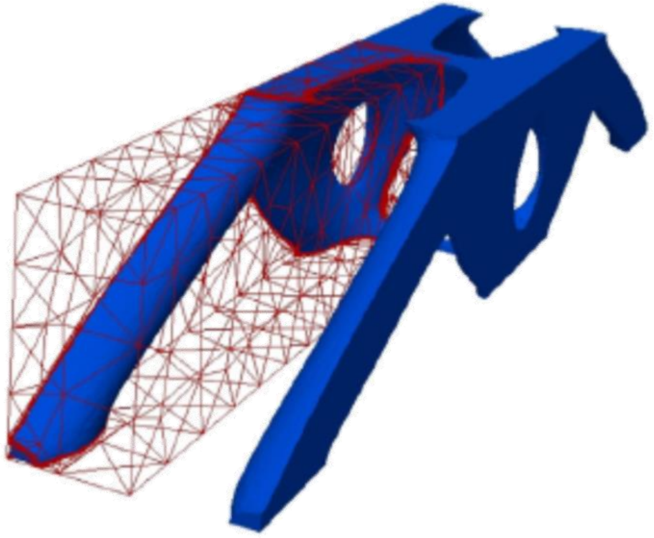






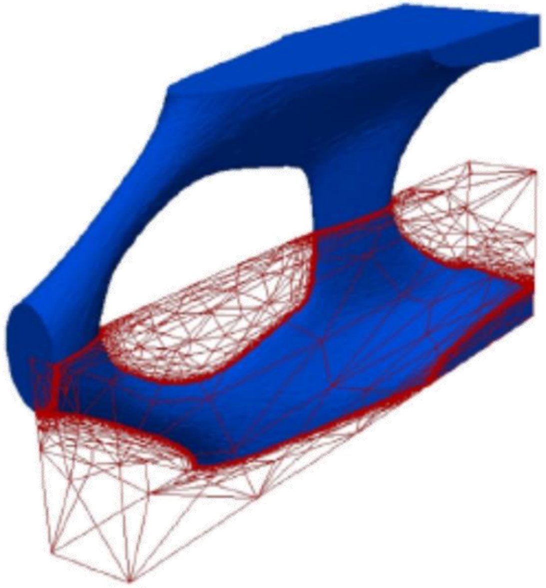
Combine shape optimization with topology optimization to obtain light and performant structures

Shape optimization



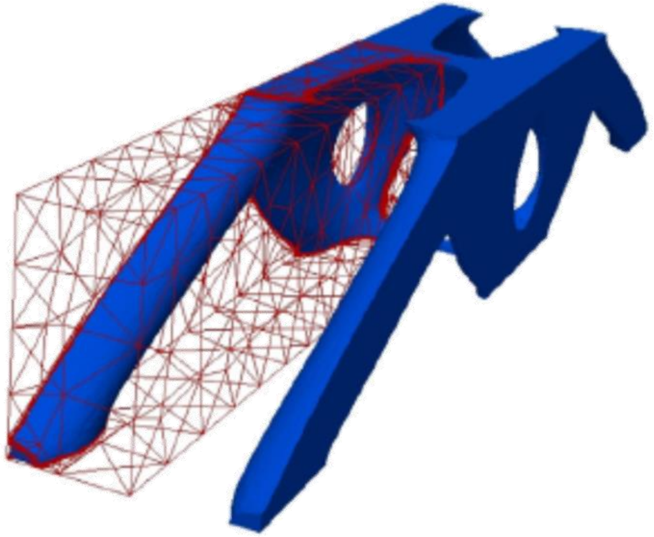
Topology optimization

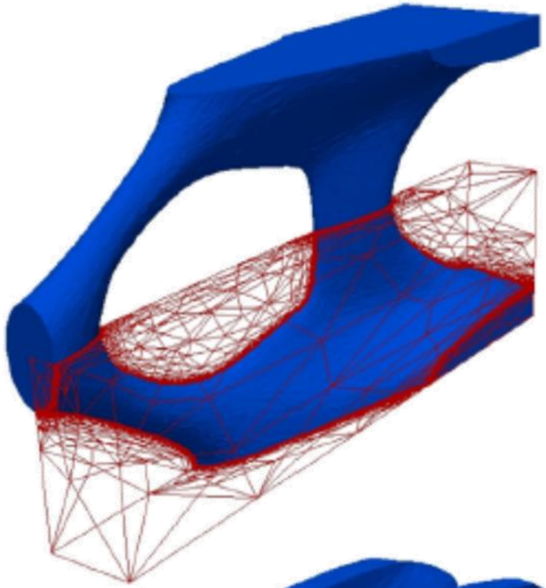
Mass ⬇ G ⬆



N.F., S. Micheletti, S. Perotto (2020)

Shape optimization



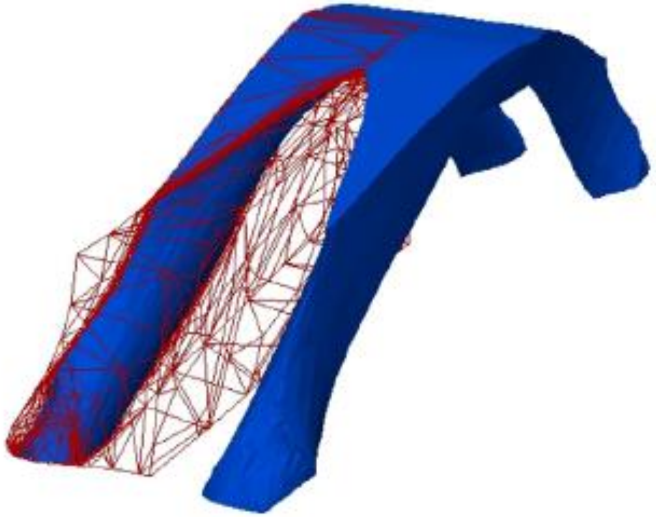


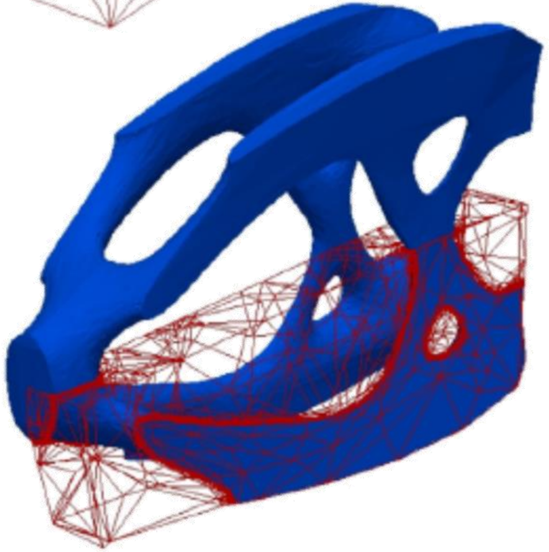
Topology optimization

Mass ↓ G ↑

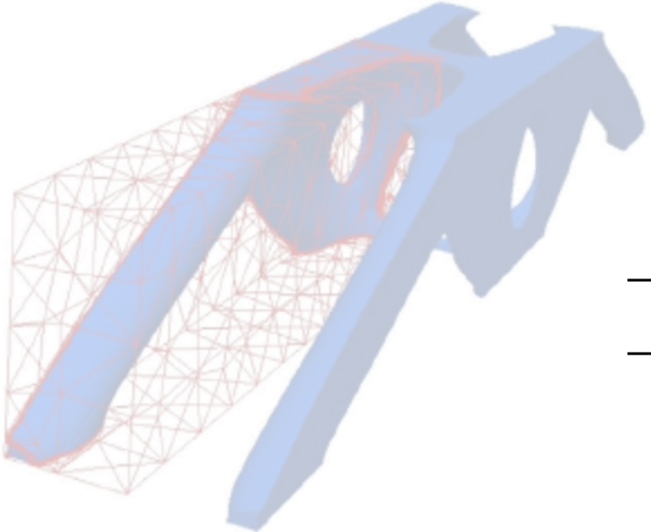
Mass ↓ G ↓

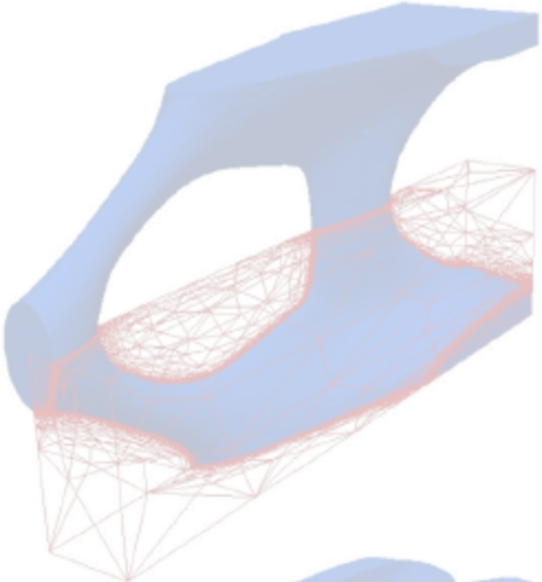
Shape + Topology optimization





Shape optimization





Topology optimization

	Bridge	Cantilever
$\mathcal{G}$ after TO	1.54e−1	4.86e−2
$\Delta_{\%}\mathcal{G}$ after TO	+65.27%	+57.28%
$\#\mathcal{T}_h$ after TO	102 293	89 994
$\mathcal{G}$ after GSTO	1.43e−1	4.08e−2
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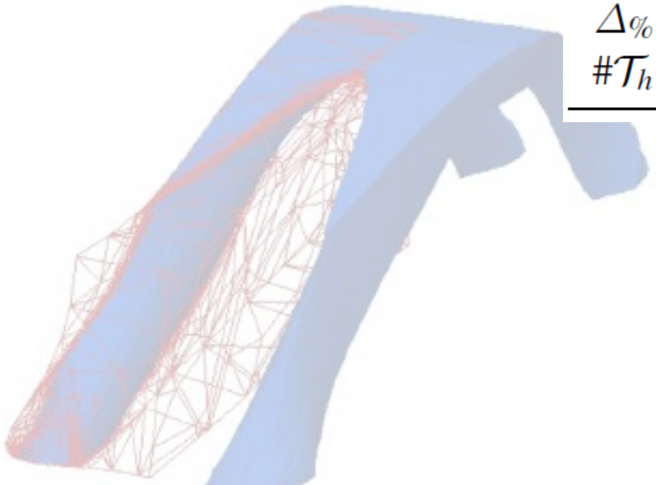
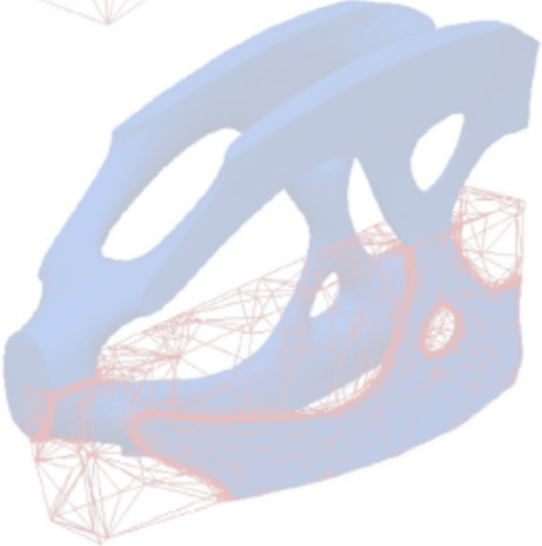
$\#\mathcal{T}_h$ after GSTO	74 074	183 153

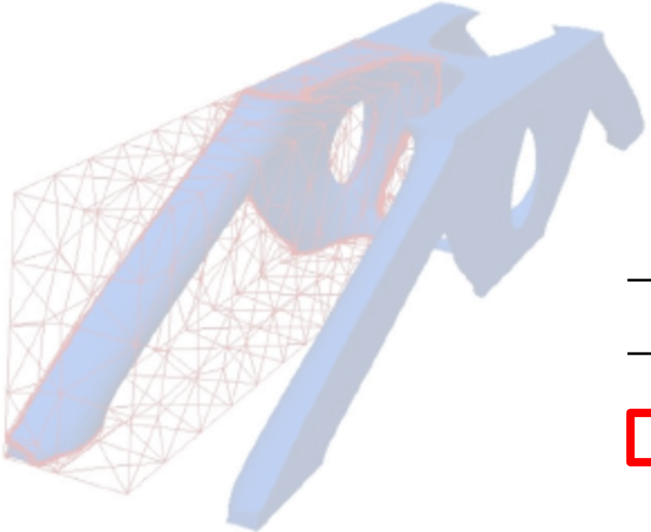
Shape + Topology optimization

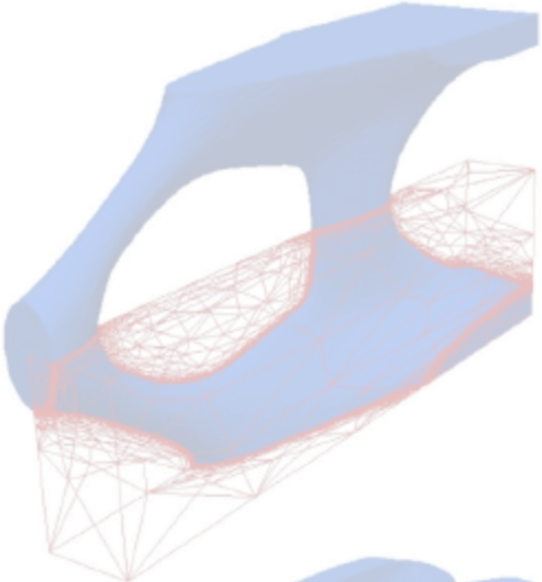




N.F., S. Micheletti, S. Perotto (2020)

Shape optimization





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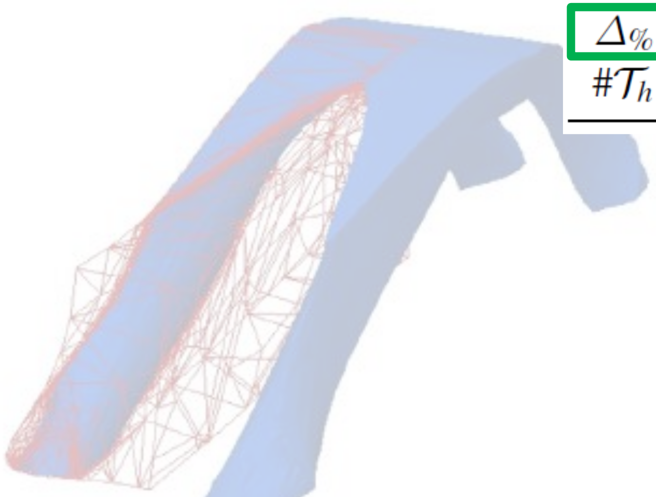

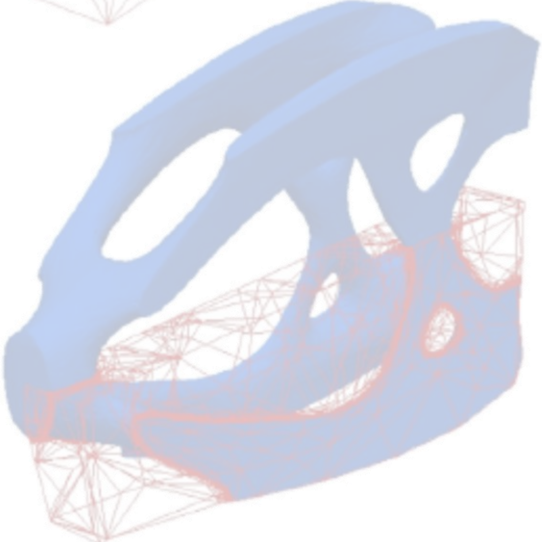

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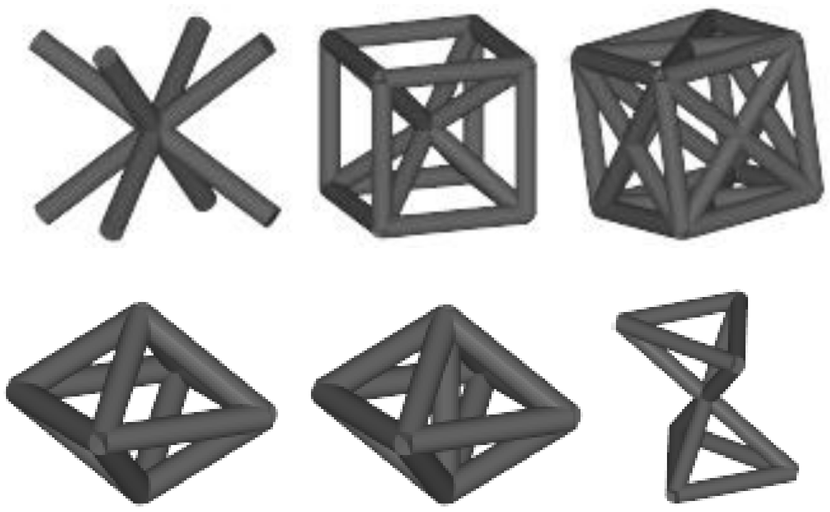
Microstructural optimization

The goal

Find the optimal microstructure that guarantees specific requirements at the macroscale, by the homogenization theory.

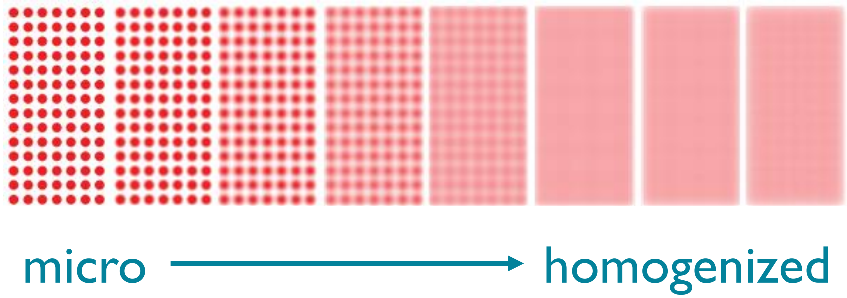


McKown et al. (2008)

Microstructural optimization

The goal

Find the optimal microstructure that guarantees specific requirements at the macroscale, by the homogenization theory.



Asymptotic expansion

$$\epsilon(x) = \bar{\epsilon} + \epsilon^{v}$$

Sánchez-Palencia (1983)

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Targeting the macroscale by observing the microscale

Objective function

$$J(\rho) = \sum_{ijkl} \left(E^{H}_{ijkl}(\rho) - E^{W}_{ijkl} \right)^2$$

$$E^{H}_{ijkl} = \frac{1}{|Y|} \int_{Y} E_{pqrs} \left(\epsilon^{0,kl}_{pq} - \epsilon^{*,klj}_{pq} \right) \left(\epsilon^{0,ij}_{rs} - \epsilon^{*,ij}_{rs} \right) dY$$

$$\int_{Y} E_{ijpq} \epsilon^{*,kl}_{pq} \epsilon_{ij}(v) dY = \int_{Y} E_{ijpq} \epsilon^{0,kl}_{pq} \epsilon_{ij}(v) dY \quad \forall v \; in \; V$$

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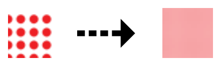

From the periodic microscale to the macroscale

Homogenized stiffness tensor

$$E^{H}_{ijkl} = \frac{1}{|Y|} \int_{Y} E_{pqrs} \left(\epsilon^{0,kl}_{pq} - \epsilon^{*,klj}_{pq} \right) \left(\epsilon^{0,ij}_{rs} - \epsilon^{*,ij}_{rs} \right) dY$$

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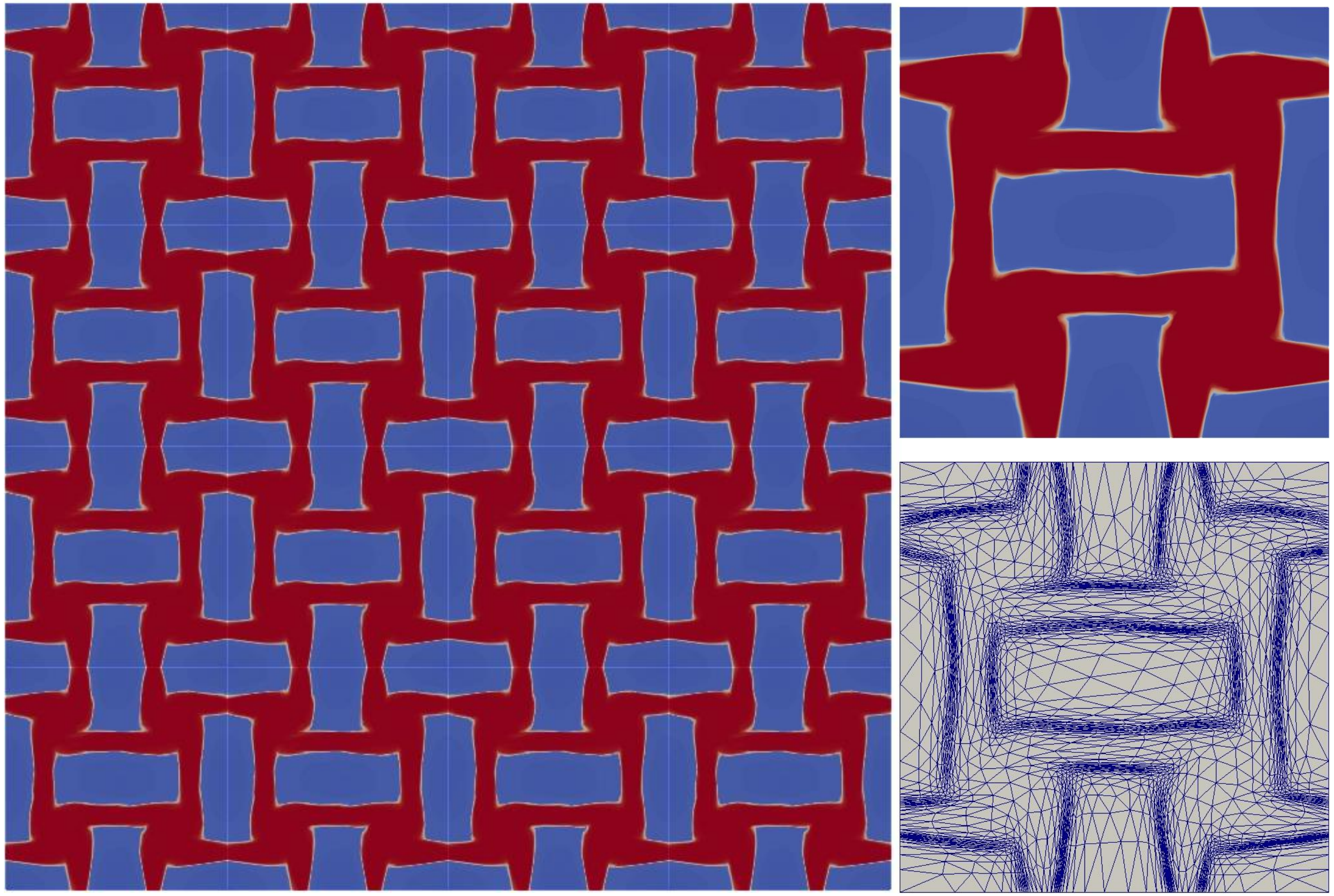
In the periodic microscale

State equations

$$\int_{Y} E_{ijpq} \epsilon^{*,kl}_{pq} \epsilon_{ij}(v) dY = \int_{Y} E_{ijpq} \epsilon^{0,kl}_{pq} \epsilon_{ij}(v) dY \quad \forall v \ in \ V$$

Numerical results



Target property – Poisson's ratio
$\nu = -0.7$

Volume fraction
0.5

Final homogenized value
$\nu = -0.54$

N.F., S. Micheletti, S. Perotto (2020)

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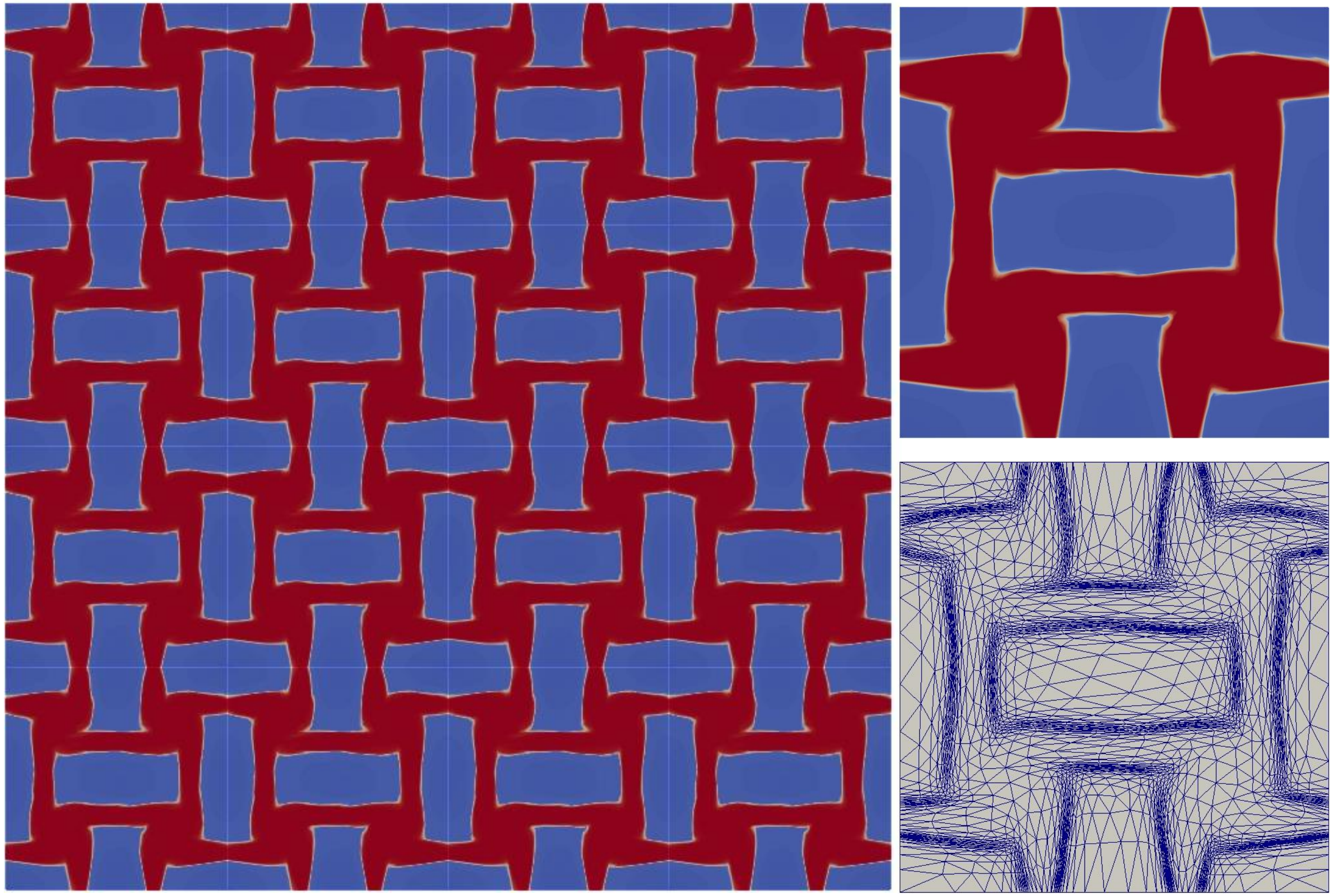

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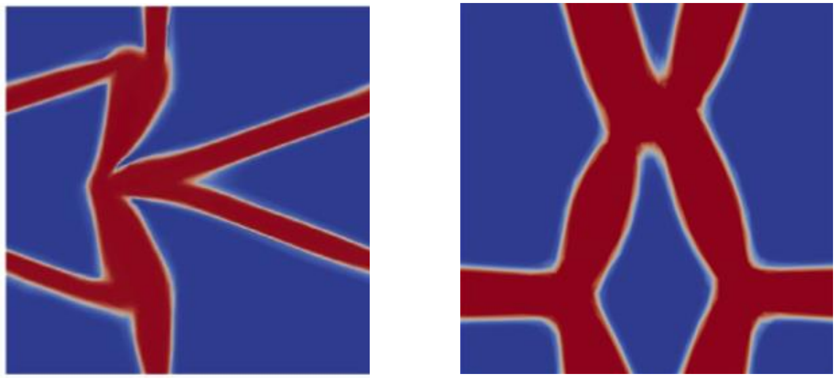

N.F., S. Micheletti, S. Perotto (2020)

Outline

The design of structures can be assisted with a mathematically robust optimization framework, involving different goals and design constraints of interest.

The mathematical modeling and the numerical discretization of this class of problem can be addressed in terms of different methods and numerical schemes.

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The methods

Topology optimization

Anisotropic mesh adaptation

The applications

Robust structural optimization

Metamaterial design

Performance-constrained design

AM-ready topology optimization

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The topology optimization problem can include several, sometimes cuncurrent, performance requirements. This eventually yields a multi-constrained and/or multi-objective framework.

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- Stress;
- Vibrational frequencies;
- …

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Inequality constraints

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Compliance

$$C(\boldsymbol{u}) = \int_{\Gamma_N} \boldsymbol{f}\cdot\boldsymbol{u}\, d\gamma$$

$$S(\boldsymbol{u}) = ||\sigma_{VM}(\boldsymbol{u})||_{L^\gamma(\Omega)}$$

$$\sigma_{VM}(\boldsymbol{u}) = \rho^p E\sqrt{\varepsilon_{11}^2 + \varepsilon_{22}^2 - \varepsilon_{11}\varepsilon_{22} + 3\,\varepsilon_{12}^2}$$

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ρ- modified von Mises stress

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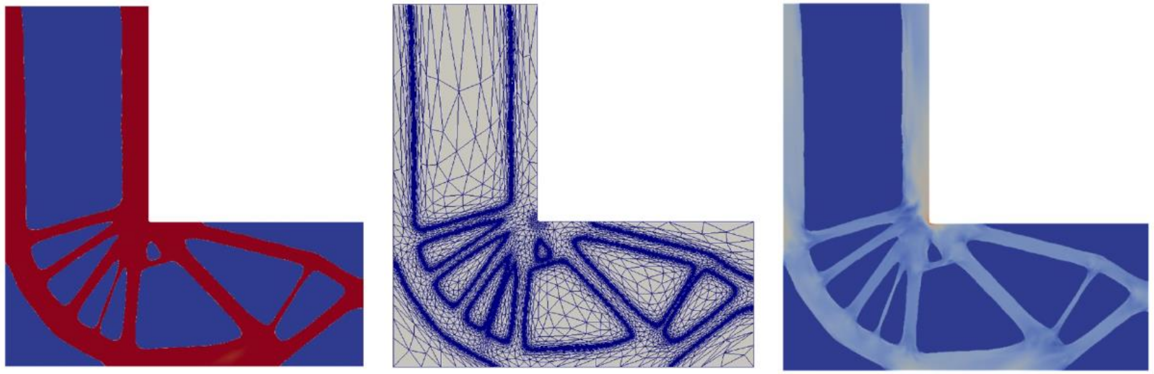
Numerical results



Compliance-constrained optimization

N.F., S. Micheletti, S. Perotto (2020)

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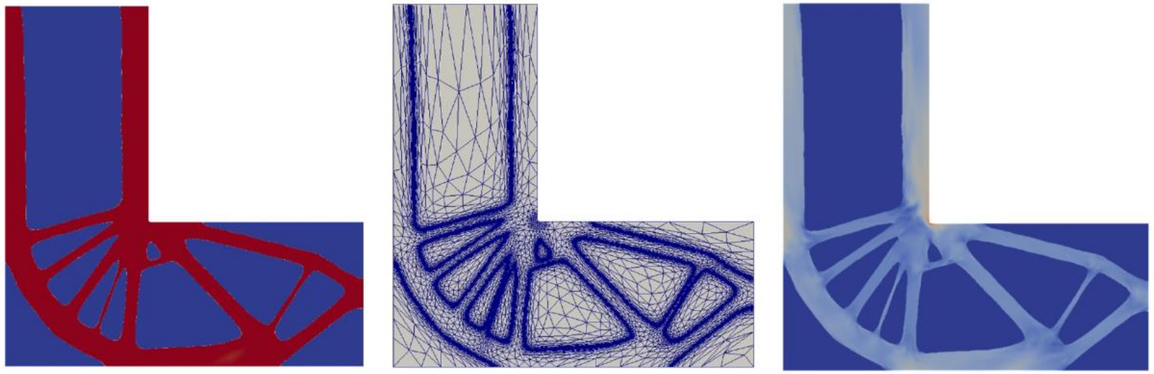

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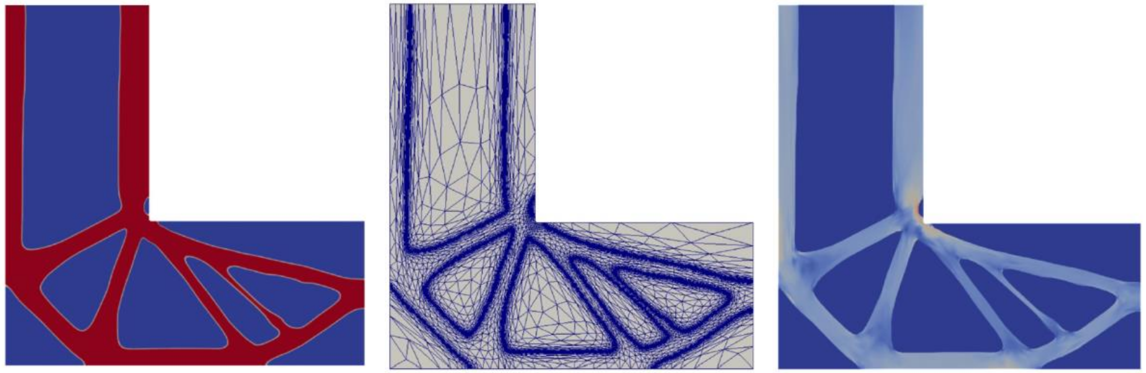

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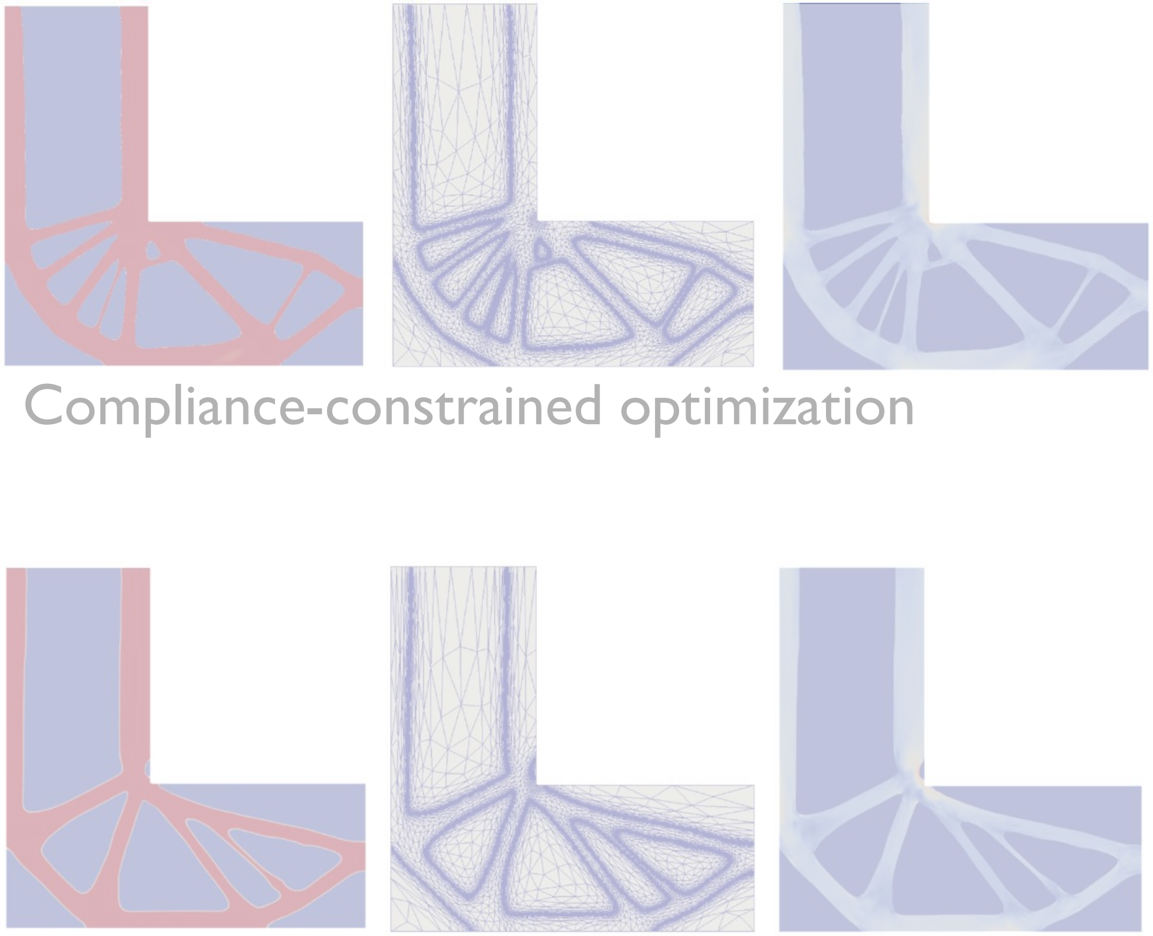
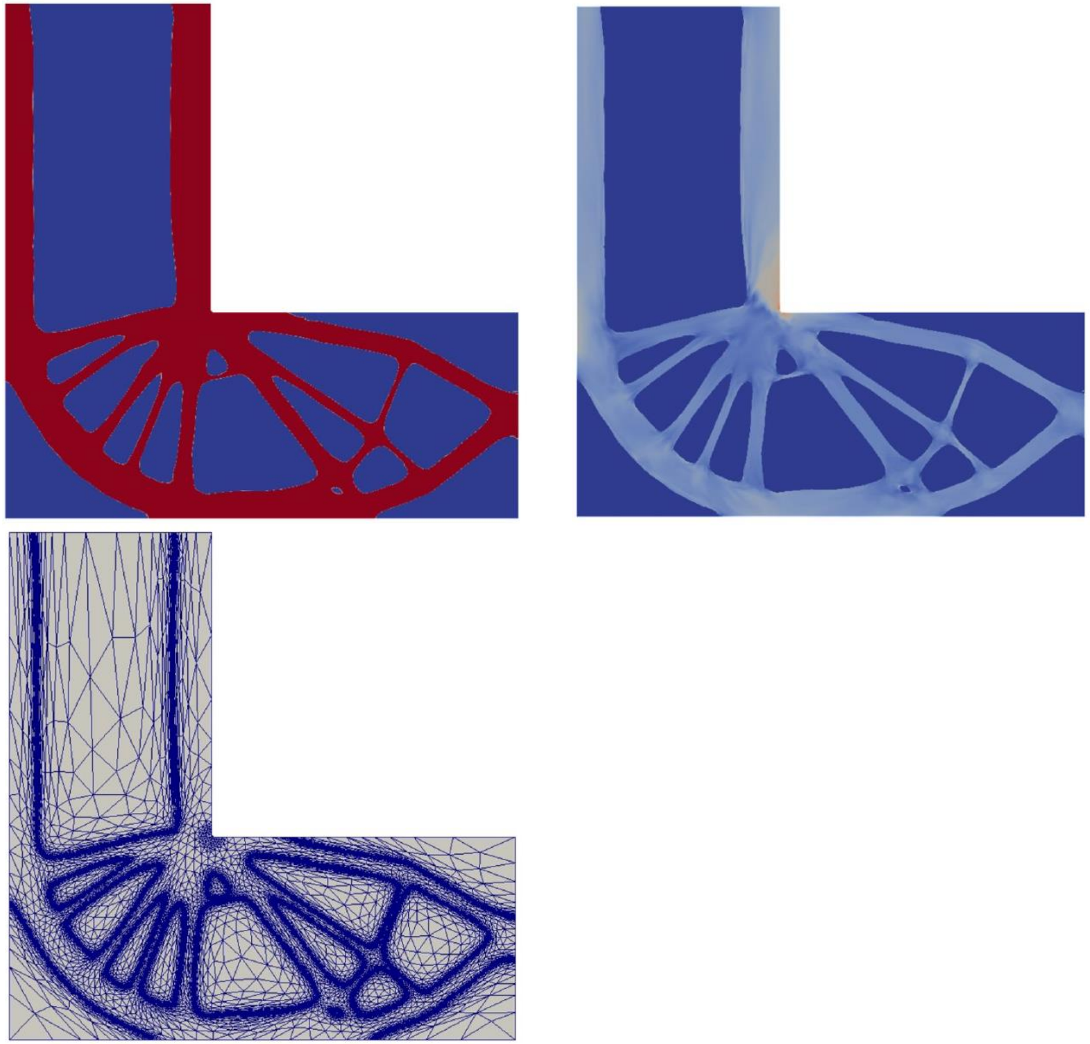


Compliance-constrained optimization

Stress-constrained optimization

Stress- and compliance-constrained optimization



N.F., S. Micheletti, S. Perotto (2020)

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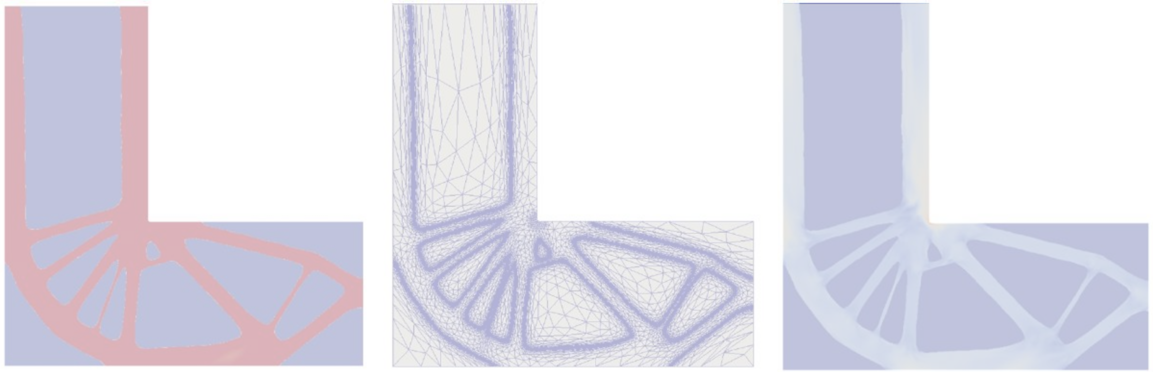

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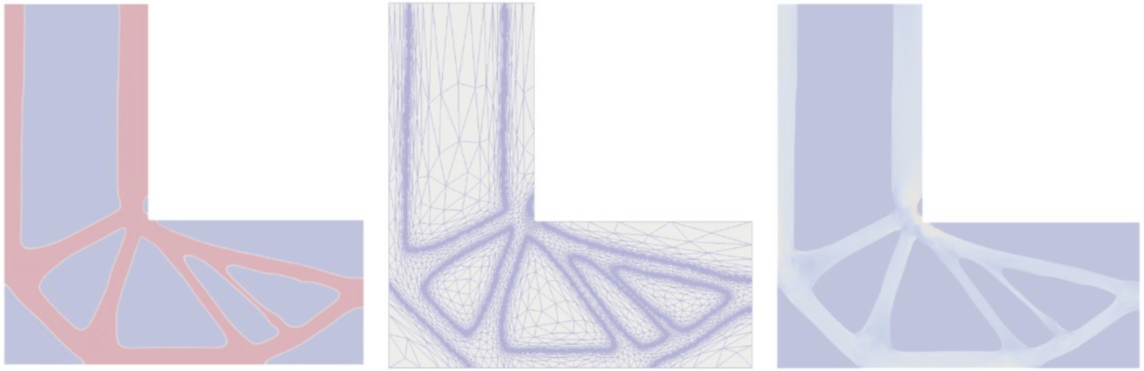

Stress-constrained optimization

N.F., S. Micheletti, S. Perotto (2020)

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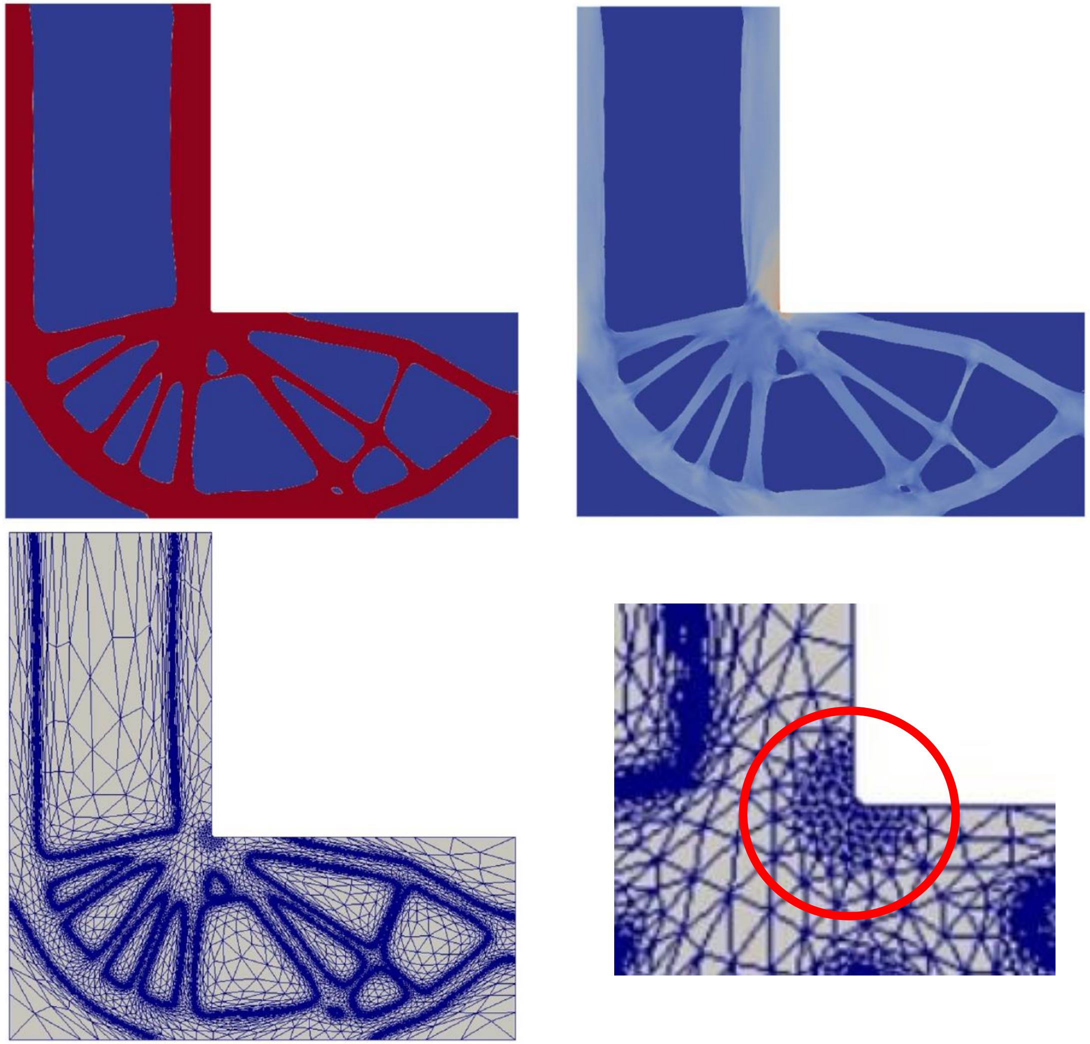

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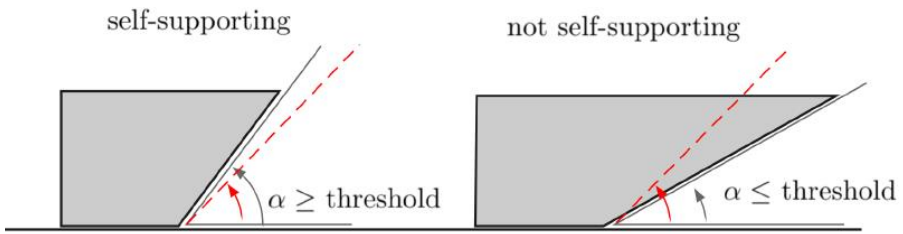
Additive manufacturing constraints should be taken into account in the design phase. This results in final layouts complying with the AM process specifications.



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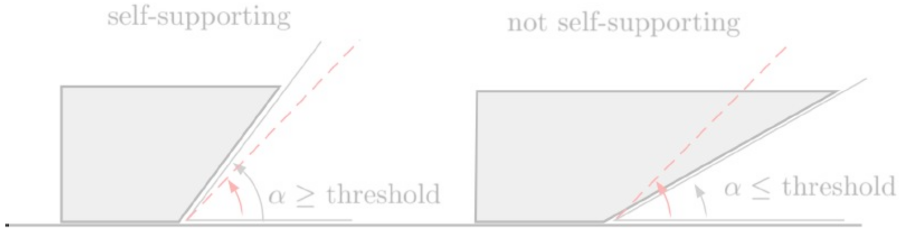

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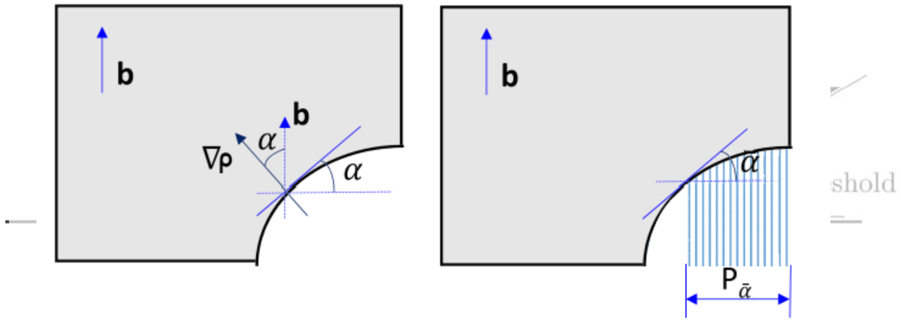

Manufacturability constraint

$$P_{\alpha} = \int_{\Omega} H_{\alpha}\left(\boldsymbol{b} \cdot \frac{\nabla \rho}{||\nabla \rho||_{L^2(\Omega)}}\right) \boldsymbol{b} \cdot \nabla \rho \leq \bar{P}$$

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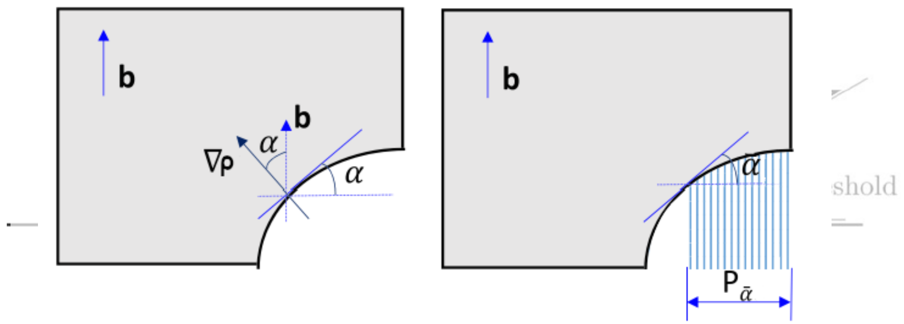

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Shifted Heaviside function

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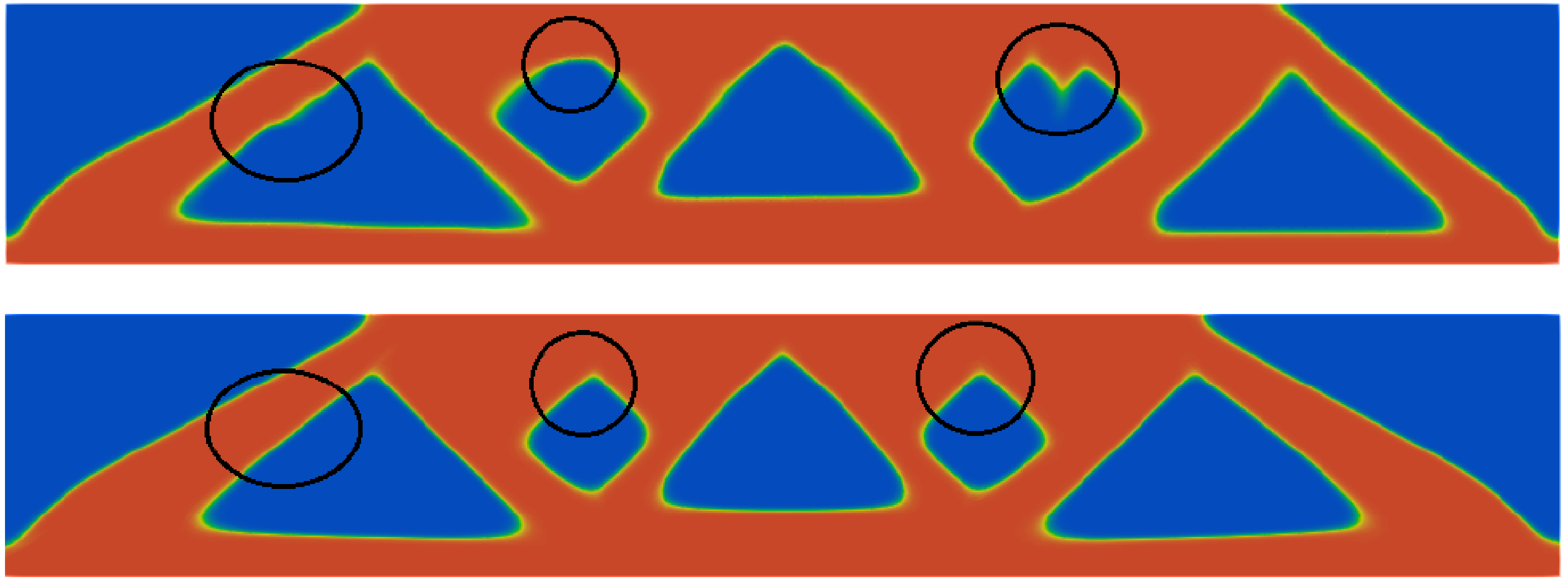
Numerical results



G. Gavinelli – Master's thesis (2020)

Summary

Anisotropic mesh adaptation has been proved to be a versatile procedure that can assist the structural design problem in different scenarios. The effectiveness has been proved on different test cases involving:

- Robust structural optimization
- Metamaterial design
- Performance-constrained design
- AM-ready topology optimization

THANK YOU FOR YOUR ATTENTION