Polarization Vision

Edwin Hancock

University of York and Beihang University.

THE ROYAL SOCIETY

Supported by a Royal Society
Wolfson Research Merit Award

Work with

Gary Atkinson

Will Smith

Nitya Subramanian

Gul e Saman

Lichi Zhang

Silvia Tozza

Hadi Dalan

Dizhong Zhu

Outline

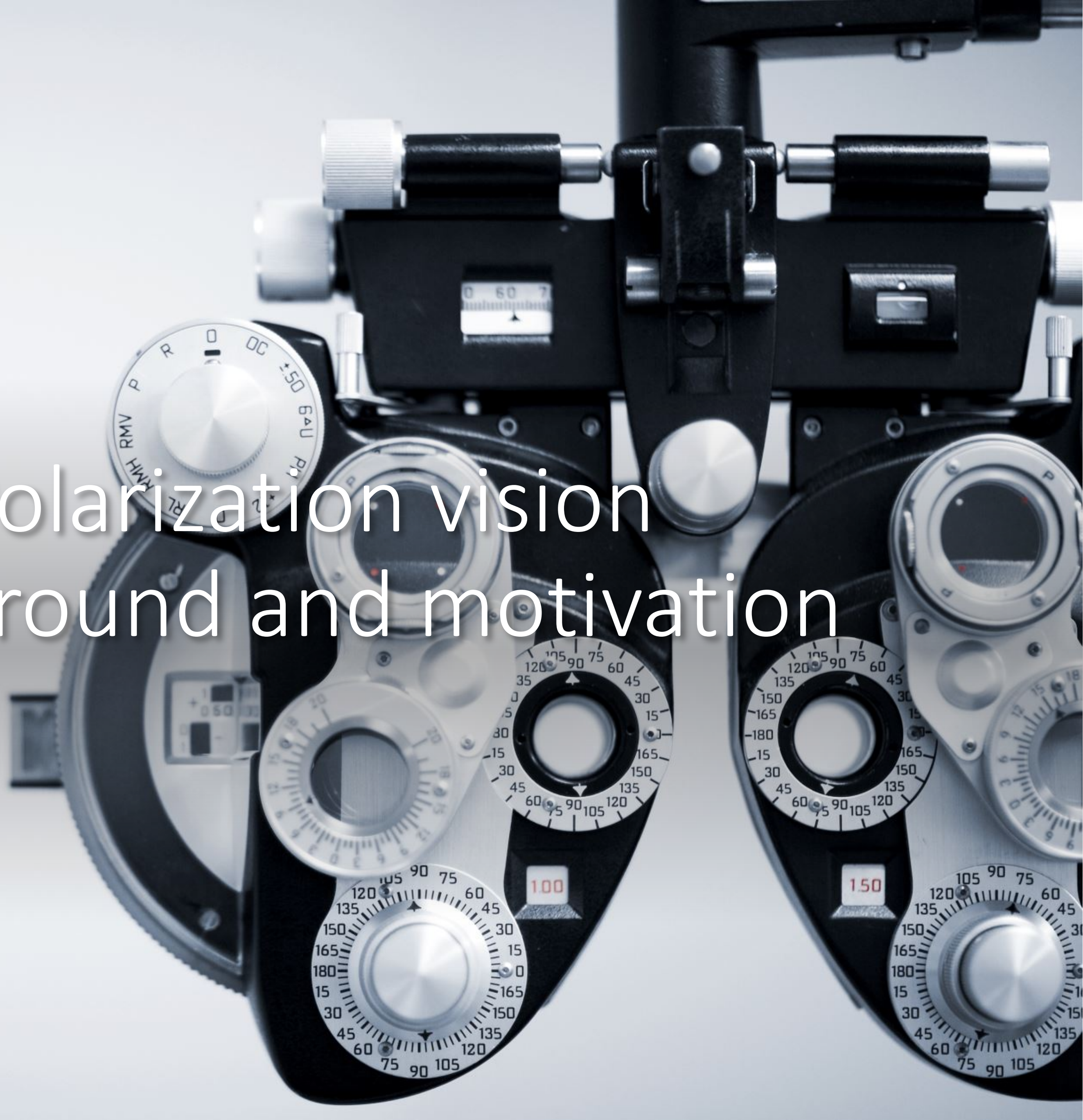
1 Motivation and background

2 Basic physics of shape-from-polarization

3 Some very old work on 3D shape recovery (Atkinson and Hancock, TPAMI 2007, TIP 2006).

4 More recent work on spectro-polarimetry (Huynh, Robles-Kelly, Hancock IJCV 2013)

5 Conclusions



Polarization vision
Background and motivation

Why polarization vision?

- Polarization cameras are becoming more widely and more cheaply available.

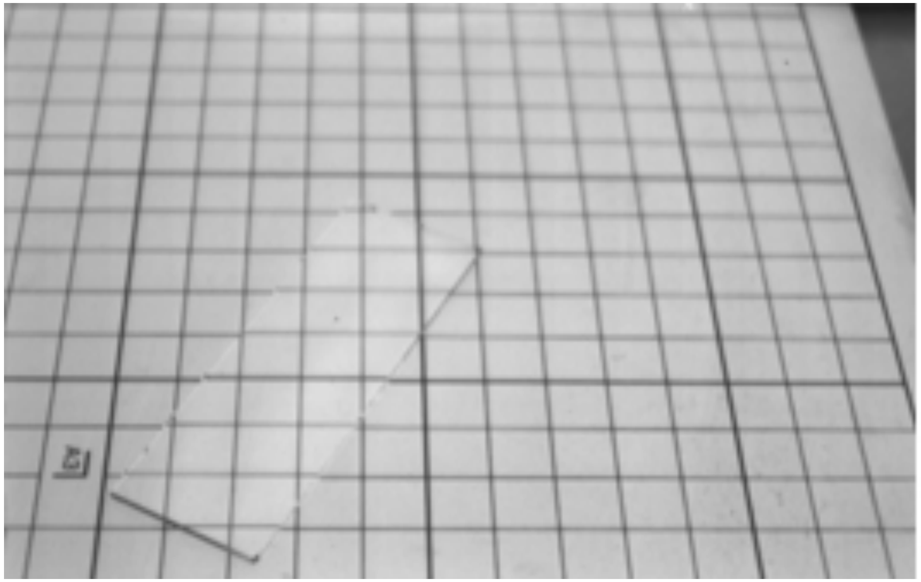
- Polarization images can be used to

  - recover surface shape
  - estimate refractive index or changes in refractive index
  - analyse the shape of transparent objects
  - probe surface substructure (e.g. layers)

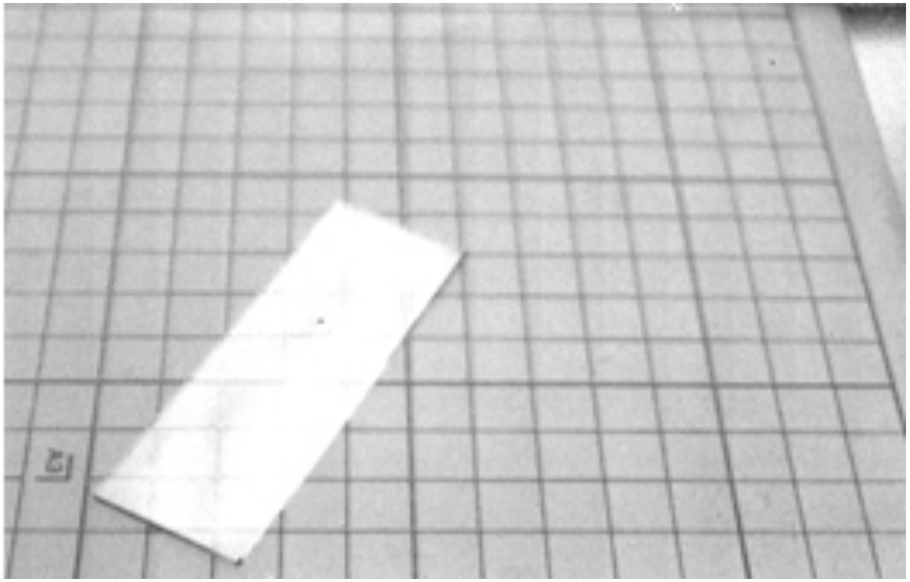
- Applications include imaging in bad atmospheric conditions and directly measuring surface properties such as refractive index.

- Used by many animals with specialist vision systems.

Ricoh



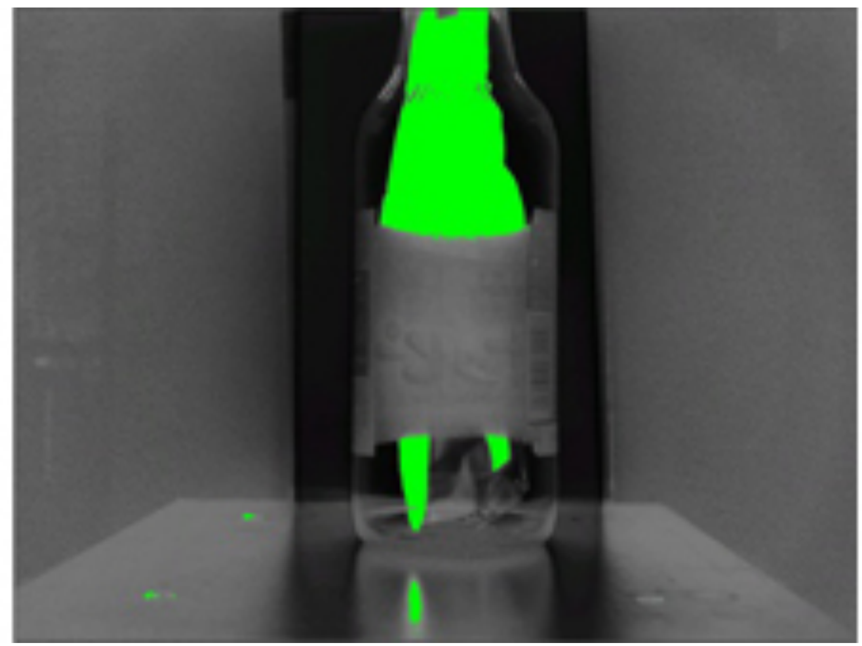
Visible light



Degree of polarization

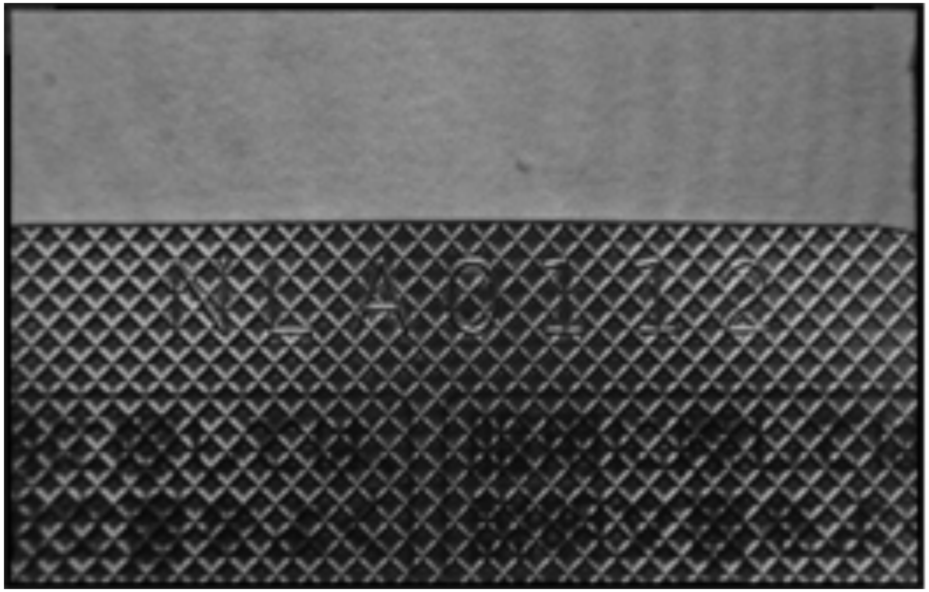
Ricoh

Visible light



Degree of polarization

Ricoh

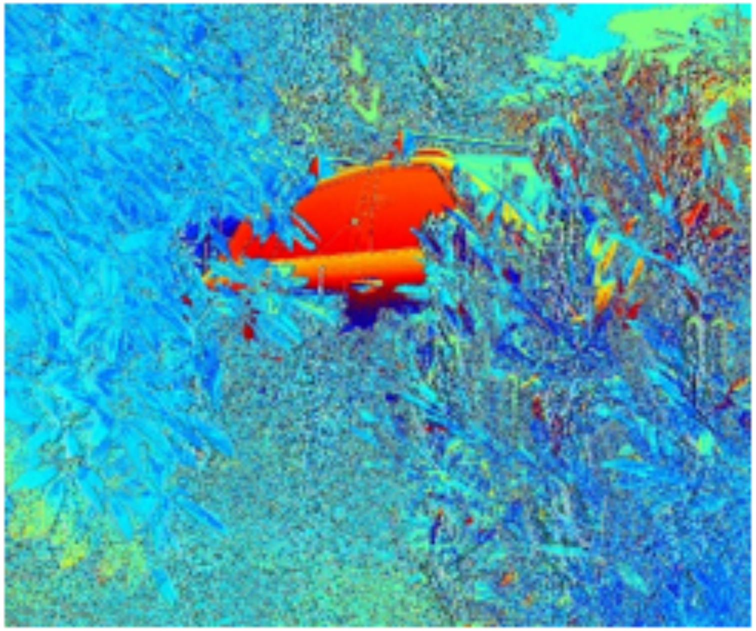


Visible light

Degree of polarization

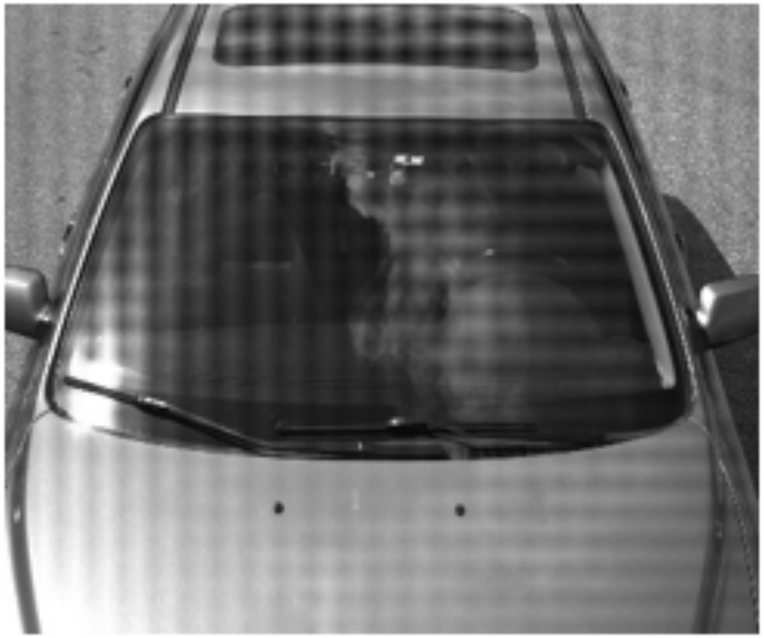
Sony

Visible light



Degree of polarization

Sony



Visible light

Degree of polarization

What does a polarization camera measure?

- Degree of polarization
- Phase
- Mean intensity

At every pixel in the image

Does light source need to polarized?

- No – light develops spontaneous polarization when it scatters from a surface.

- Depends on the refractive index of the surface and the angle reflection.

What do polarization images reveal?

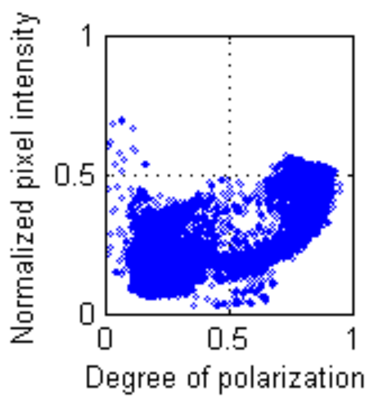
- Single polarization image: surface normals and hence 3D shape of an object provided refractive index known.

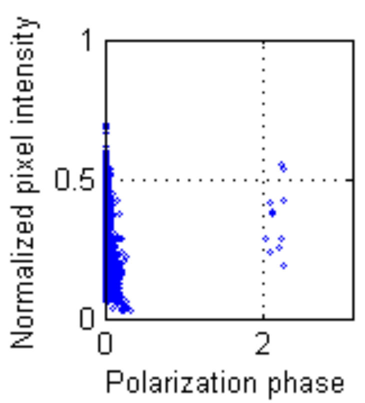
- Multiple polarization images: (from different viewing angles or light source direction) shape, refractive index, albedo (intrinsic surface texture).

- Spectro-polarimetric images: (polarization images at different wavelengths) shape, albedo and refractive inex at different wavelengths.

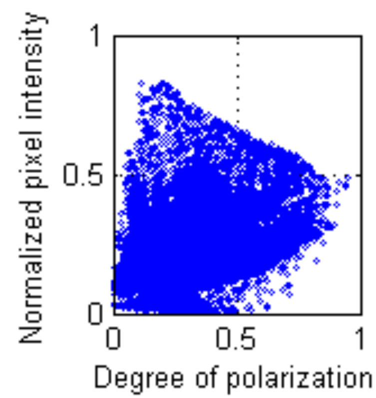
Reflectance Distributions

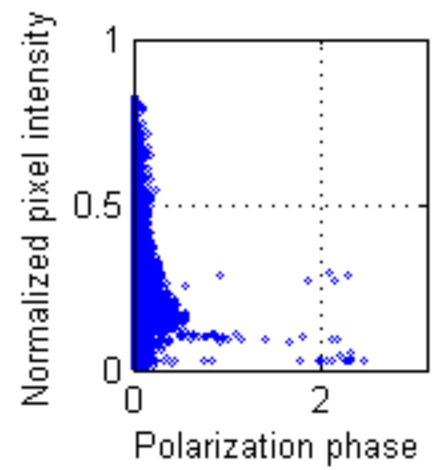
Natural leaves





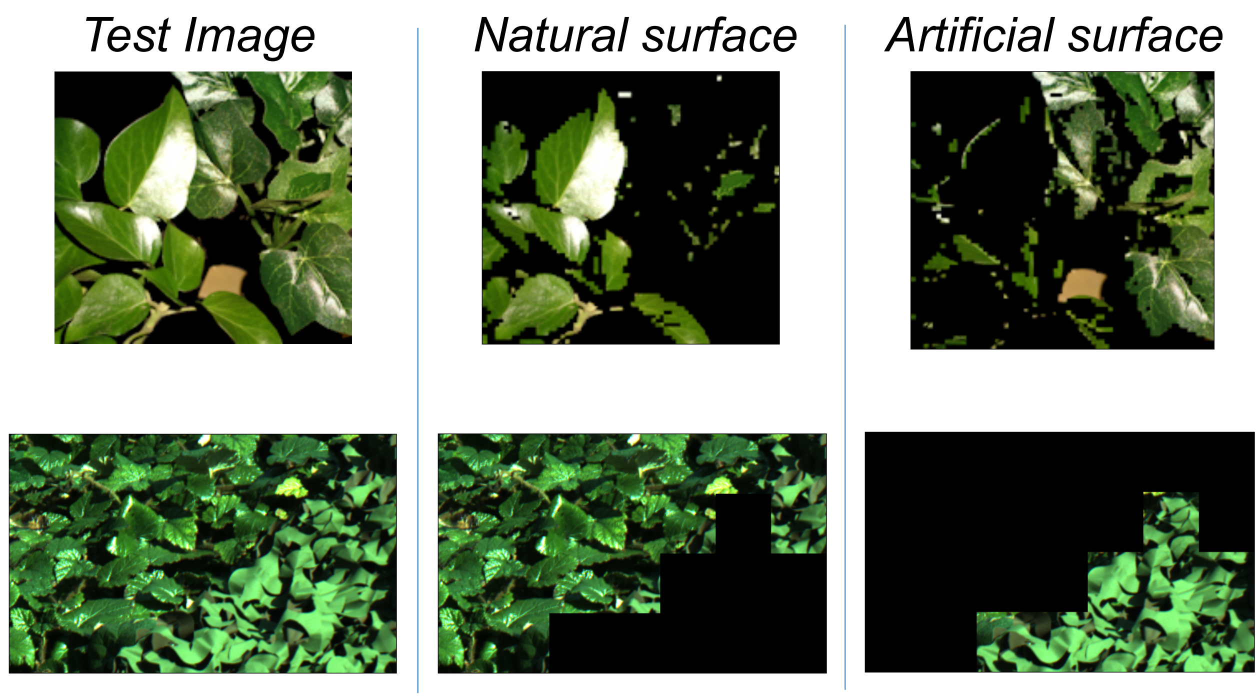
Plastic leaves





Reflectometry

- Distinguishing natural and man-made (camouflage) surfaces

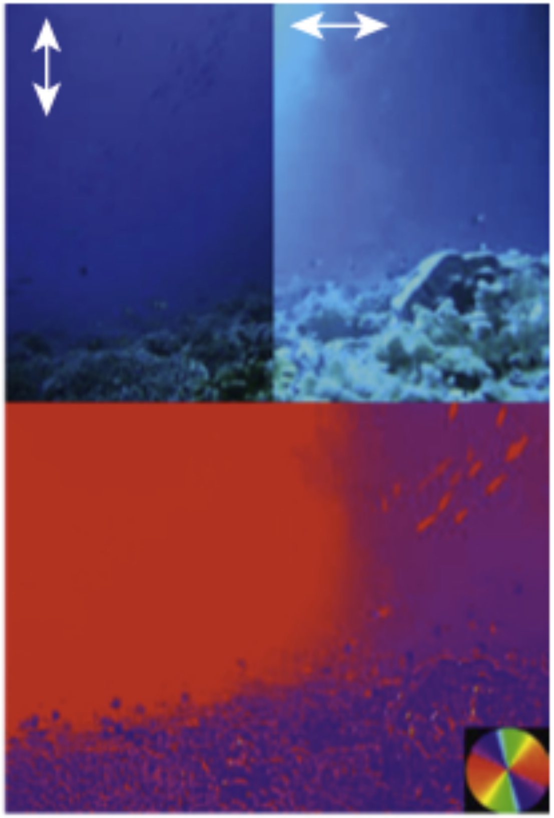


Polarization vision in nature

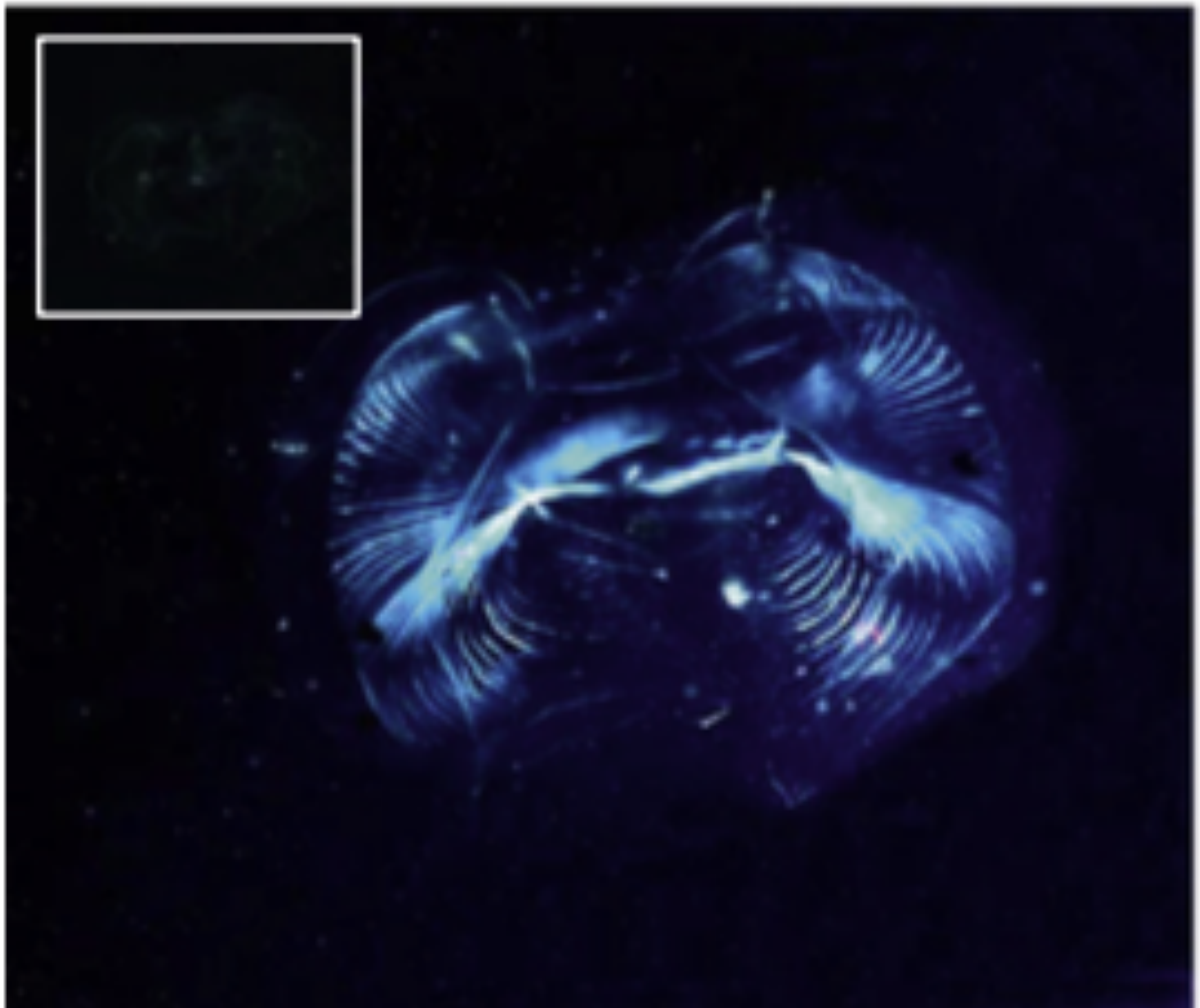
- Evidence that both insects and aquatic creatures (e.g. shrimps, crabs, cuttlefish) exhibit polarization vision, with eyes having up to 16 channels.

- Most animals lack optical polarizing filters. Instead, their individual photoreceptors are sensitive to polarized light.

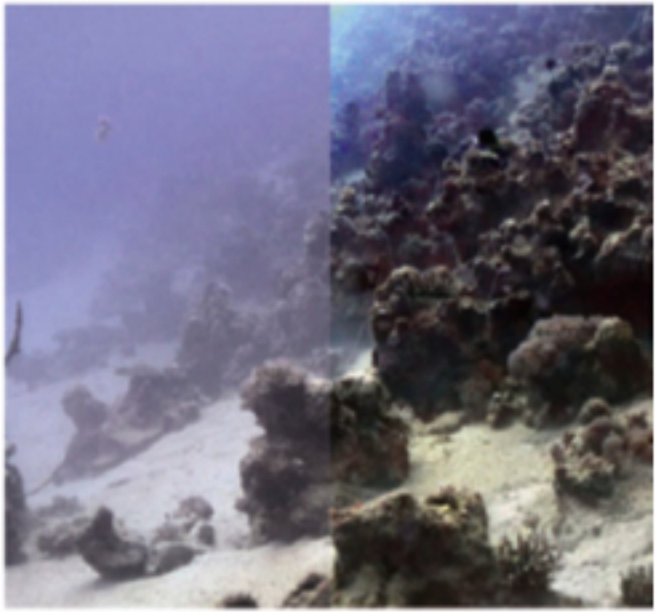
Underwater vision: phase of polarization reveals structure of scene



Degree of polarization: reveals translucent objects (via variations in refractive index).



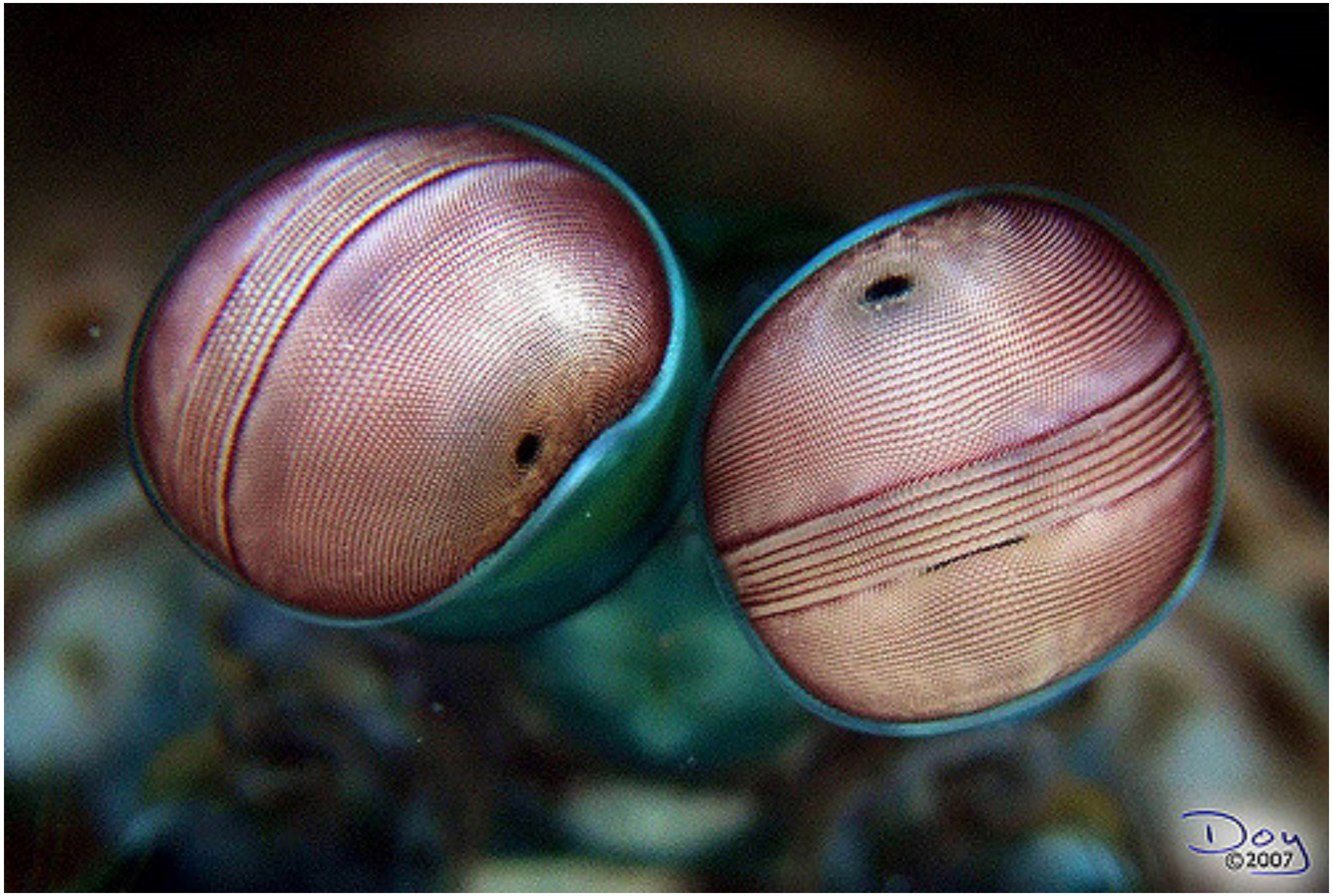
Can be used to dehaze scenes.



An end user

The mantis shrimp has 12-16 visual channels which it uses for polarization vision.

Mantis shrimp eyes



Highly mobile with multiple colour and polarization channels.

Publications relevant to work

- Use diffuse polarization measurements to estimate surface orientation (IEEE TIP 06).
- Extend to multiple views to resolve ambiguities and extend object coverage (PAMI 07).
- Use method to estimate BRDF’s for surfaces composed of different materials (CVIU 08).
- Spectro-polarimetry (fixed view, multiple wavelengths) IJCV 2013.
- Direct height estimation from multiple polarization images (ICCV 2017).

Polarization Vision and Applications

Shape from polarization

Polarization vision — Wolff and Boult TPAMI ’91

Shape recovery — Miyazaki et al ICCV ’03, TPAMI ’04

Drbohlav and Šára SPIE ’99

Rahmann and Canterakis CVPR ’01

Polarization Vision and Applications

Other early uses of polarization

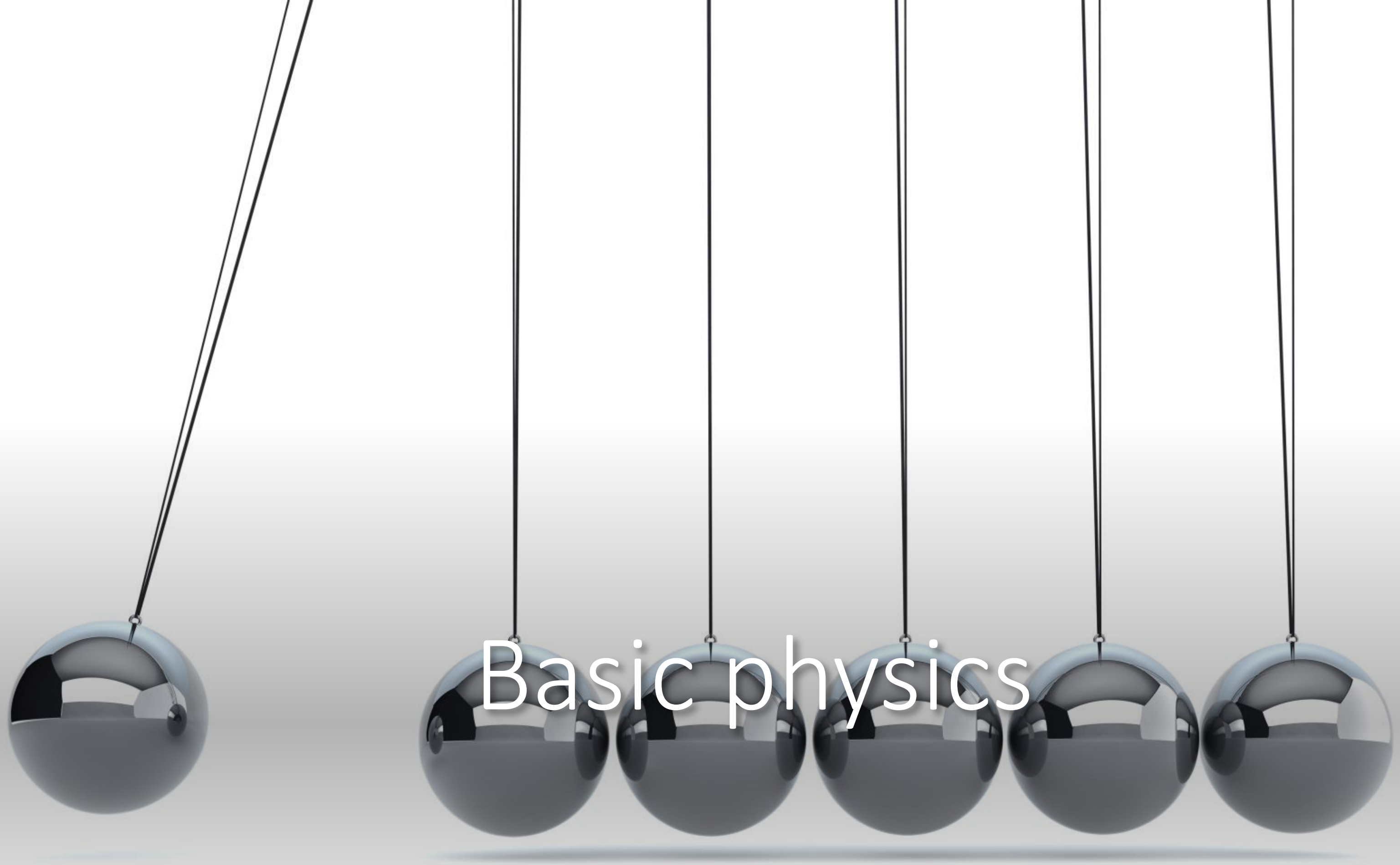
Reflection components	Umeyama TPAMI ’04
Photometric stereo	Drbohlav and Šára ICCV ’01
Range scanning	Clark, Trucco and Wolff IVC ’97
Marine vision	Schechner and Karpel CVPR ’03
BRDF estimation	Shibata et al SPIE ’05
Segmentation / classification	Chen and Wolff IJCV ’98

Polarization Vision and Applications

Revival on interest in graphics

Graphics	Ghosh SIGGRAPH Asia 2012
Cues for coarse depth maps	Kadamba ICCV 2015
Planar Surface Polarimetry	Riviere SIGGRAPH 2017
Polarimetry in the wild	Gosh SIGRAPH Asia 2017
Transparent object reconstruction.	Wu ACM Tgraphics 2018
Polarimetric reflectance estimation	Baek ACM TGraphics 2020
Skin Reflectance	Riviere ACM Tgraphics 2020
Holographic VR	Maimone ACM TGraphics 2019

Basic physics

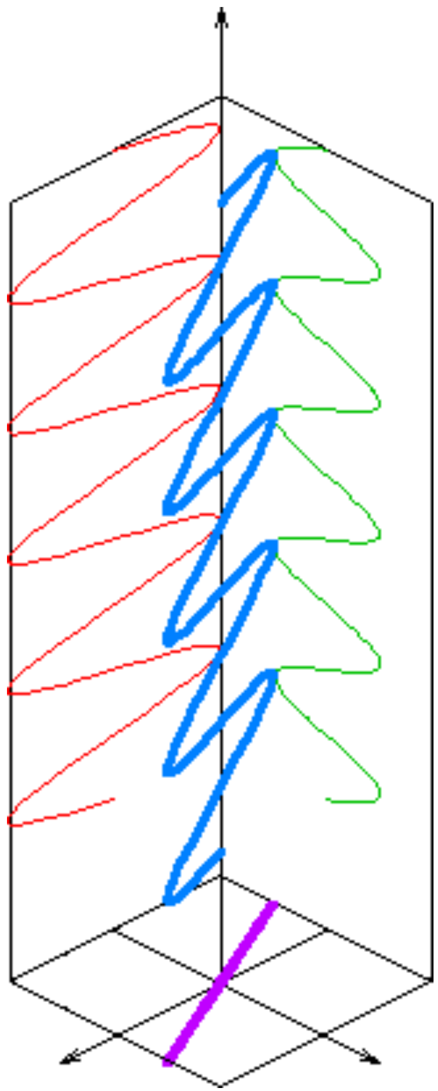


Polarization of Light

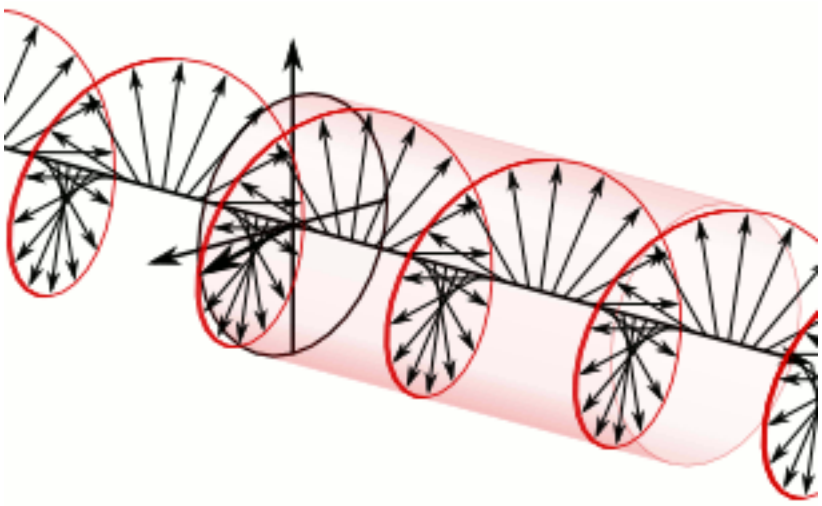
- Linear polarization: confinement of the electric field vector or magnetic field vector to a given plane along the direction of propagation.

- Circular polarization: the electric field of the wave has a constant magnitude but its direction rotates with time at a steady rate in a plane perpendicular to the direction of the wave.

Linear polarization



Circular polarization



Origins of polarization

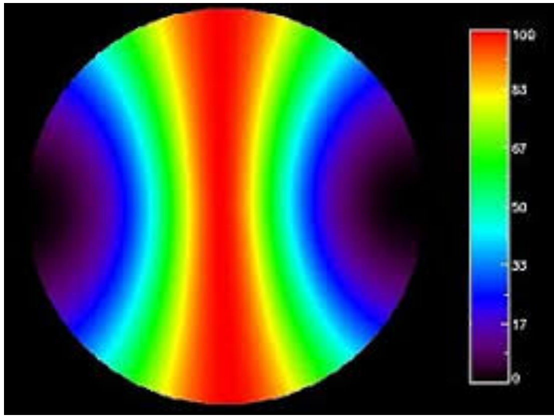
- When scattered or passed through a dichroic medium, light in different polarization states experience different absorption.

- Results in spontaneous polarization on scattering.

Origins of polarization

- Sunlight is unpolarized (when it leaves the sun).

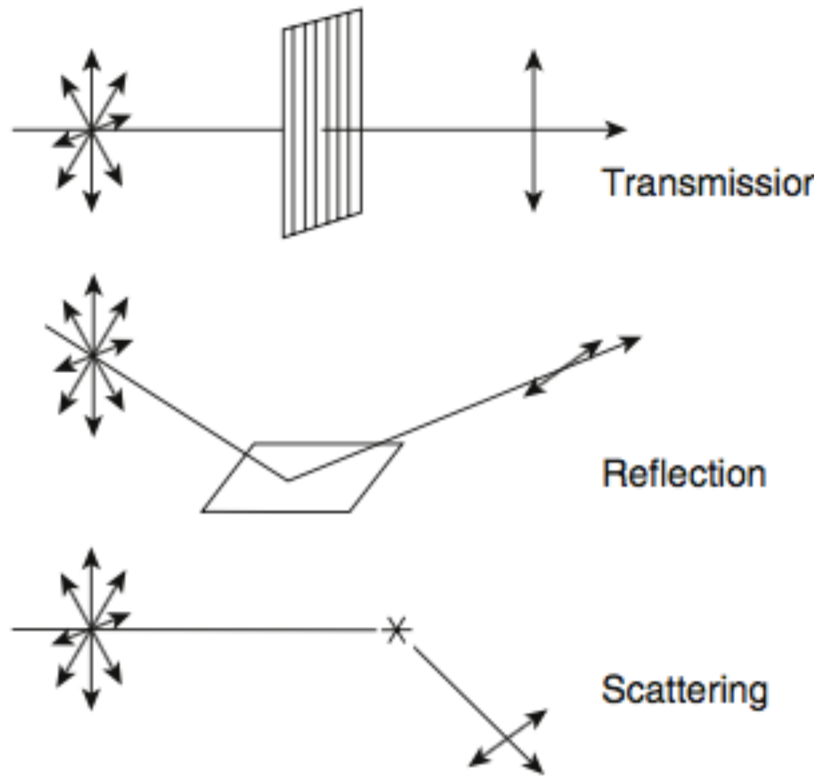
- Atmospheric Rayleigh scattering from air molecules, water, dust, and aerosols causes the sky's light to have a defined polarization pattern. The same elastic scattering processes cause the sky to be blue.



- Pattern of polarization depends on angle. Some insects use this for navigation.

Origins of polarization

Occurs when light is reflected from boundary between layers of different refractive index


Transmissior
Reflection
Scattering

Physics of polarization

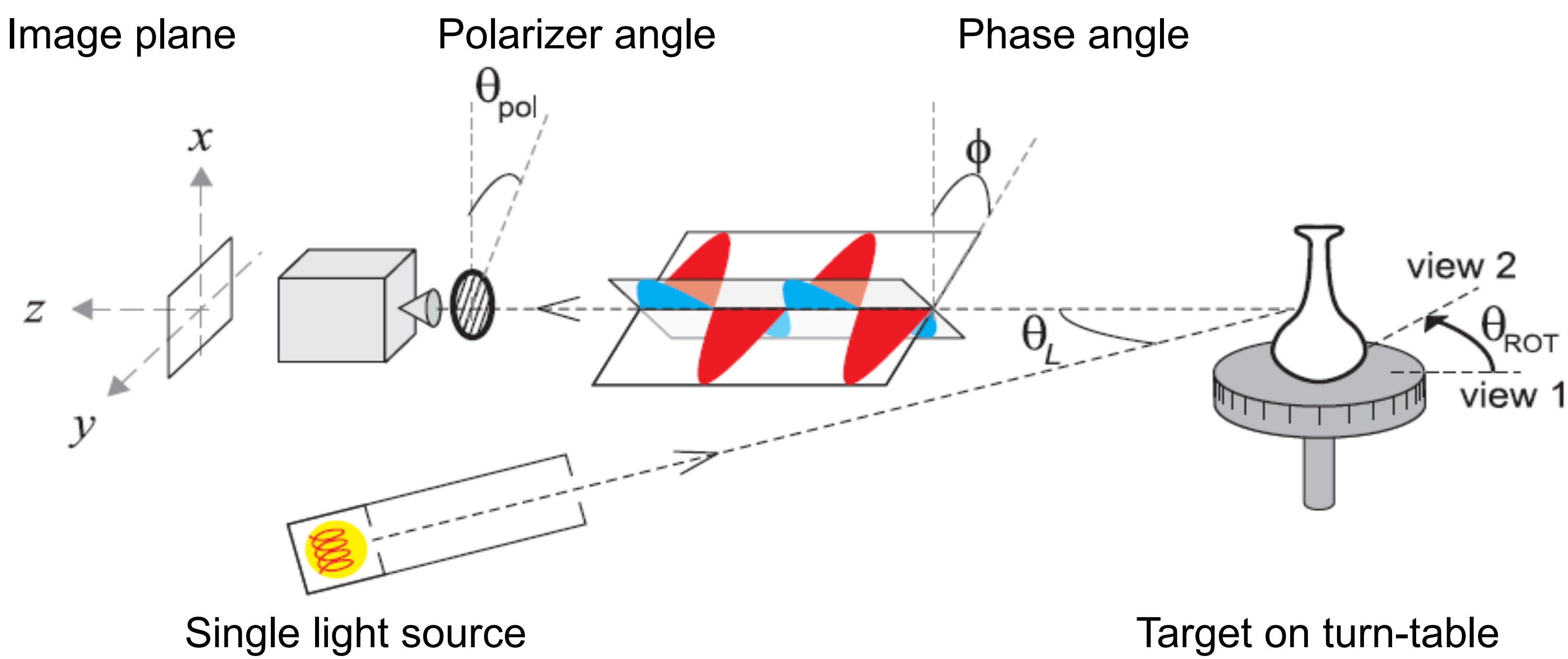
- Degree of polarization depends on angle of incidence of scattered light.

- Also determined by refractive index of scattering surface.

- Polarsation can hence be used to determine surface shape and surface composition.

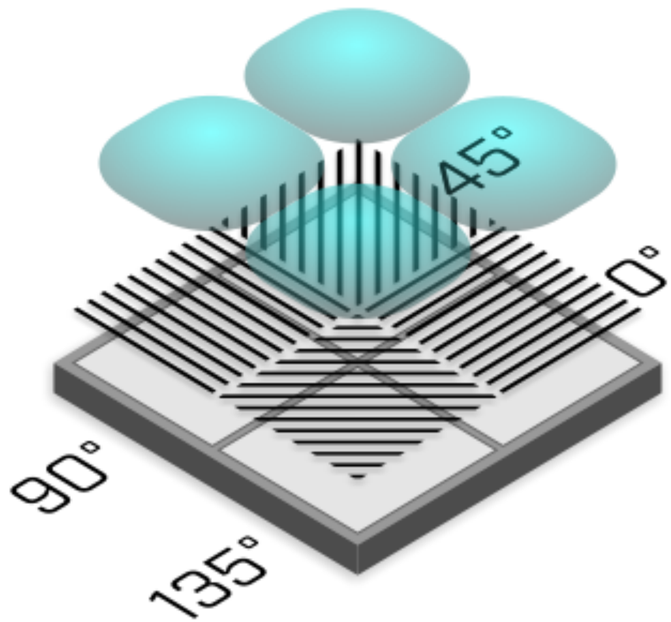
Polarization Camera

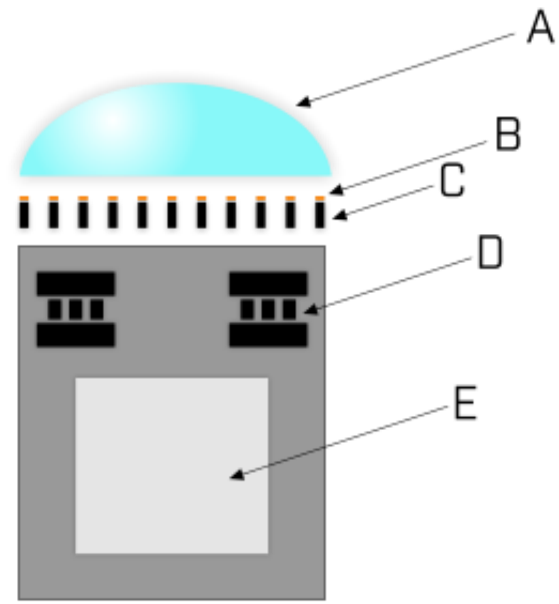
1. Acquire polarization images with light souce, camera and object fixed while polarizer rotates



Note: incident light is unpolarized.

Commercial polarization camera





Polarization filters with different orientations arranged in 2x2 pixel blocks. Some cameras allow readout of Stokes vector.

Polarisation representations

- Transmission radiance sinusoid – mean-intensity, degree of polarization, phase.

- Stokes vector – distinguishes different states of linear and circular polarization, plus unpolarized case. Spherical representation of polarization parameters. Related to components of electric field vector.

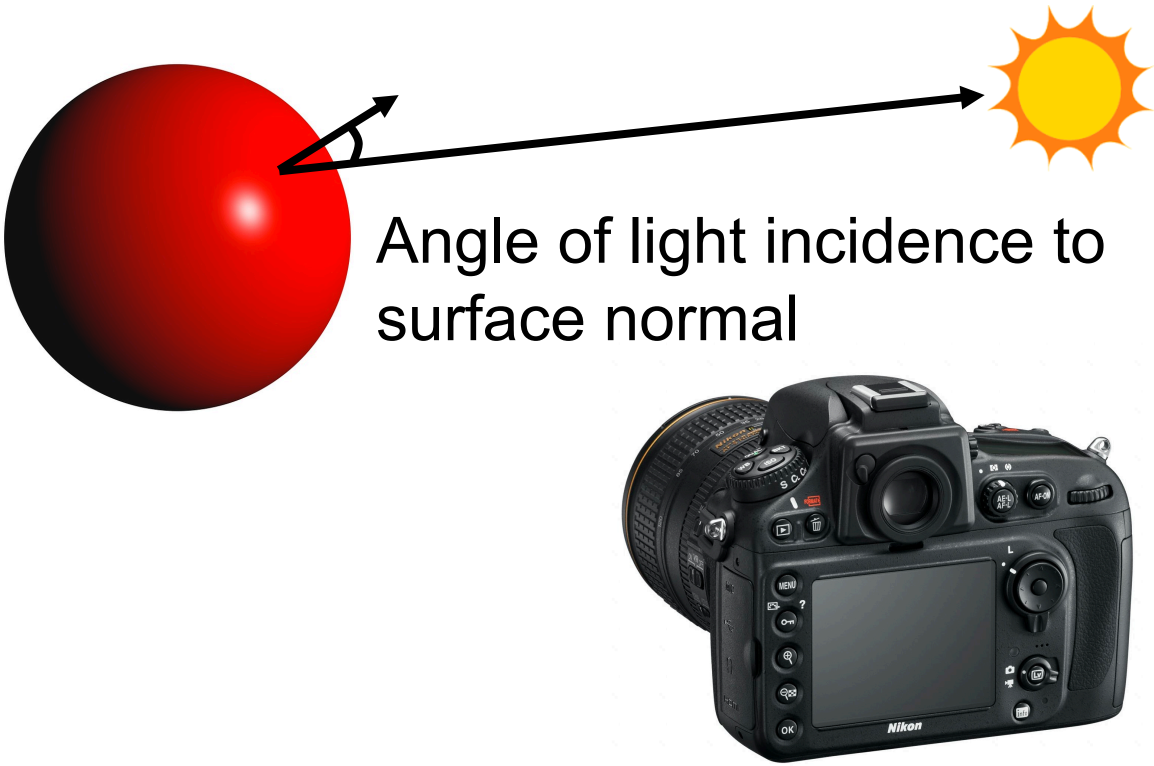
- Jones matrices - model effects of optical medium on fully polarized light represented using Stokes vectors.

- Mueller matrices – additionally model effects of optical medium on Stokes vectors for randomly or partially polarized or incoherent light.

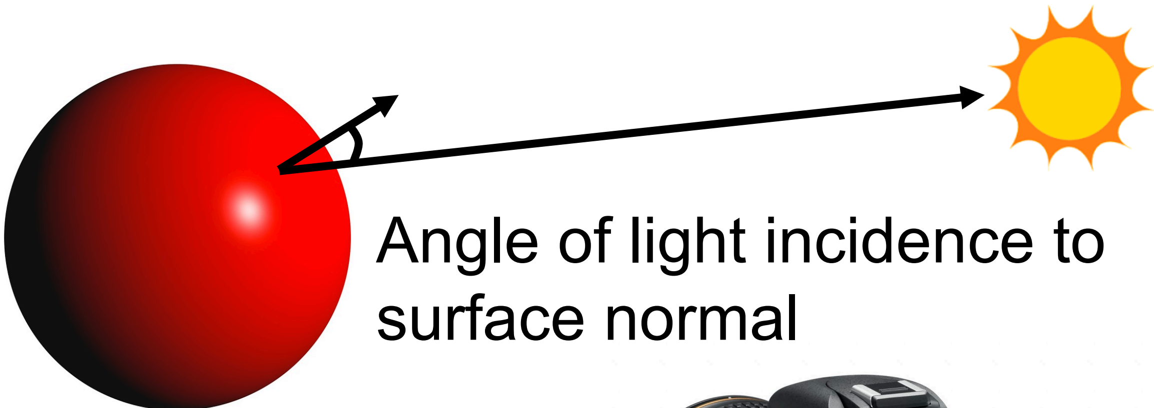
Settings

- Single polarization image: surface normals of a constant albedo uniform refractive index surface.
- Single polarization and brightness images: normals, albedo and variations in refractive index. (Polarimetric stereo)
- Multiple polarization and brightness images: height (directly), albedo, refractive index.
- Multiple polarization images at different wavelengths and fixed direction: surface normals, albedo, variations of refractive index. (Spectro-polarimetry)

Shape shading and polarization compared

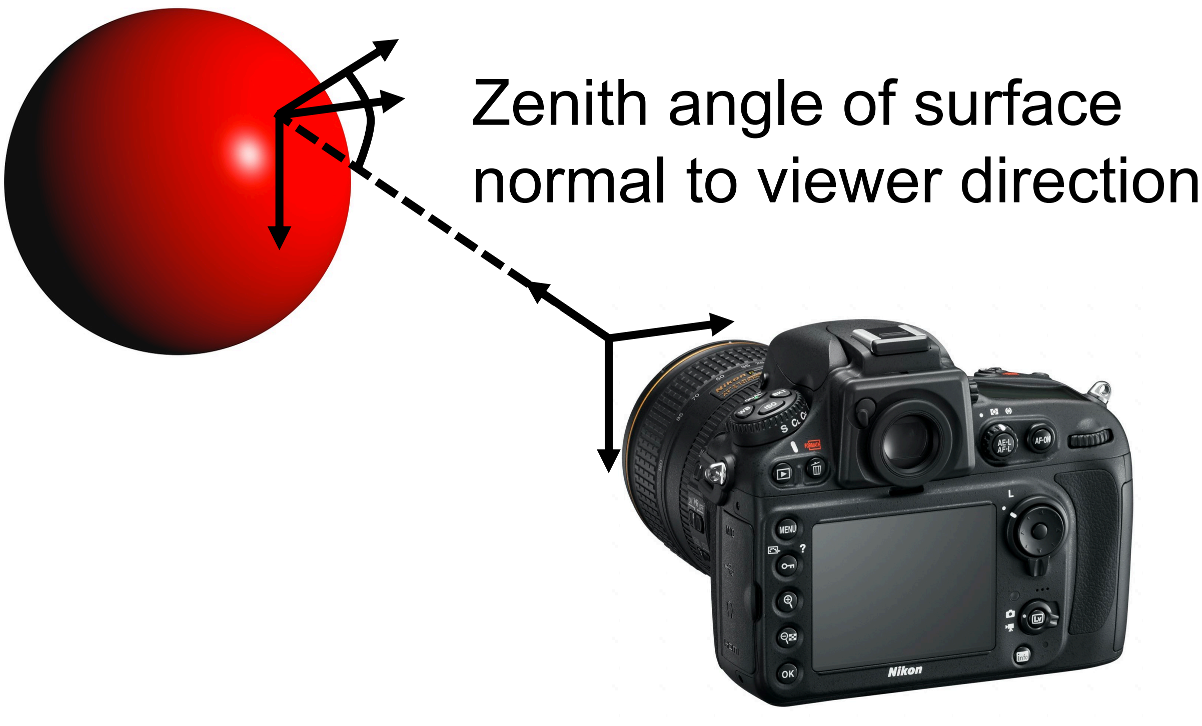


Shape shading and polarization compared

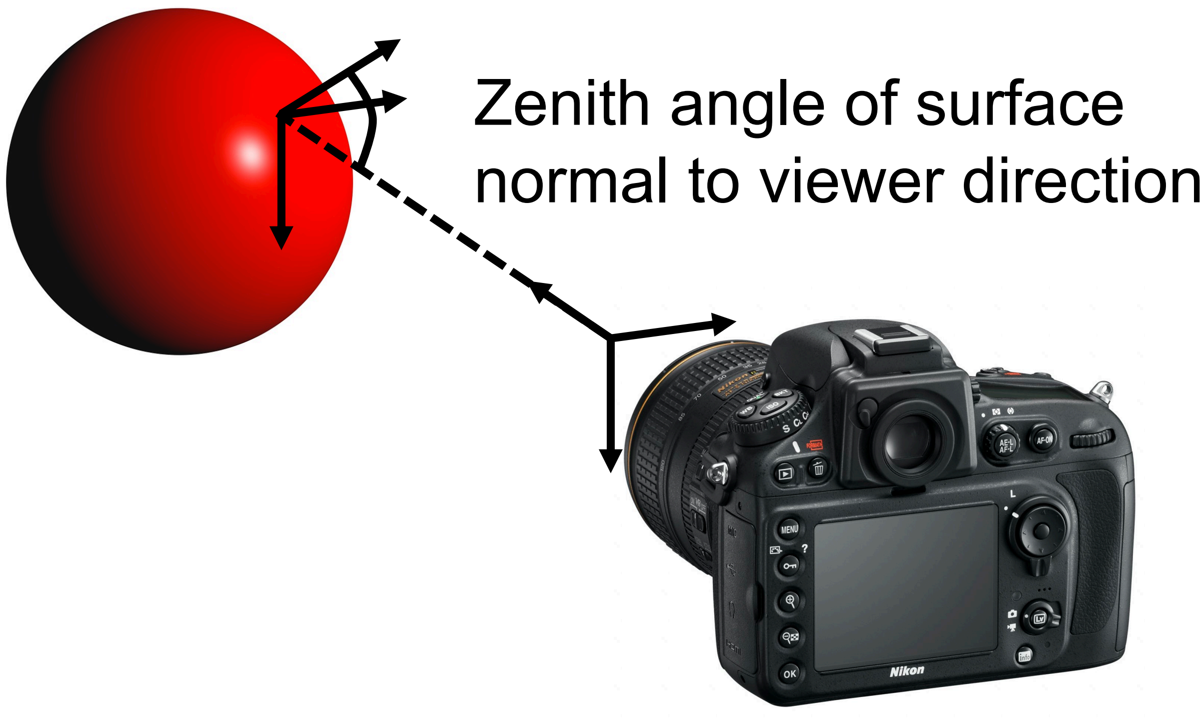


Brightness measurement determines direction between surface normal and light source direction.

Shape shading and polarization compared



Shape shading and polarization compared



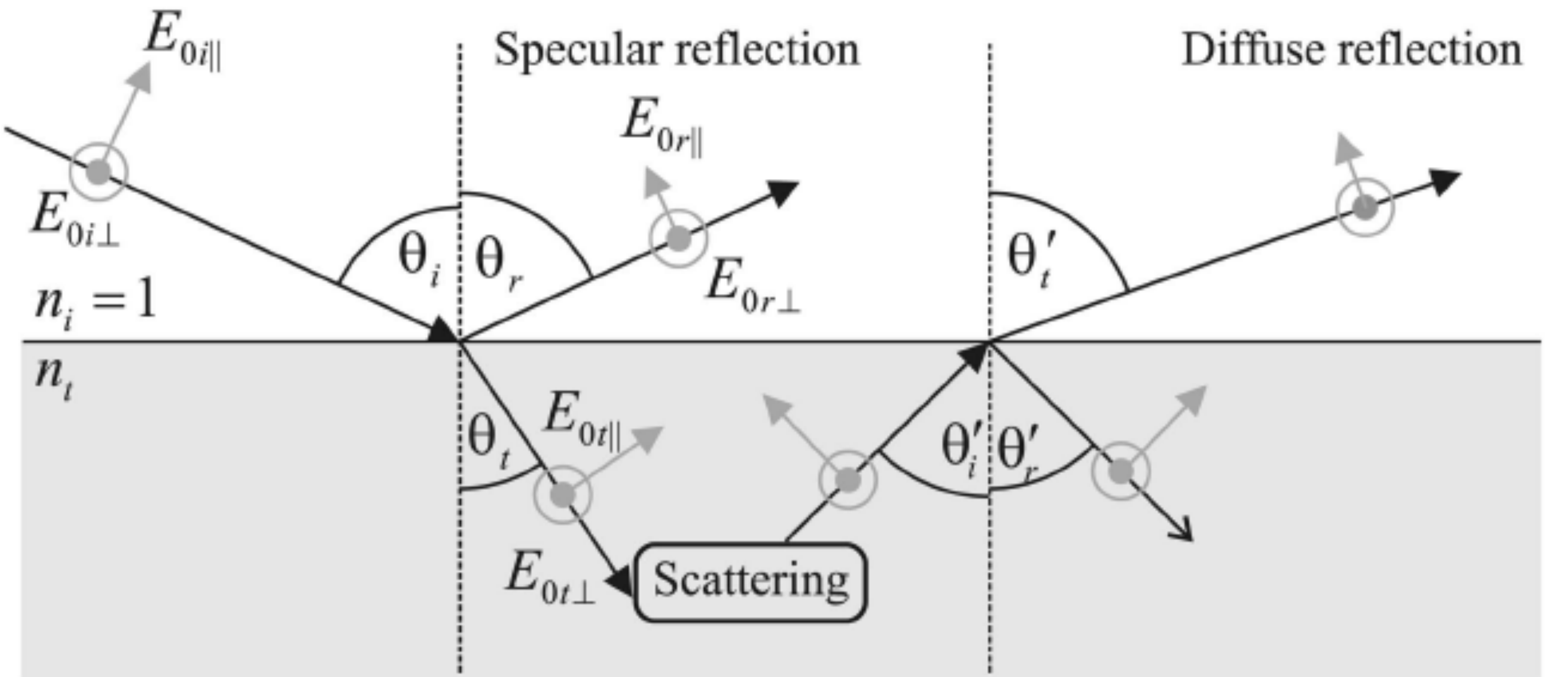
Degree of polarization determines zenith angle between surface normal and viewer direction.
Phase determines azimuth angle about normal up to an ambiguity of 180 degrees.

Theoretical background

Fresnel theory

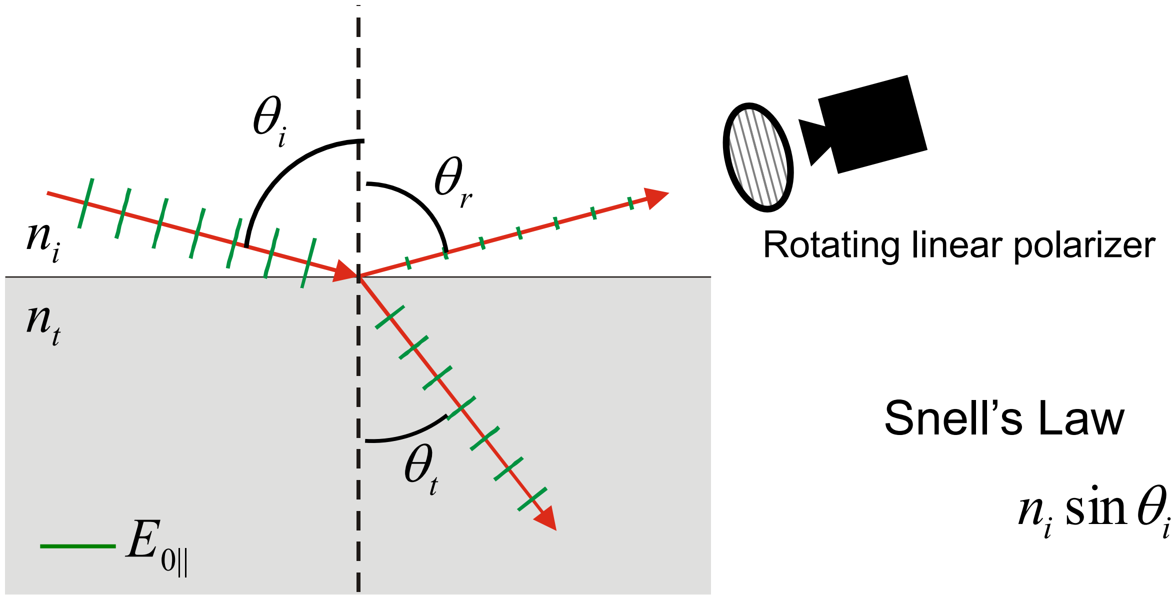
Augustin-Jean Fresnel (1788-1827)

Basic concepts



Theory: Physical Origins of Polarization by Reflection

Specular reflection



Snell's Law

$$n_i \sin\theta_i = n_t \sin\theta_t$$

Fresnel Coefficients

Perp to incidence plane

$$r_\perp \equiv \frac{E_{0r\perp}}{E_{0i\perp}} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \qquad R_\perp = r_\perp^{\,2}$$

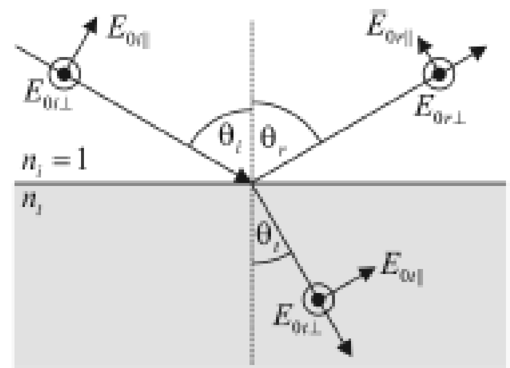
Parallel to incidence plane

$$r_\parallel \equiv \frac{E_{0r\parallel}}{E_{0i\parallel}} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t} \qquad R_\parallel = r_\parallel^{\,2}$$

Polarization for specular reflection

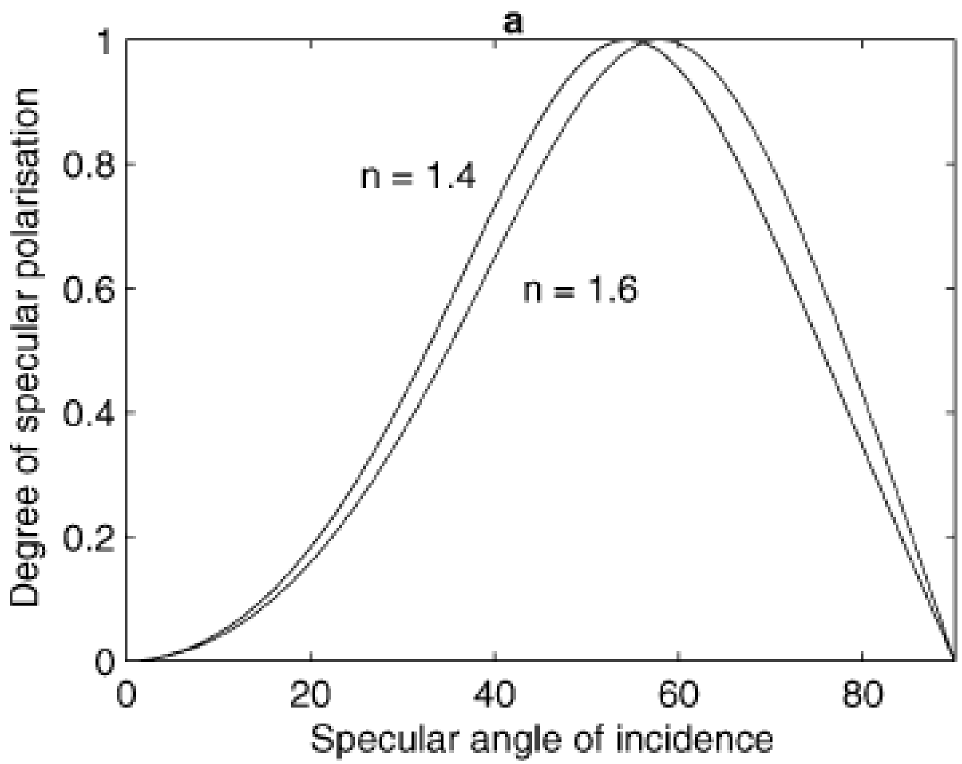
Defined in terms of reflection coefficients
for different planes of polarization.

$$\rho_s = \frac{R_\perp(n,\theta_i) - R_\parallel(n,\theta_i)}{R_\perp(n,\theta_i) + R_\parallel(n,\theta_i)}$$



$$\rho_s = \frac{2\sin^2\theta\cos\theta\sqrt{n^2 - \sin^2\theta}}{n^2 - \sin^2\theta - n^2\sin^2\theta + 2\sin^2\theta}$$

Specular polarization versus incidence angle



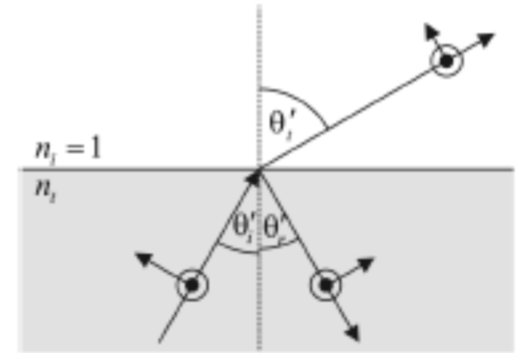
Because of Brewster angle, for a measured polarization there are two possible incidence angles.

Brewster angle increases with refractive index.

Some representative refractive indices			
Material	λ (nm)	*n*	Ref.
Vacuum		1 (by definition)	
Air at STP		1.000277	
Gases at 0 °C and 1 atm			
Air	589.29	1.000293	[1]
Carbon dioxide	589.29	1.001	[2][3][4]
Helium	589.29	1.000036	[1]
Hydrogen	589.29	1.000132	[1]
Liquids at 20 °C			
Arsenic trisulfide and sulfur in methylene iodide		1.9	[5]
Benzene	589.29	1.501	[1]
Carbon disulfide	589.29	1.628	[1]
Carbon tetrachloride	589.29	1.461	[1]
Ethanol (ethyl alcohol)	589.29	1.361	[1]
Water	589.29	1.330	[1]
10% Glucose solution in water	589.29	1.3477	[6]
20% Glucose solution in water	589.29	1.3635	[6]
60% Glucose solution in water	589.29	1.4394	[6]
Solids at room temperature			
Silicon carbide (Moissanite; 6H form)	589.29	2.65	[7]
Titanium dioxide (rutile phase)	589.29	2.614	[8][9]
Diamond	589.29	2.417	[1]
Strontium titanate	589.29	2.41	[10]
Amber	589.29	1.55	[1]
Sodium chloride	589.29	1.544	[11]
Fused silica (a pure form of glass, also called fused quartz)	589.29	1.458	[1][12]

Polarization for diffuse reflection

Defined in terms of transmission rather than reflection: T=1-R

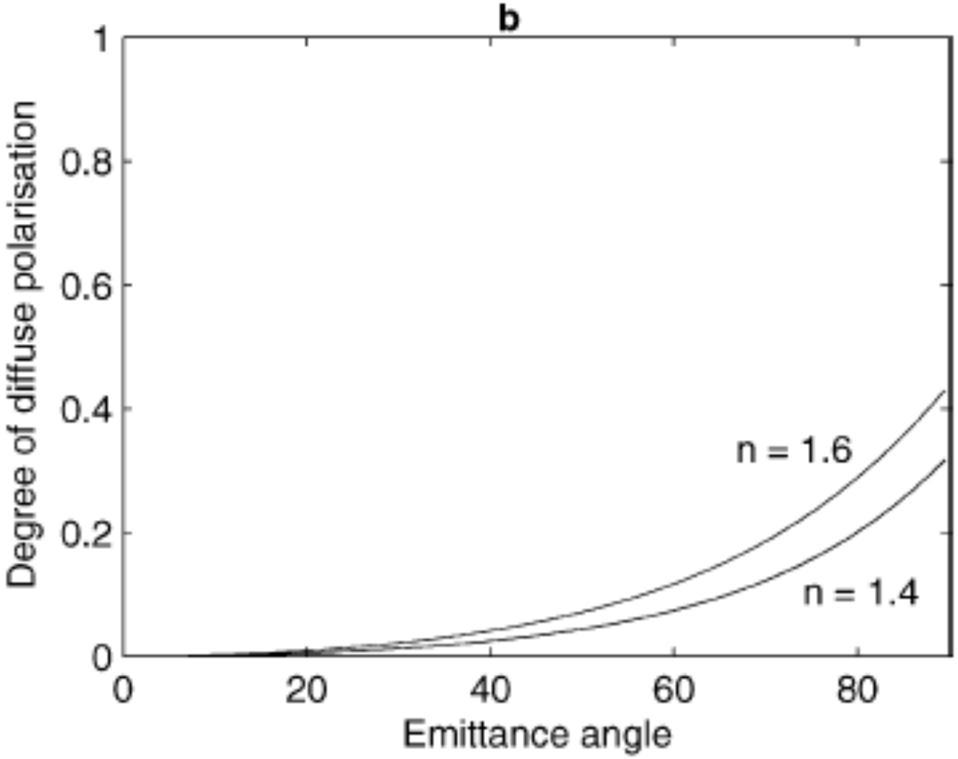


$$\rho_d = \frac{T_{\parallel}(1/n, \theta_i^{'}) - T_{\perp}(1/n, \theta_i^{'})}{T_{\parallel}(1/n, \theta_i^{'}) + T_{\perp}(1/n, \theta_i^{'})} = \frac{R_{\perp}(1/n, \theta_i^{'}) - R_{\parallel}(1/n, \theta_i^{'})}{2 - R_{\perp}(1/n, \theta_i^{'}) - R_{\parallel}(1/n, \theta_i^{'})}$$

Use Snell's law to re-express in terms of emittance angle

$$\rho_d = \frac{(n - 1/n^2)\sin^2\theta}{2 + 2n^2 - (n - 1/n^2)\sin^2\theta + 4\cos\theta\sqrt{n^2 - \sin^2\theta}}$$

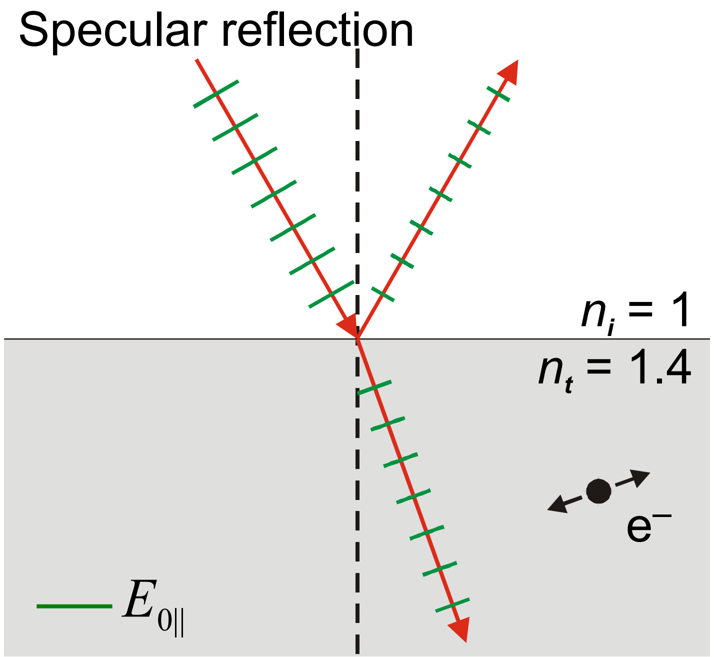
Diffuse polarization versus emmittance angle



No Brewster angle for diffuse polarization. Single measurement of polarization gives a single emittance angle.

Polarization stronger the larger refractive index.

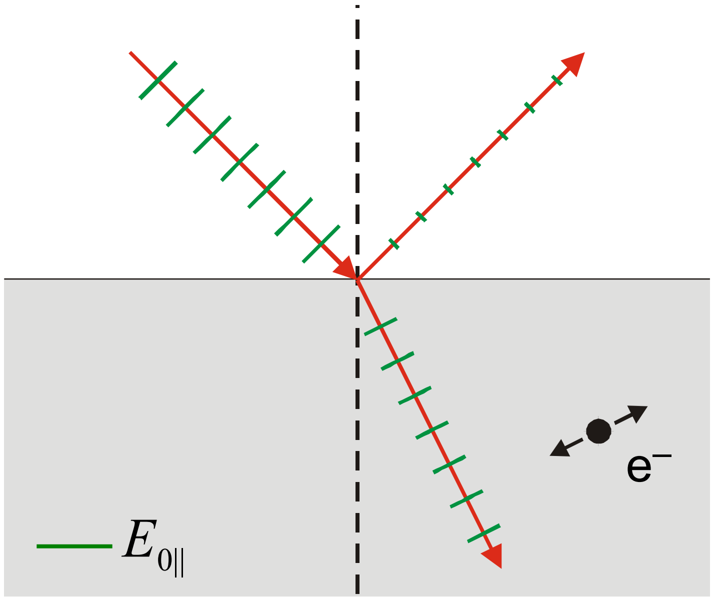
Theory: Polarization by Reflection



Low polarization

Theory: Polarization by Reflection

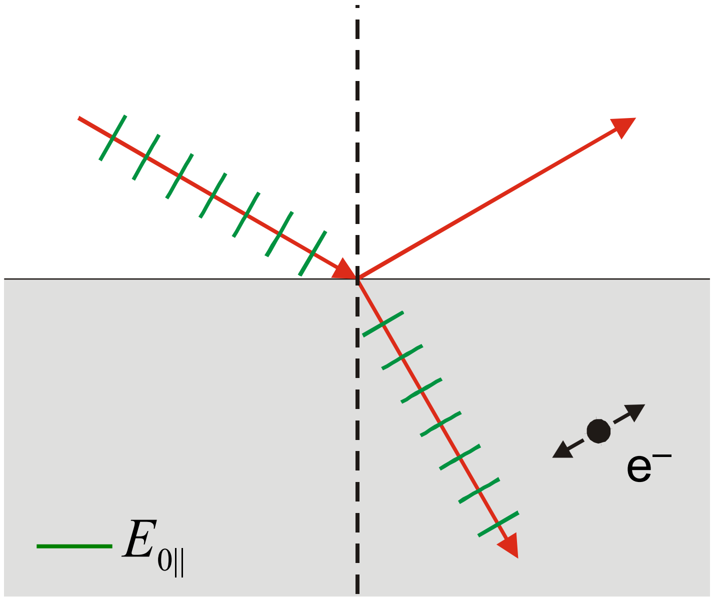
Specular reflection



High polarization

Theory: Polarization by Reflection

Specular reflection

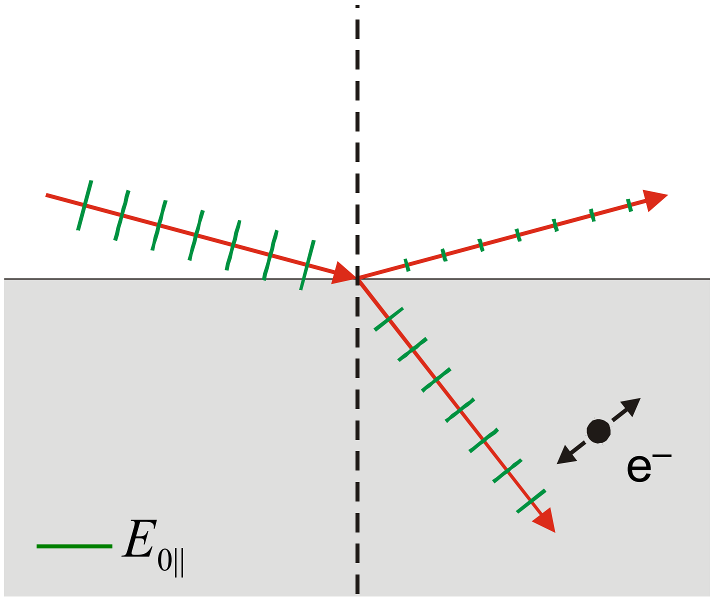


Complete polarization:
Brewster Angle

Reflected light totally extinguished by rotating polarizer.

Theory: Polarization by Reflection

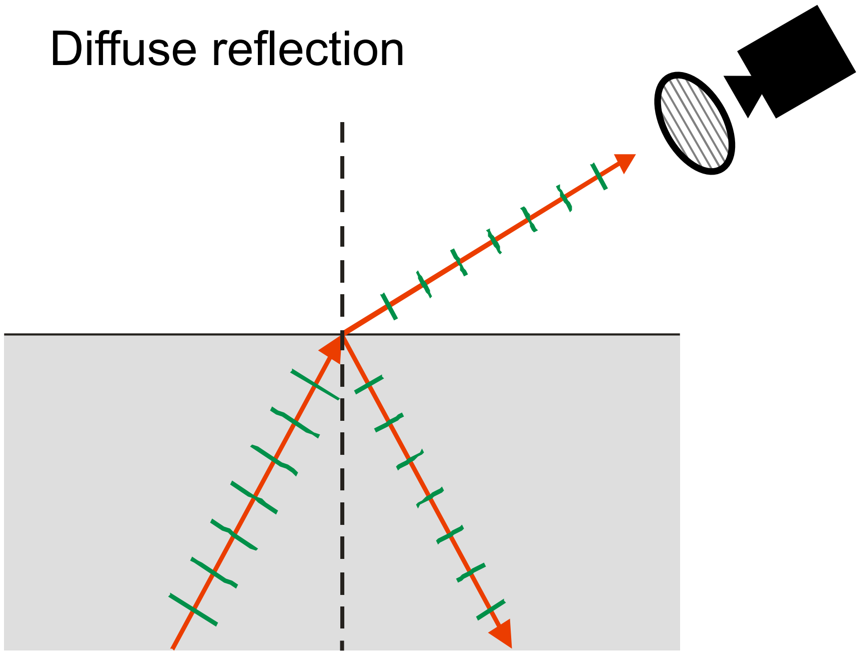
Specular reflection



High polarization

Theory: Shape from Diffuse Polarization

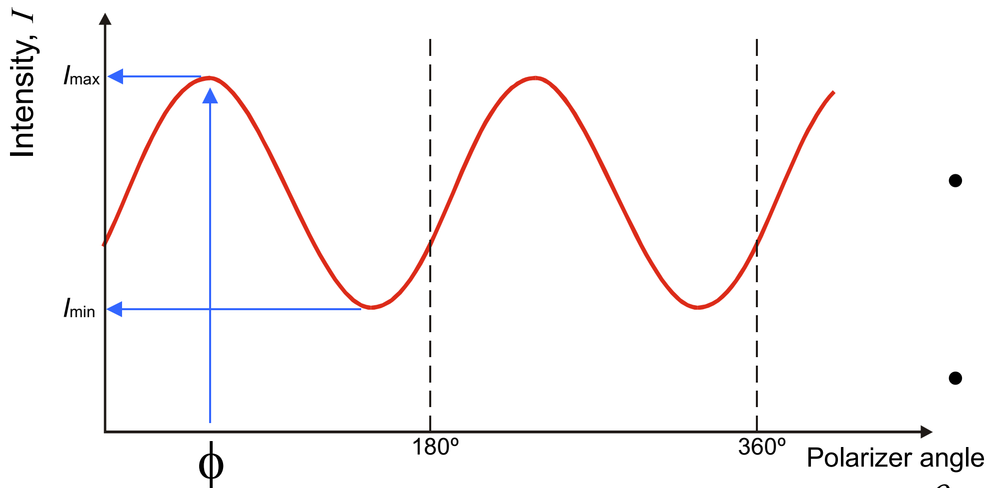
Diffuse reflection



Diffuse component emerges after subsurface scattering

Polarization measurements

Measured intensity variation



Degree of polarization

$$\rho = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

- Rotate polarizer and measure brightness at each pixel with camera, light source and object fixed.
- Brightness varies sinusoidally with polarizer angle.
- Fit to recover maximum and minimum brightness together with phase of sinusoid at each pixel.
- Compute polarization from max and min brightneses.

Polarization Image

- Composed of brightness, phase and polarization

Brightness

Phase

Polarization

Single view shape reconstruction

Use estimates of zenith and azimuth angles to recover surface normals. Reconstruct object shape using surface integration.

Single View Shape Recovery: Overview

1. Acquire polarization images

2. Estimate zenith angles from degree of polarization

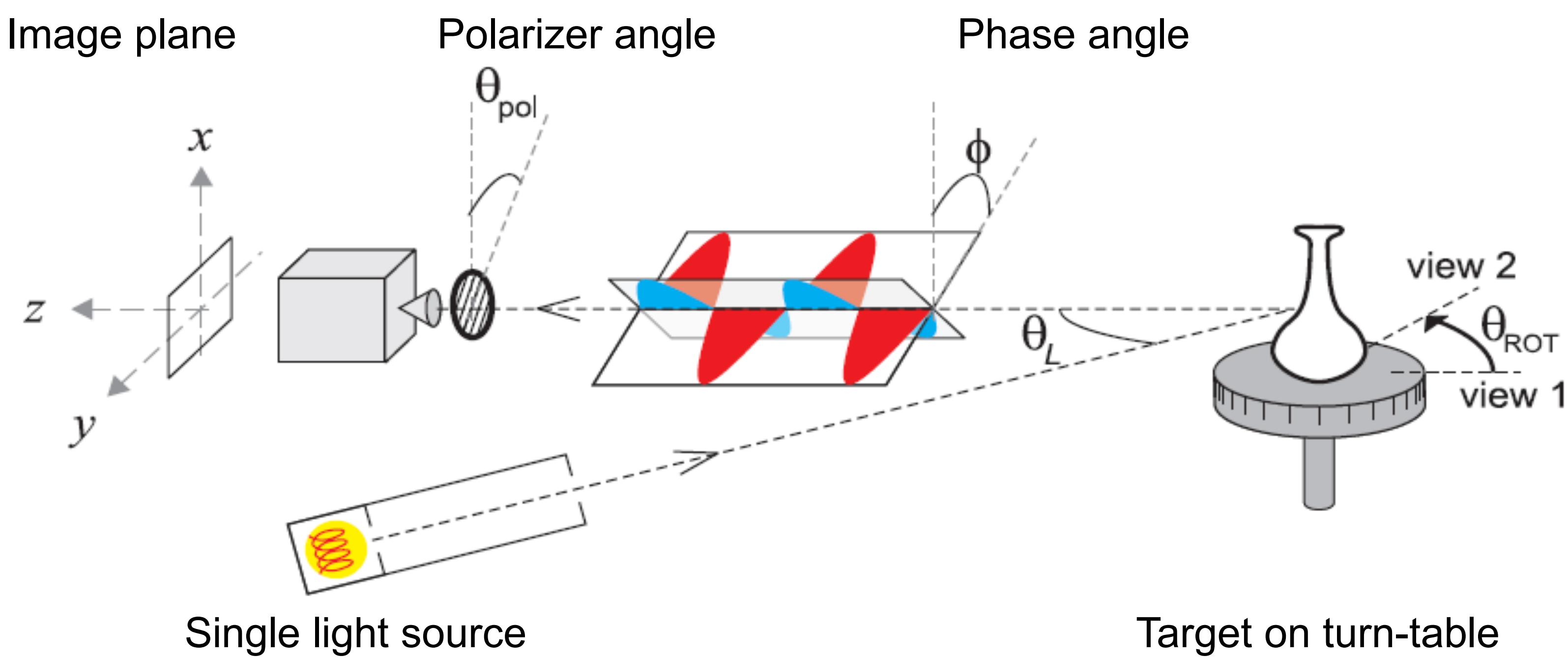
3. Ambiguously estimate azimuth angles

4. Disambiguate azimuth angles

5. Integrate normals using Frankot-Chellappa method [TPAMI ’88]

Single View Vision: Apparatus

1. Acquire polarization images



Single View Vision: Method

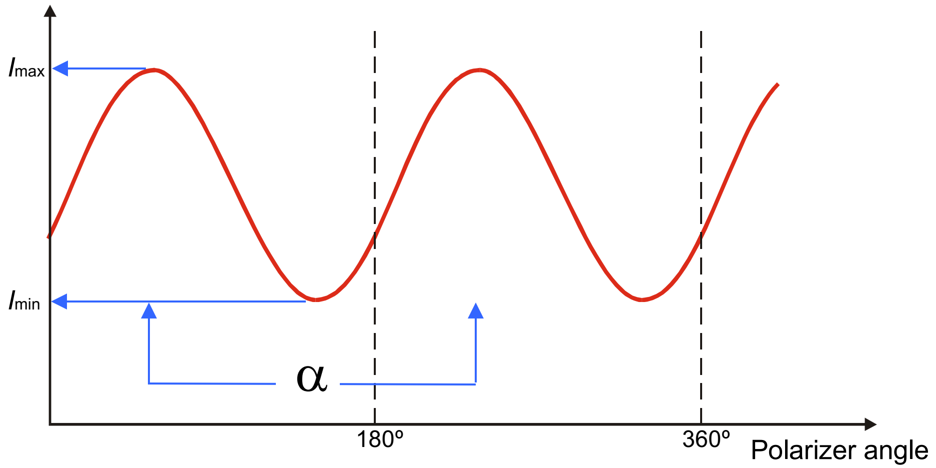
A: Estimate zenith angles from degree of polarization

$$\rho_d = \frac{(n - 1/n)^2 \sin^2\theta}{2 + 2n^2 - (n + 1/n)^2 \sin^2\theta + 4\cos\theta\sqrt{n^2 - \sin^2\theta}}$$

Single real solution since polarization increases monotonically with emittance angle. i.e. there is no Brewster angle for diffuse polarization.

Single View Vision: Method

B: Ambiguously estimate azimuth angles from measured phase



$$\alpha = \begin{cases} \phi \\ \phi + \pi \end{cases}$$

Azimuth angle of surface normal is orientation of projection of surface normal onto image plane. Light is reflected most efficiently when polarized parallel to plane containing surface normal and reflected ray. Hence, phase of polarized light is equivalent to azimuth angle of surface normal up to an ambiguity of 180 degrees.

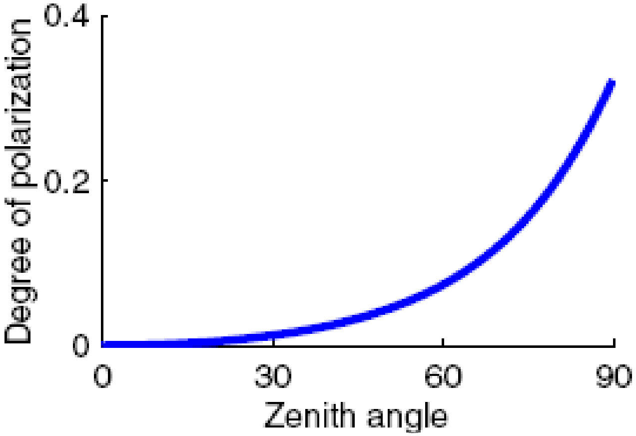
Disambiguation

- On boundary select azimuth angle that is closest to that of occluding boundary normal.

- Propagate constraint as brush-fire into interior of object.

- For small zenith angles allow aburpt changes of azimuth angle.

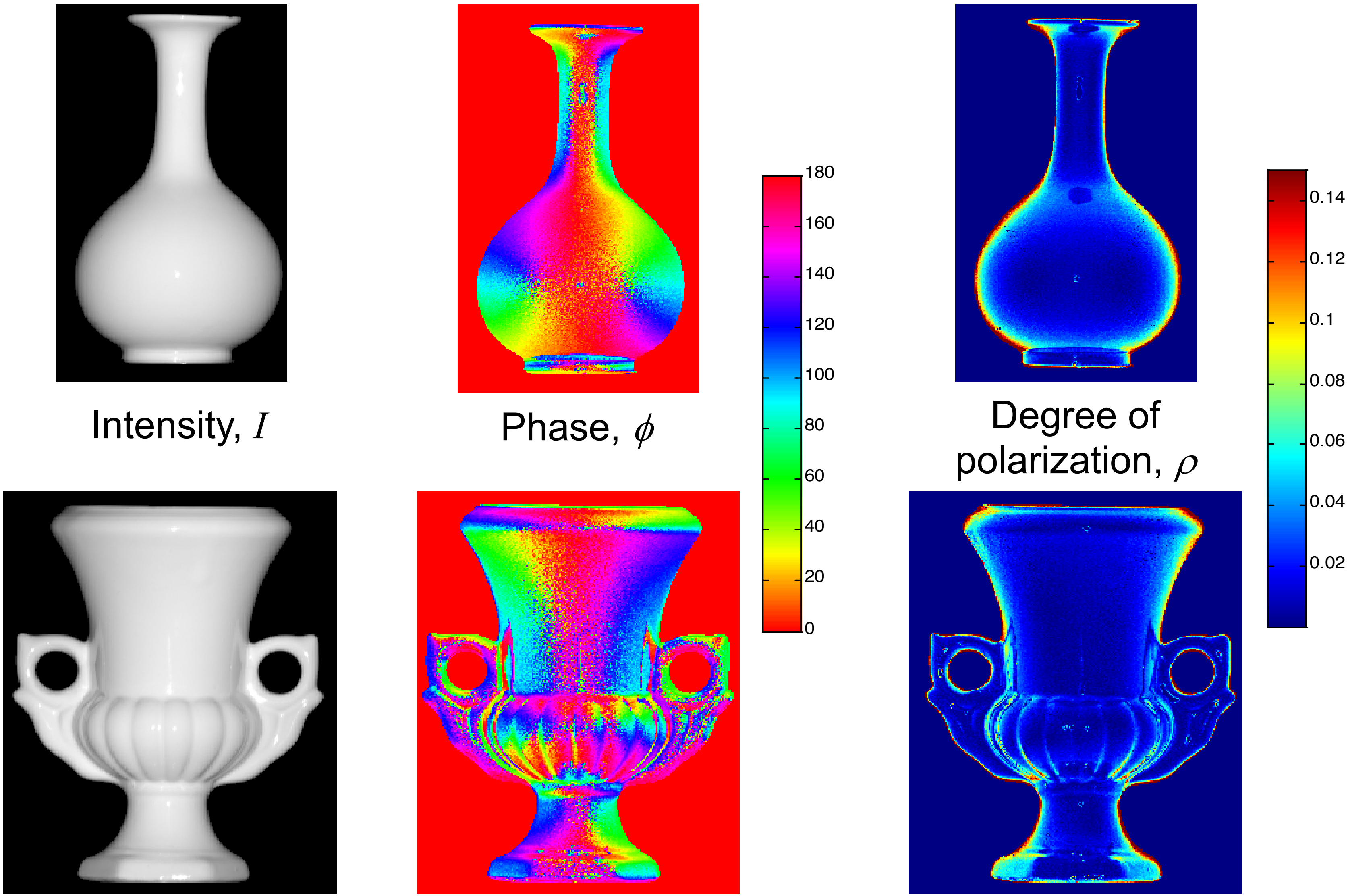
- Diffuse polarization solved for surface normal zenith angle (unambiguously)



- Analogous to shape-from-shading, where Lambert's law allows zenith angle to be determined from measured image brightness

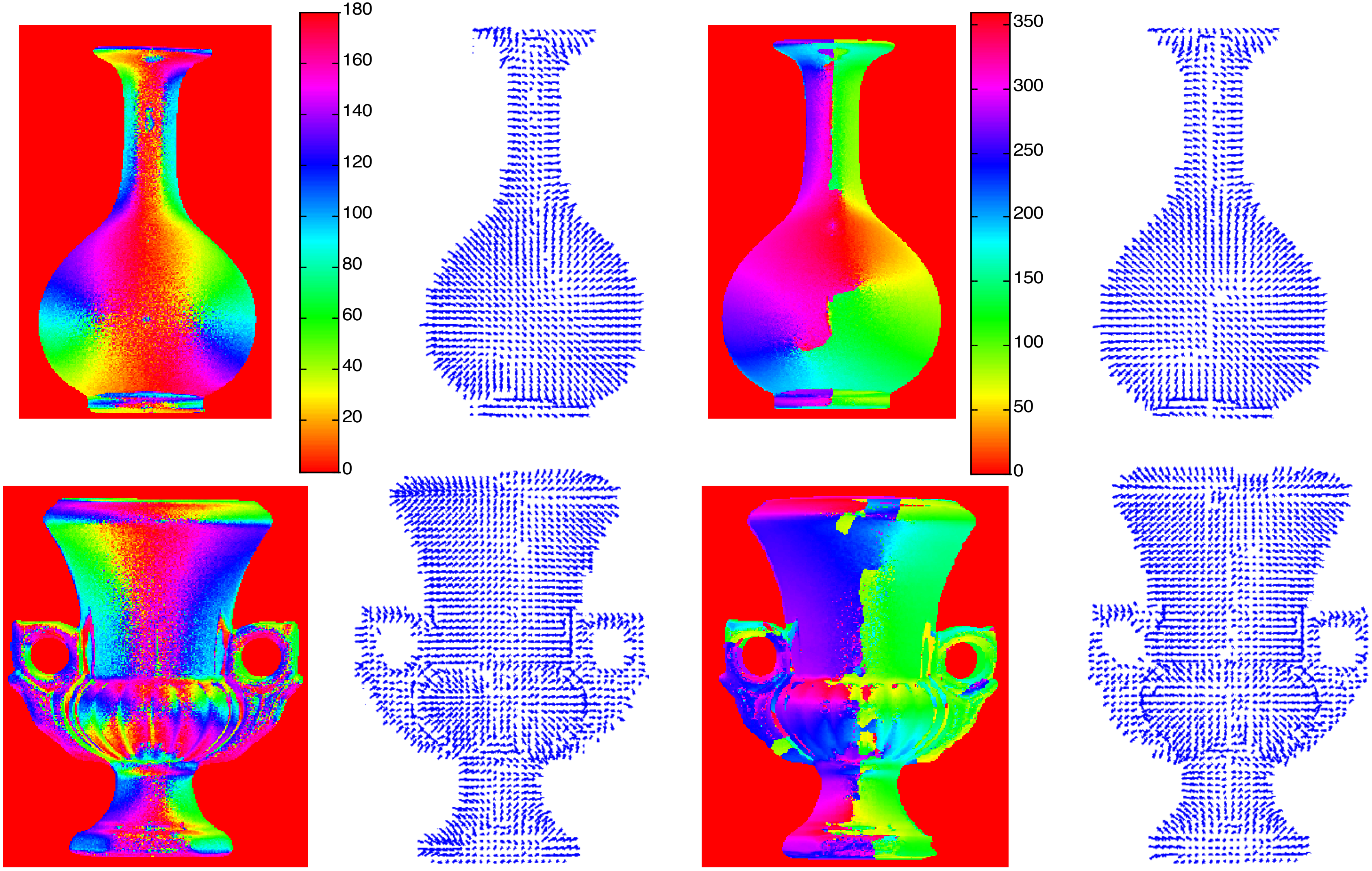
$$L = n.s = \cos\theta$$

Single View Vision: Method

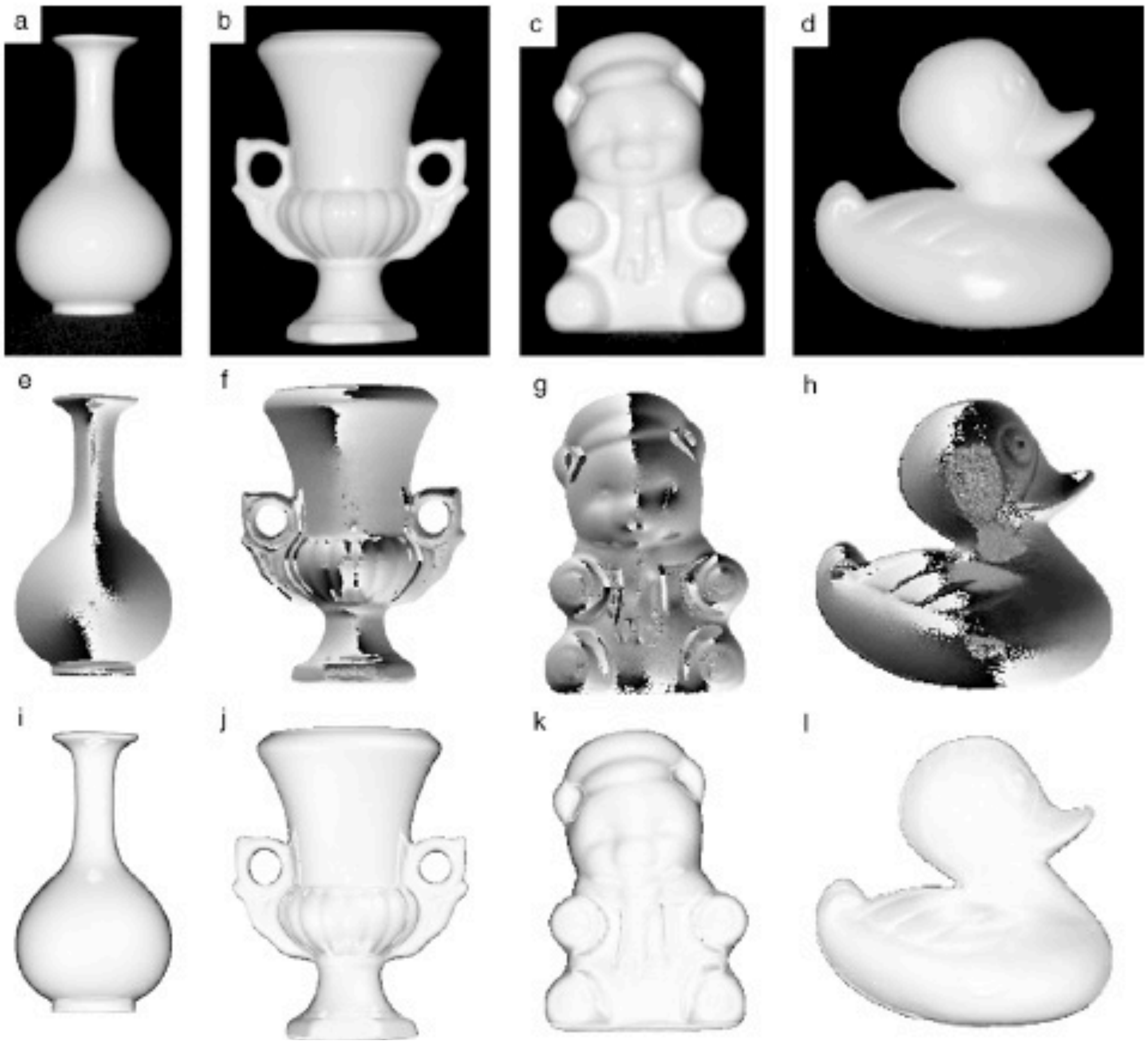


Single View Vision: Method

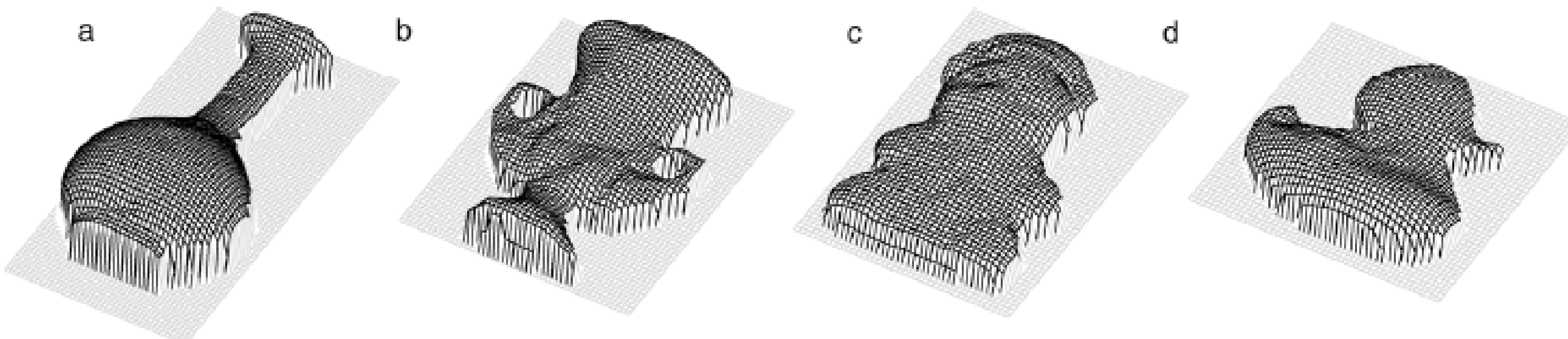
4. Disambiguate azimuth angles



Examples



Height functions



Shape and Refractive Index from Spectro-polarimetric Imagery

Idea

- Multiple polarization images from a single viewpoint and different wavelengths
- Additional constraints on a) wavelength and b) surface integrability.
- Solved using optimisation method.

Physics

From the Fresnel equations

$$\frac{I_{min}}{I_{max}} = \left(\frac{\cos\theta(u)\sqrt{\eta^2(u,\lambda) - \sin^2\theta(u)} + \sin^2\theta(u)}{\eta(u,\lambda)} \right)^2 \quad = R(u,\lambda)^2$$

Solve for zenith angle

$$\sin\theta(u) \equiv \frac{\eta(u,\lambda)\sqrt{1 - R^2(u,\lambda)}}{\sqrt{\eta^2(u,\lambda) - 2R(u,\lambda)\eta(u,\lambda) + 1}}.$$

Material Dispersion Equations

Need model of wavelength dependence of refractive index (sometimes varies by as much as 10% over visible spectrum).

Cauchy

$$\eta(u, \lambda) = \sum_{m=1}^{M} C_m(u) \lambda^{-2(m-1)},$$

Sellmeier

$$\eta^2(u, \lambda) = 1 + \sum_{m=1}^{M} \frac{B_m(u)\lambda^2}{\lambda^2 - D_m(u)},$$

Cost Function

- At each pixel allow refractive index to vary with wavelength, but zenith and azimuth angles remain fixed with wavelength.

- Objective function is the squared difference between measured and predicted values of the max/min intensity ratio plus a smoothness (regularisation term) that ensures the surface normal field is integrable.

- Minimise with respect to the Cauchy/Sellmeir parameters and the surface normal directions.

Integrability constraint on zenith and azimuth angles

$$\cos\alpha(u)\frac{\partial \tan\theta(u)}{\partial y} = \sin\alpha(u)\frac{\partial \tan\theta(u)}{\partial x}.$$

$$\cos\alpha(u)\theta_y(u) = \sin\alpha(u)\theta_x(u),$$

Recover intensity ratio and surface normal to minimize cost function

$$\mathcal{E} = \int_{\mathcal{S}} \int_{\mathcal{W}} \big(R(u, \lambda) - r(u, \lambda)\big)^2 \, d\lambda \, du + \beta(u) \int_{\mathcal{S}} \big(\cos\alpha(u)\theta_y(u) - \sin\alpha(u)\theta_x(u)\big)^2 \, du,$$

Minimize equivalent cost function with data closeness of azimuth angle

$$\mathcal{E}_1 = \int_{\mathcal{S}} \int_{\mathcal{W}} \big(\theta(u) - \varphi(u, \lambda)\big)^2 \, d\lambda \, du$$
$$+ \beta(u) \int_{\mathcal{S}} \left(\cos\alpha(u) \frac{\partial \theta(u)}{\partial y} - \sin\alpha(u) \frac{\partial \theta(u)}{\partial x} \right)^2 du,$$

$$\varphi(u, \lambda) = \arcsin\left(\frac{\eta(u, \lambda)\sqrt{1 - r^2(u, \lambda)}}{\sqrt{\eta^2(u, \lambda) - 2r(u, \lambda)\eta(u, \lambda) + 1}} \right).$$

Intensity ratio for the transmission sinusoid

Theoretical value

$$\frac{I_{min}}{I_{max}} = \left(\frac{\cos\theta(u)\sqrt{\eta^2(u,\lambda) - \sin^2\theta(u)} + \sin^2\theta(u)}{\eta(u,\lambda)} \right)^2.$$

Empirical value

$$r(u,\lambda) \triangleq \sqrt{\frac{I_{min}}{I_{max}}}$$

Estimating zenith angle $\theta(u)$

Focus on minimizing the reduced cost function

$$\mathcal{E}_1 = \int_{\mathcal{S}} \int_{\mathcal{W}} \big(\theta(u) - \varphi(u, \lambda)\big)^2 \, d\lambda \, du + \beta(u) \int_{\mathcal{S}} \left(\cos\alpha(u) \frac{\partial\theta(u)}{\partial y} - \sin\alpha(u) \frac{\partial\theta(u)}{\partial x} \right)^2 du,$$

Solution given by

$$\theta(u) = \frac{1}{K} \int_{\mathcal{W}} \varphi(u, \lambda) \, d\lambda + \frac{\beta(u)}{K} \big(\sin^2\alpha(u)\theta_{xx}(u) - \sin 2\alpha(u)\theta_{xy}(u) + \cos^2\alpha(u)\theta_{yy}(u)\big).$$

Recover refractive index

With zenith angle $\theta(u)$ and intensity ratio r(u,λ) at pixel u known, we solve the following quadratic equation for refractive index, selecting smoothest physically plausible root

$$\left(\cos^2\theta(u) - r^2(u,\lambda)\right) \times \eta^2(u,\lambda) + 2r(u,\lambda)\sin^2\theta(u) \times \eta(u,\lambda) - \sin^2\theta(u) = 0.$$

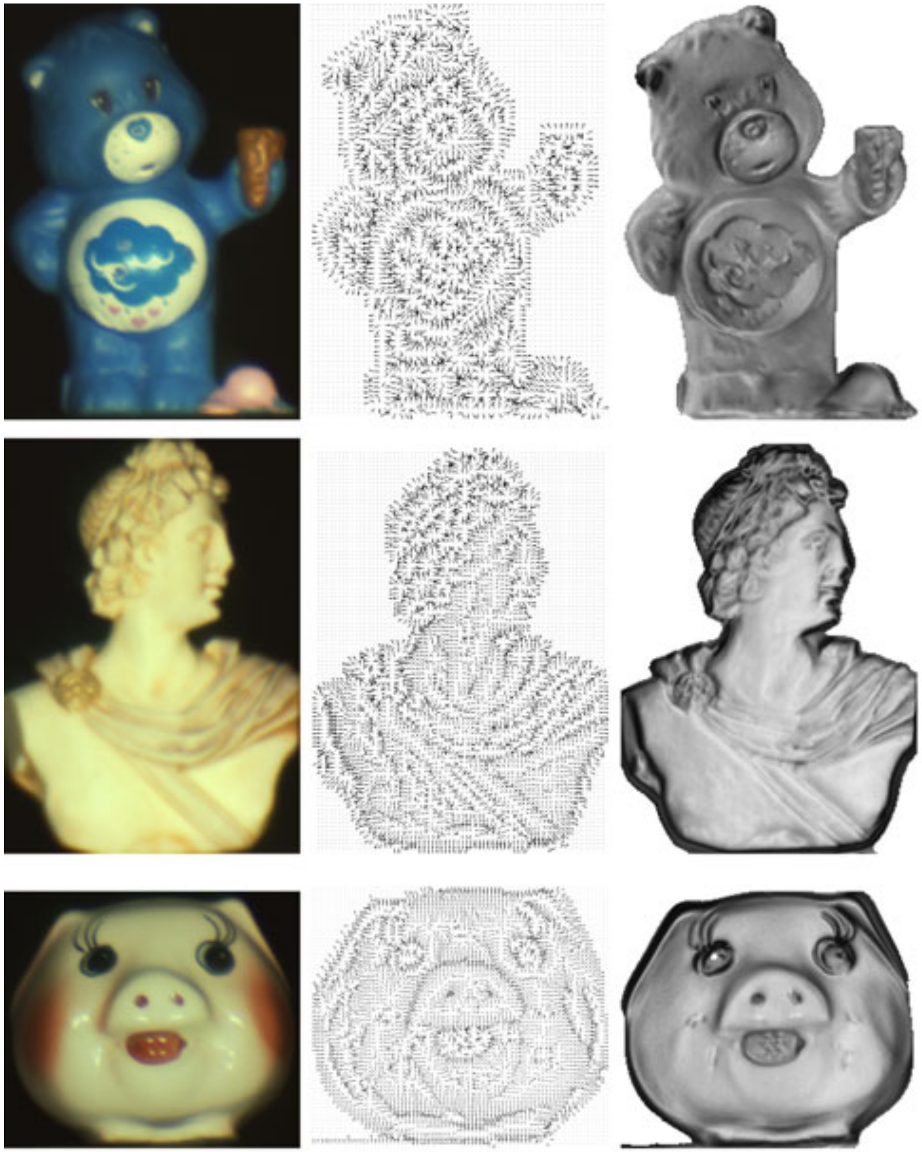
Method

- Collect images with fixed viewpoints at different light source, polarizer and wavelength settings.

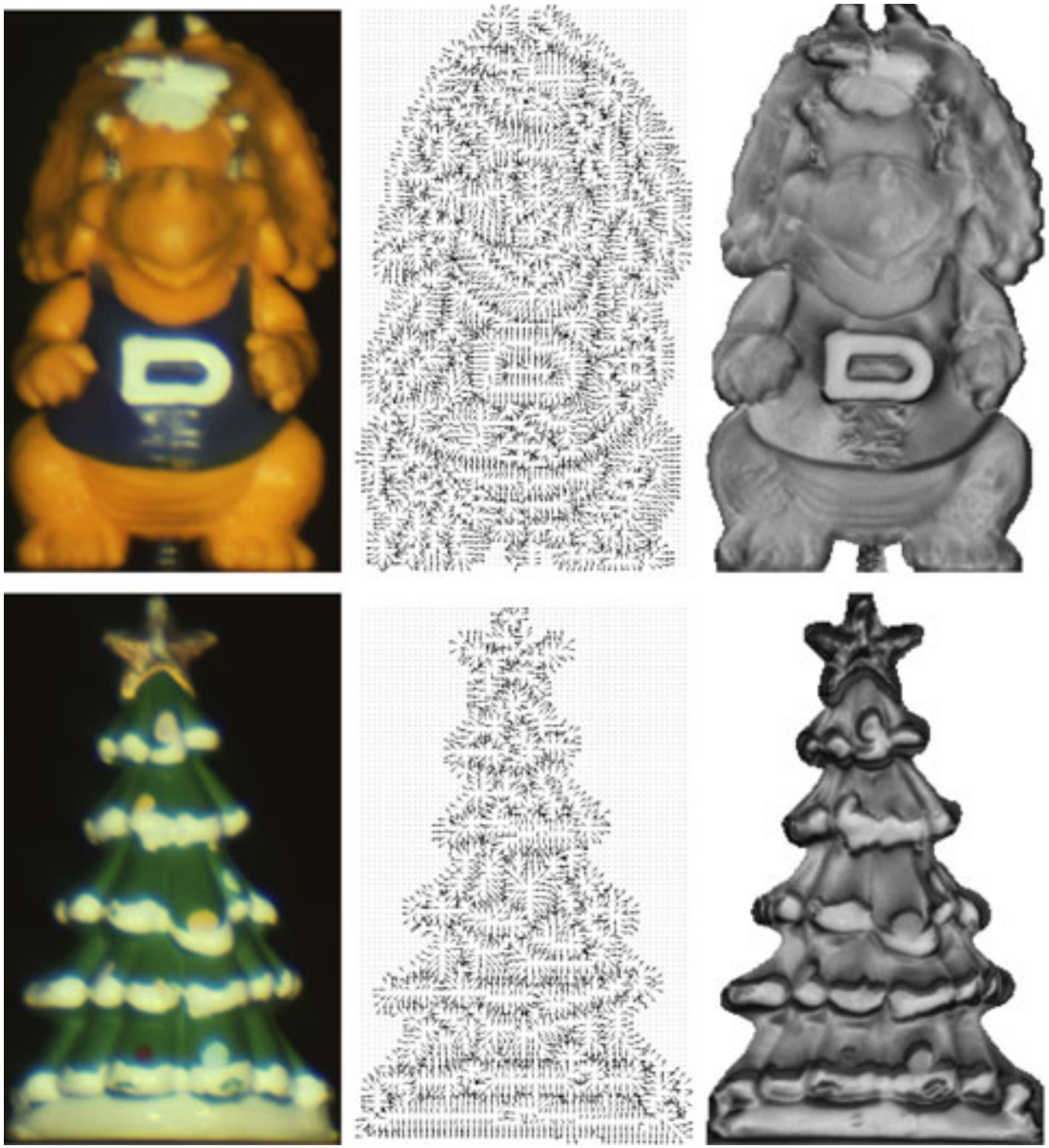
- Solve minimisation problems for refractive index and zenith angle

- Use wavelength dependant phase information for resolve azimuth anlge ambiguity.

- Reconstruct depth using Frankot and Chellappa Fourier domain surface integration method.



Input normals albedo



Input normals albedo

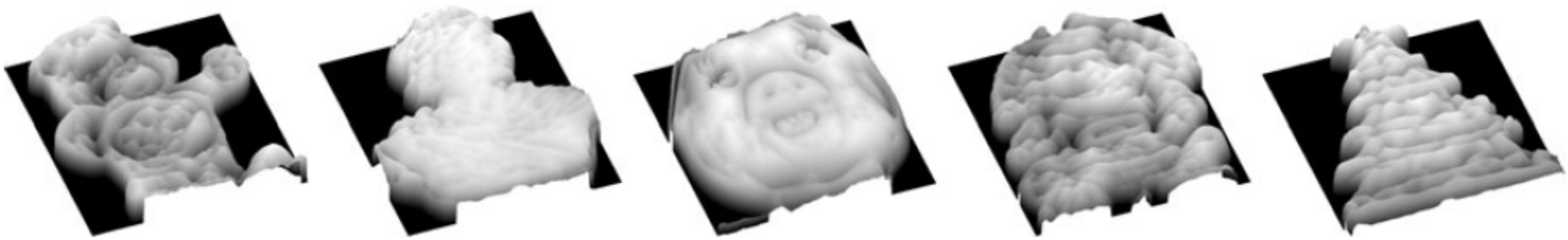
Angular errors

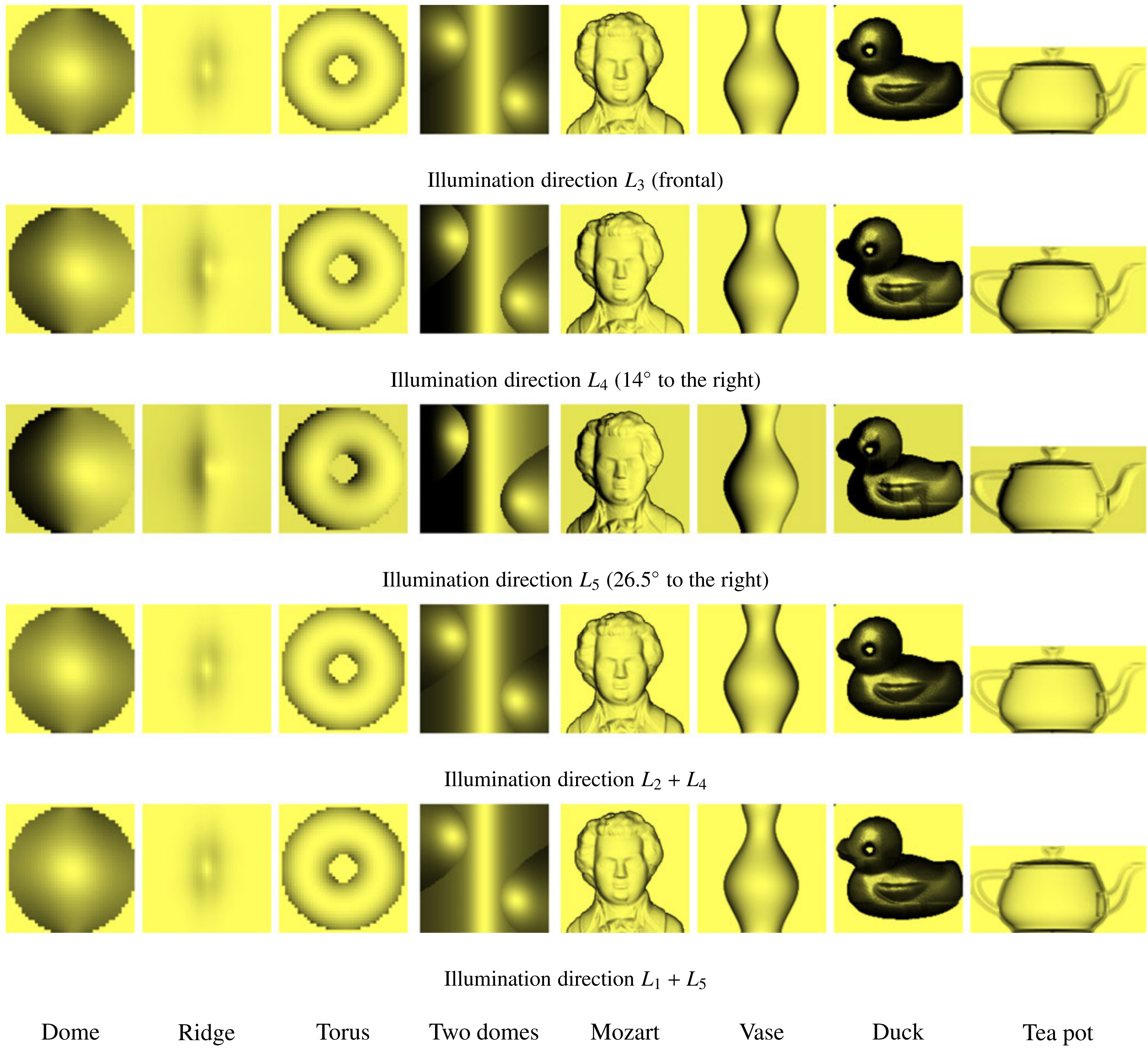
Table 7 The angular deviation (in degrees) between the spectral reflectance images rendered for the frontal viewing direction and the ground truth images. The mean and standard deviation of these errors across pixels are reported for each image

	L_3	L_4	L_5	$L_1 + L_5$	$L_2 + L_4$
Bear	11.63 ± 2.90	12.22 ± 3.67	12.94 ± 5.95	12.65 ± 4.71	11.48 ± 3.17
Statue	12.32 ± 3.43	14.11 ± 3.24	14.18 ± 4.02	15.75 ± 4.03	13.46 ± 2.90
Pig	10.70 ± 3.40	11.78 ± 3.47	12.94 ± 4.14	12.87 ± 4.43	10.53 ± 2.91
Dinosaur	10.67 ± 3.76	12.19 ± 6.94	14.01 ± 8.15	9.02 ± 3.94	8.27 ± 3.60
Pine Tree	10.82 ± 2.69	11.33 ± 3.51	14.92 ± 4.81	13.05 ± 3.94	10.35 ± 3.99

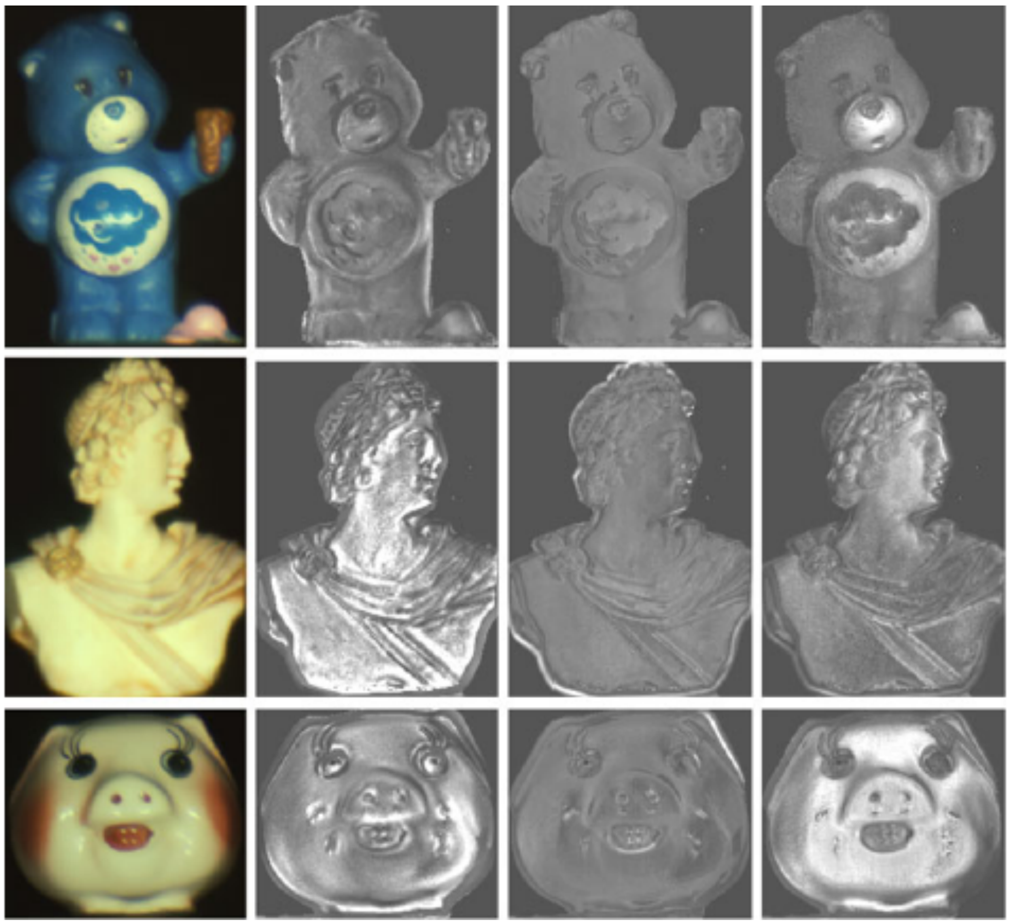
L_3=frontal, L_4= 14 degrees right, L_5= 26.5 degrees

Depth maps



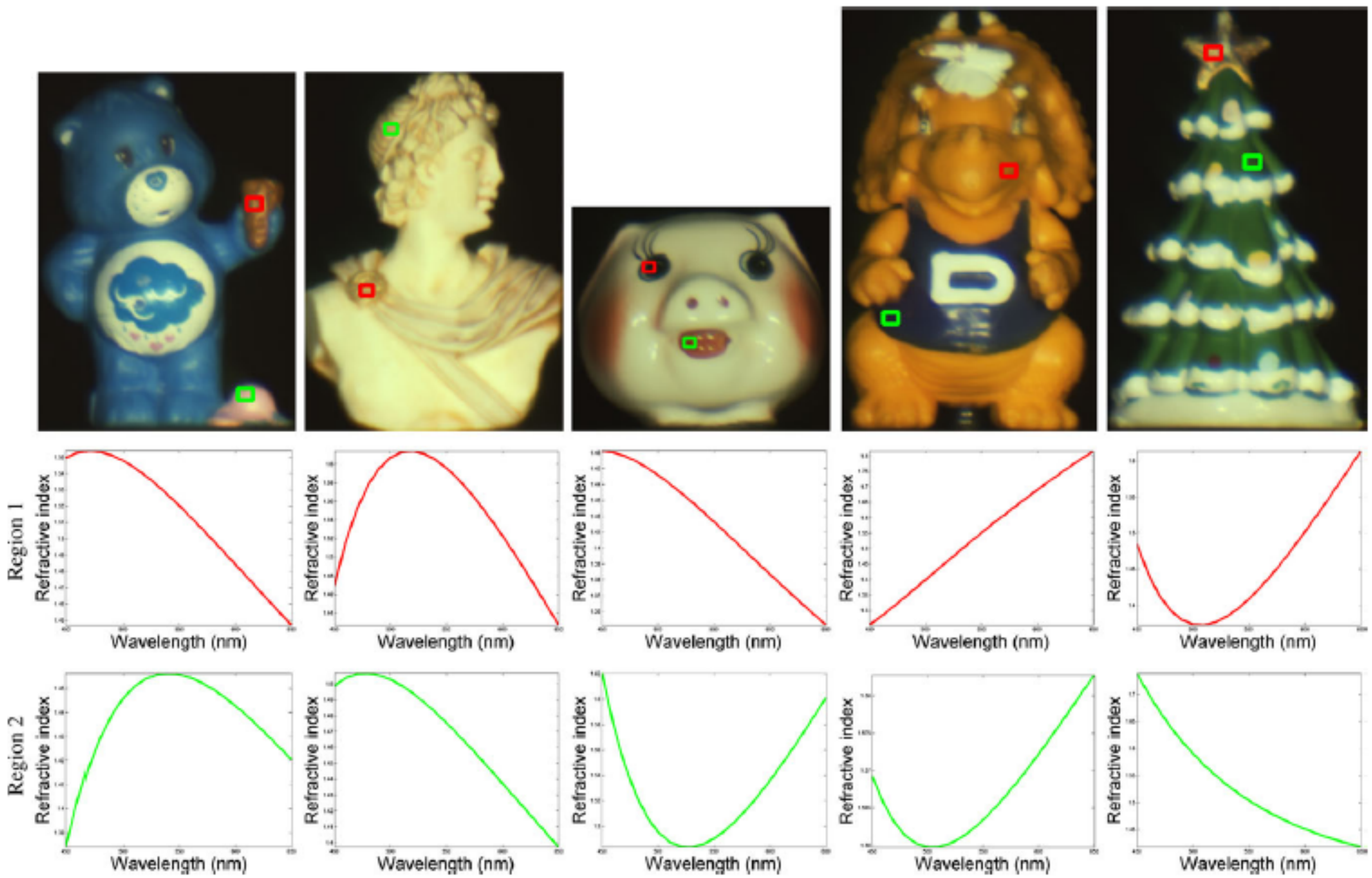

Illumination direction L_3 (frontal)
Illumination direction L_4 (14° to the right)
Illumination direction L_5 (26.5° to the right)
Illumination direction $L_2 + L_4$
Illumination direction $L_1 + L_5$
Dome
Ridge
Torus
Two domes
Mozart
Vase
Duck
Tea pot

Refractive Index Variation with wavelength



Input 450nm blue 550nm green 650nm red

Wavelength Dependence of Refractive Index



Conclusions

- Demonstrated potential of diffuse polarization for shape-recovery from single and multiple polarization images.
- Gives reliable shape recovery, and could be the basis of a range imaging camera design.
- Can be used to estimate material characteristics of surface (refractive index, complex refractive index).