Learning geometry

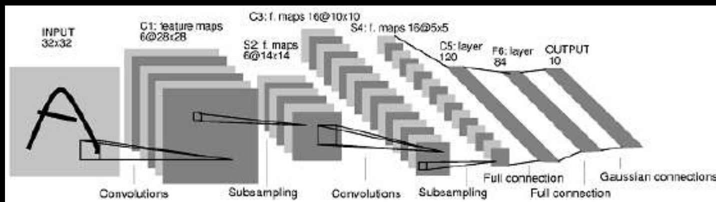
Ron Kimmel

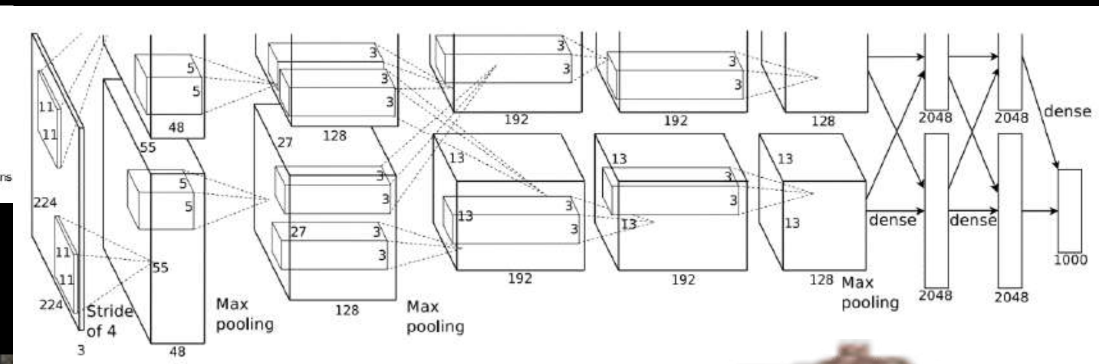
Geometric Image Processing Lab.

Technion – Israel Institute of Technology

Mathematical Methods for Objects Reconstruction: from 3D Vision to 3D Printing

INdAM Workshop. Rome Feb. 10, 2021


INPUT 32x32
C1: feature maps 6@28x28
S2: f. maps 6@14x14
C3: f. maps 16@10x10
S4: f. maps 16@5x5
C5: layer 120
F6: layer 84
OUTPUT 10
Convolutions
Subsampling
Convolutions
Subsampling
Full connection
Full connection
Gaussian connections


224
224
3
11
11
Stride of 4
55
48
5
Max pooling
27
128
3
Max pooling
13
192
192
128
Max pooling
dense
dense
dense
2048
2048
1000


Yann LeCunn
Geoff Hinton
Yoshua Bengio

3DMM

Vetter & Blanz, A morphable model for the synthesis of 3D faces, Siggraph 1999

Kemelmacher & Basri, 3D face reconstruction from a single image … PAMI 2011

Learning using Axiomatic Knowledge

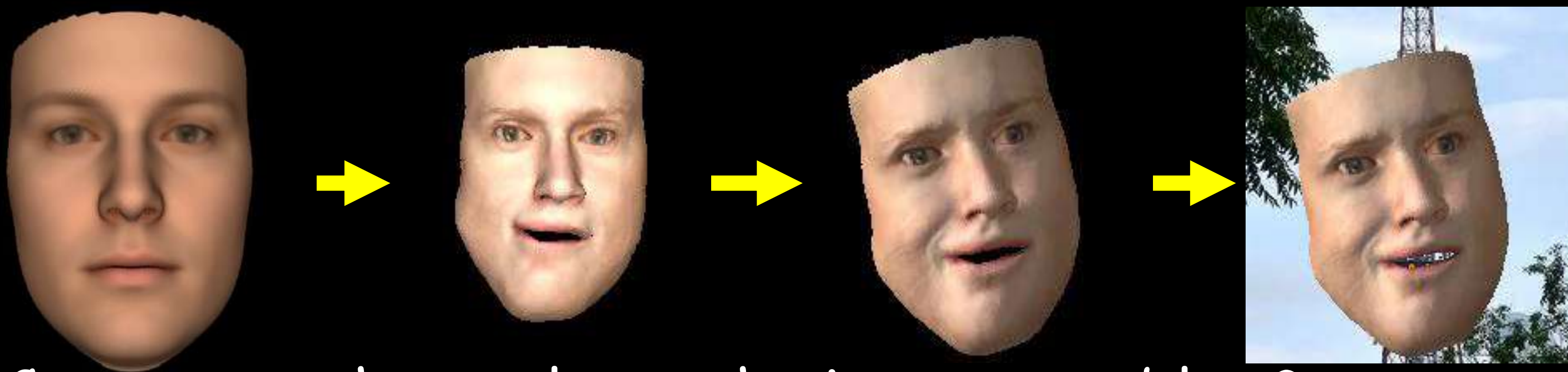
Richardson Sela Or-El K. CVPR'17

Learning using Axiomatic Knowledge

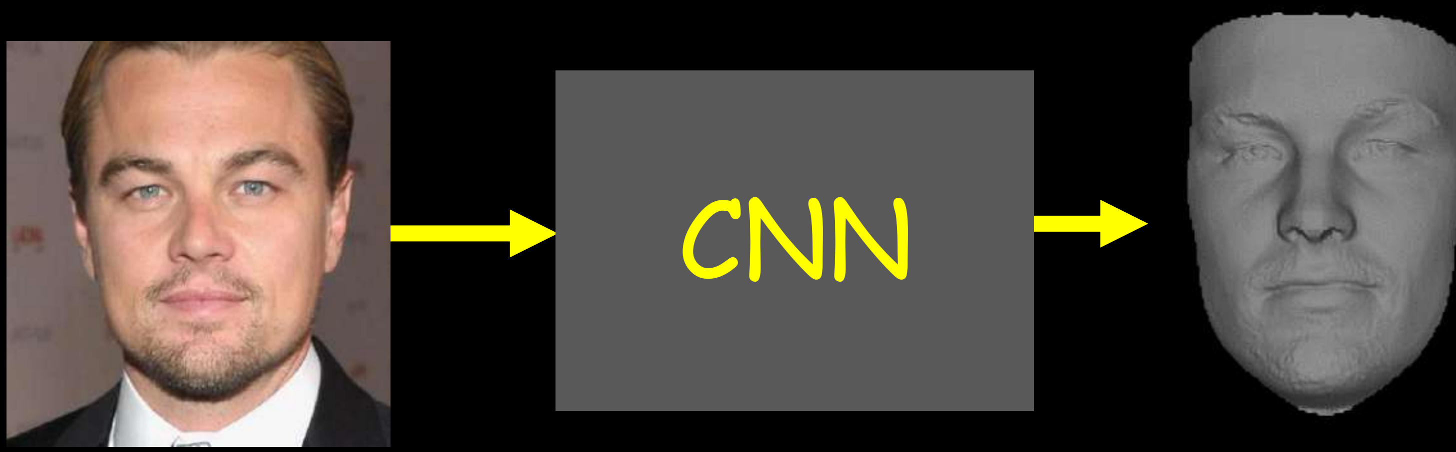
Richardson Sela Or-El K. CVPR'17

Learning using Axiomatic Knowledge

We know how to model faces

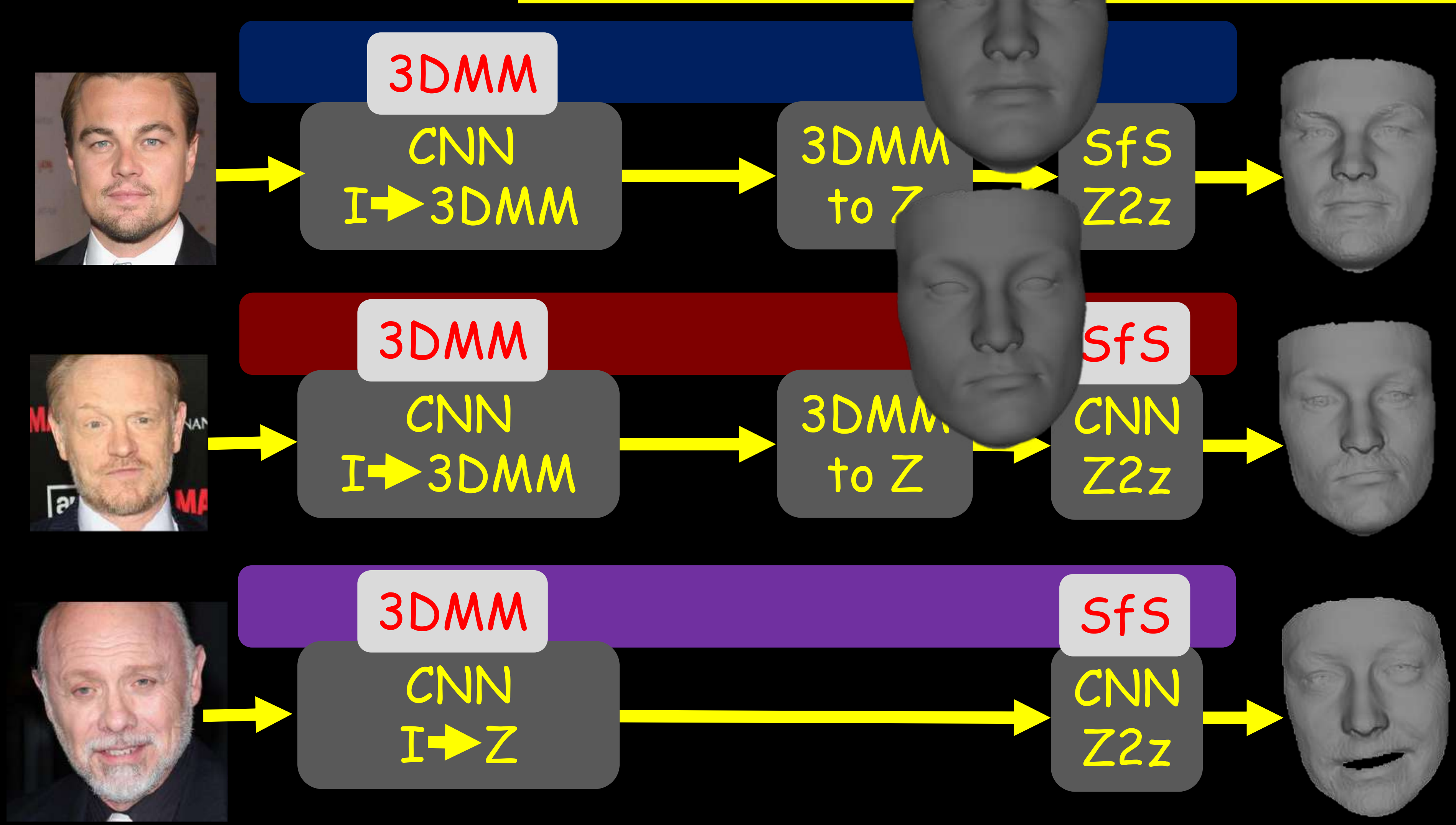


Can we use that to learn the inverse problem?

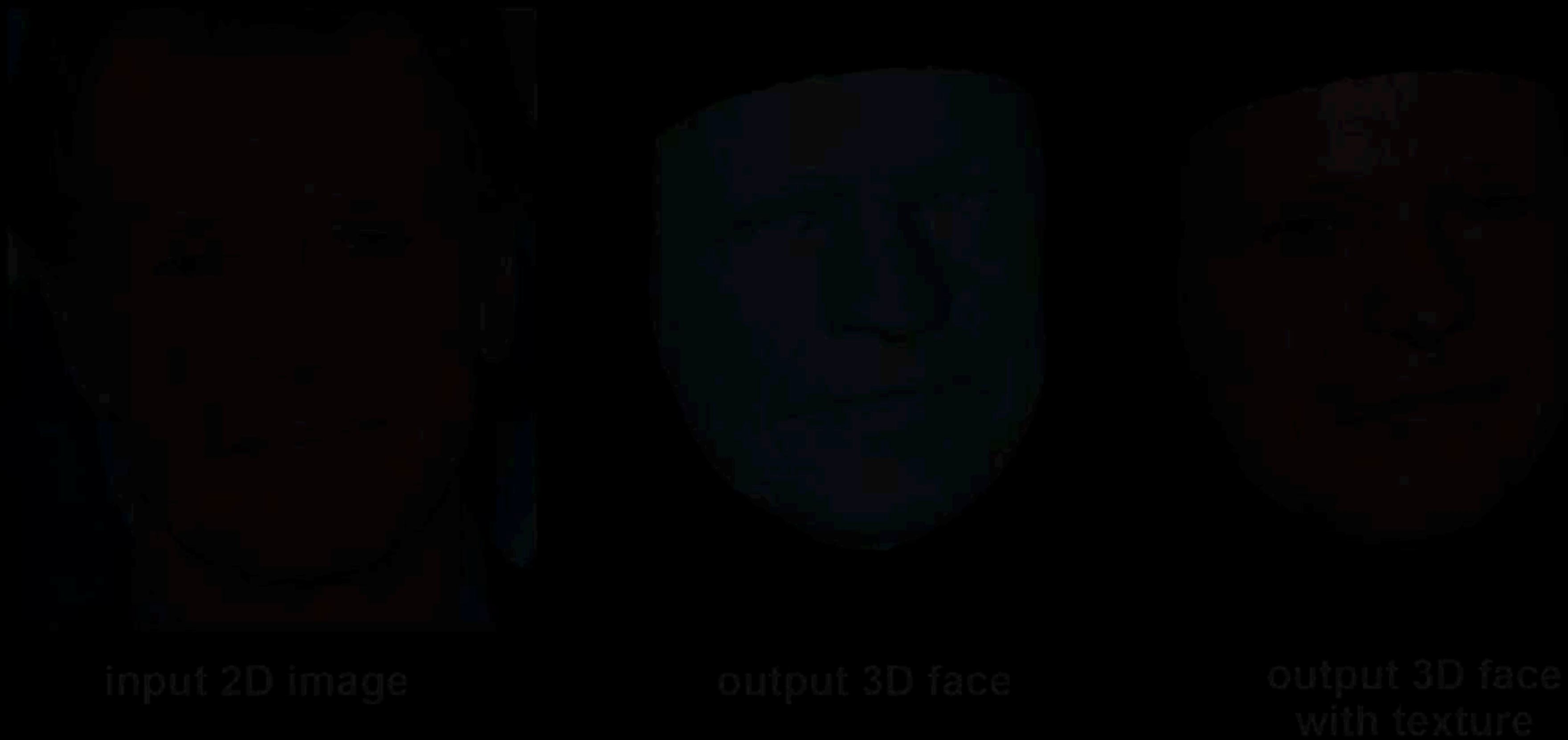


Richardson Sela Or-El K. CVPR'17

Face reconst… n evolution



Richardson Sela K. 3DV'16/ Richardson Sela Or-El K. CVPR'17/ Richardson Sela K. ICCV'17



Richardson Sela K. ICCV 2017

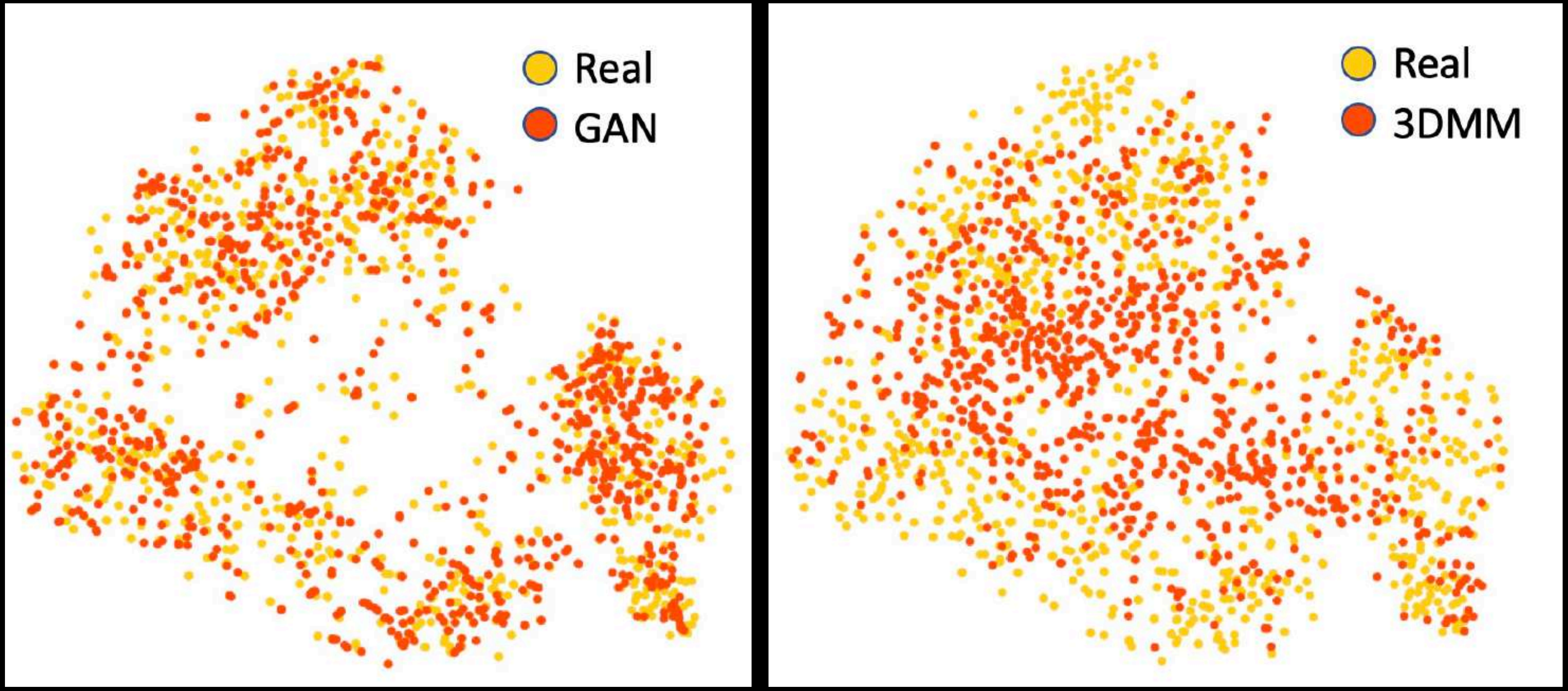
Shamai, Slossberg, K. 2018/2019 Synthesizing photometries and geometries with GANs &

Synthesizing Expressions

Shamai, Slossberg, K. 2018/2019 Synthesizing photometries and geometries with GANs &

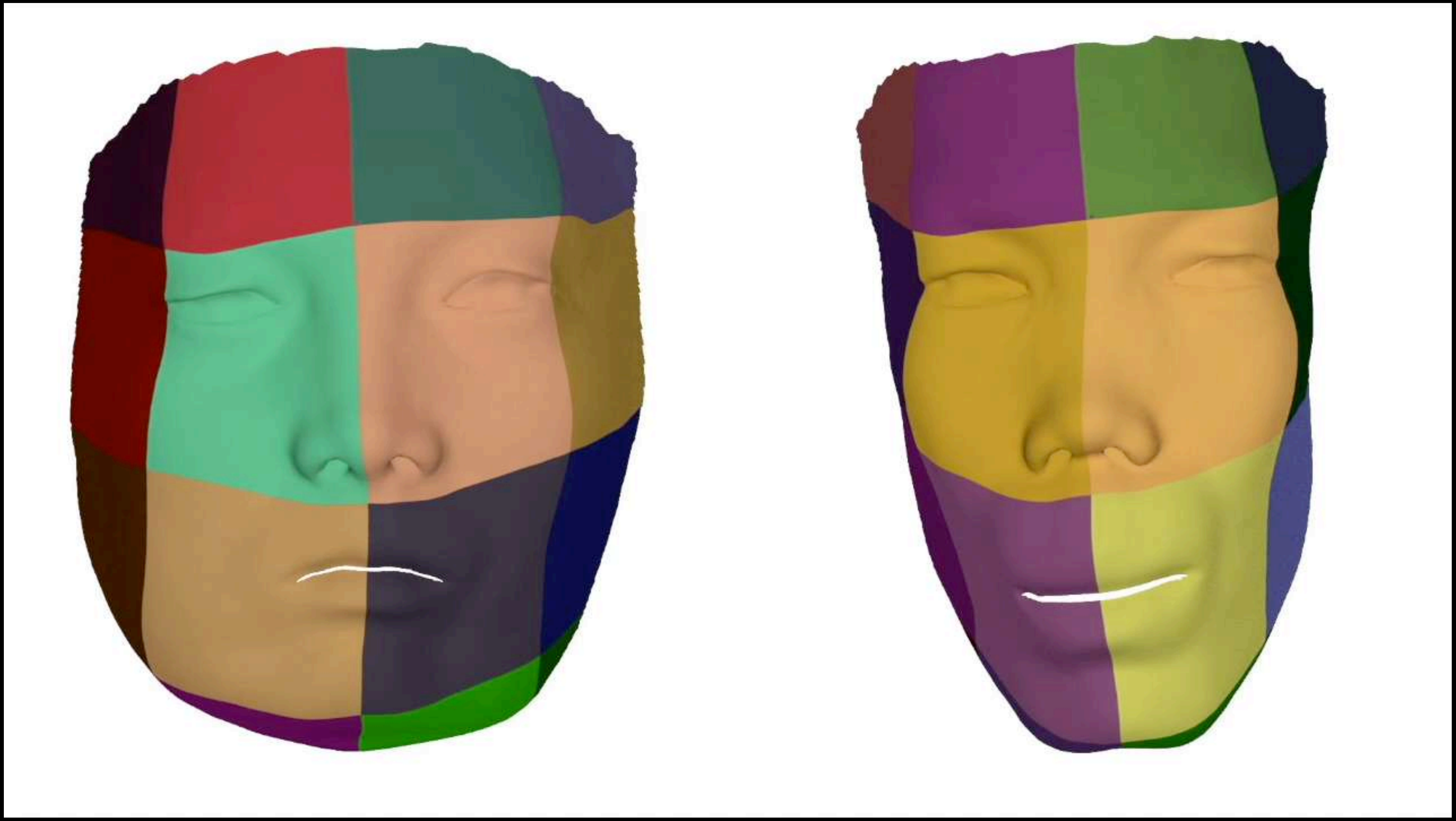
Shamai, Slossberg, K. 2018/2019 Synthesizing photometries and geometries with GANs &

2D embedding identities



Shamai, Slossberg, K. 2018/2019 Synthesizing photometries and geometries with GANs &

Training iterations (GAN)



Shamai, Slossberg, K. 2018/2019 Synthesizing photometries and geometries with GANs &

ISRAEL SCIENCE FOUNDATION

Ministry of Science, Technology and Space
Israel

SCHMIDT FUTURES

The Lokey Center

Artificial Intelligence Algorithms to Assess Hormonal Status From Tissue Microarrays in Patients With Breast Cancer

Gil Shamai Yoav Binenbaum Ron Slossberg Irit Duek Ziv Gil Ron Kimmel






Technion
Israel Institute of Technology

Tel-Aviv Sourasky Medical Center

RAMBAM

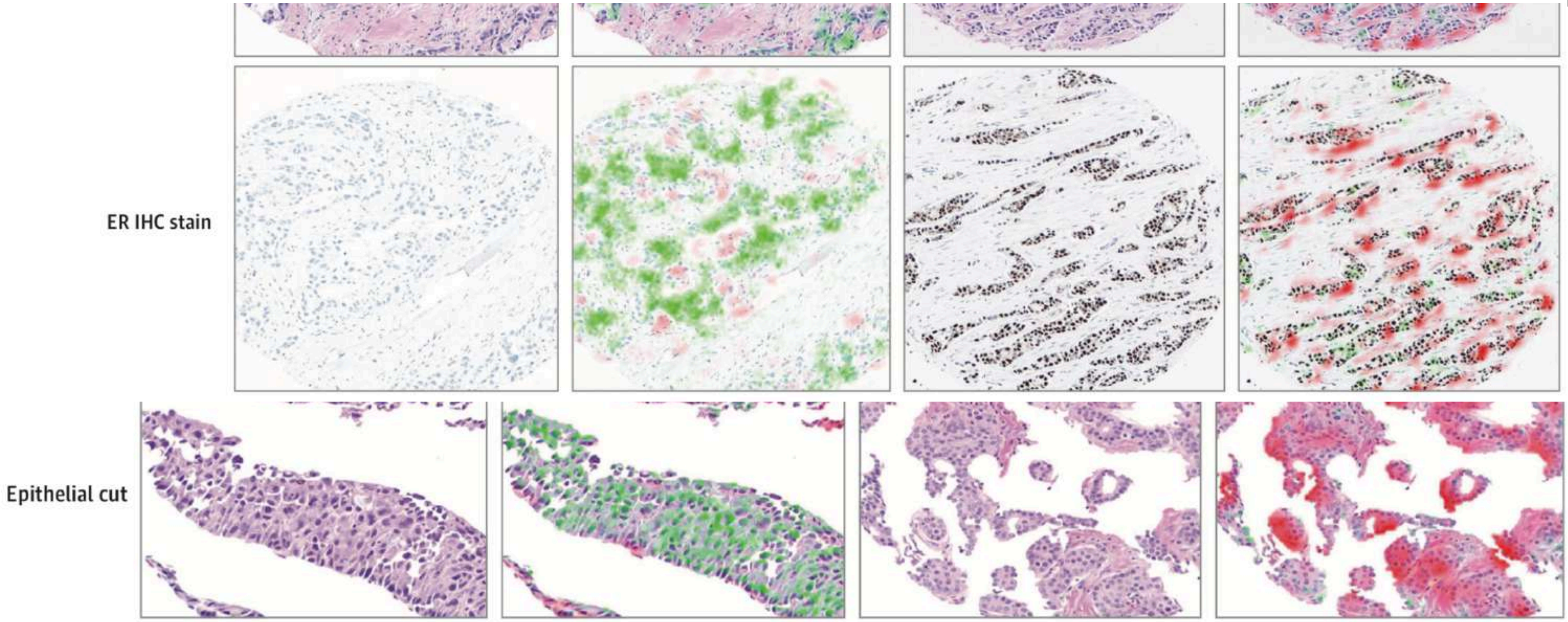
Geometric Image Processing lab

JAMA Network Open

Original Investigation | Oncology

Artificial Intelligence Algorithms to Assess Hormonal Status From Tissue Microarrays in Patients With Breast Cancer

Gil Shamai, MSc; Yoav Binenbaum, MD, PhD; Ron Slossberg, MSc; Irit Duek, MD; Ziv Gil, MD, PhD; Ron Kimmel, DSc



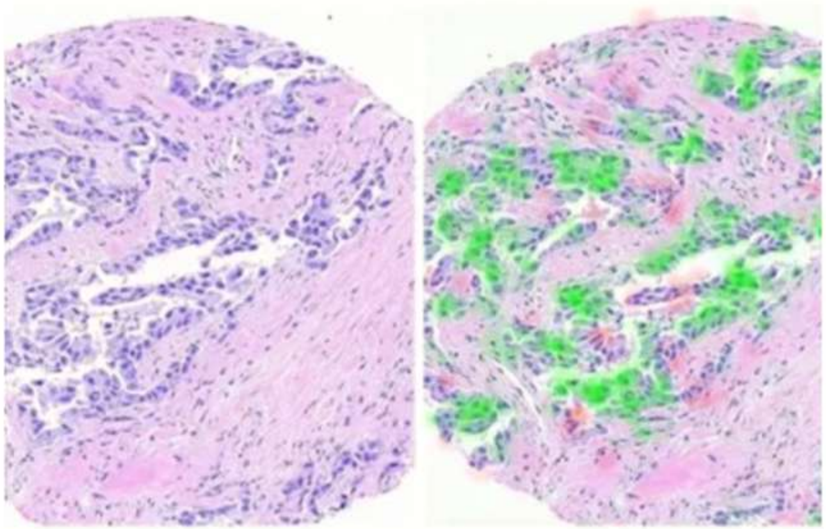
THE JERUSALEM POST

GROUNDBREAKING AI-BASED CANCER TREATMENT DEVELOPED BY ISRAELI RESEARCHERS

2 minute read.

By LEON SVERDLOV

Share



The original scan (left) and the areas where information was extracted (in red and green, right) using the technology developed at the Technion (photo credit: TECHNION SPOKESPERSON'S OFFICE)

The new technology allows AI to identify molecular features of cancer

www.news.cn 新华网 News www.xinhuanet.com

Israeli research
technology to i

Source: Xinhua | 2019-08-19 22

JERUSALEM, Aug. 19 (
(DL) technology that is
the northern Israel Inst

This is a method for
breast cancer patie

The new method,
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WORLD NIGERIA OPINI

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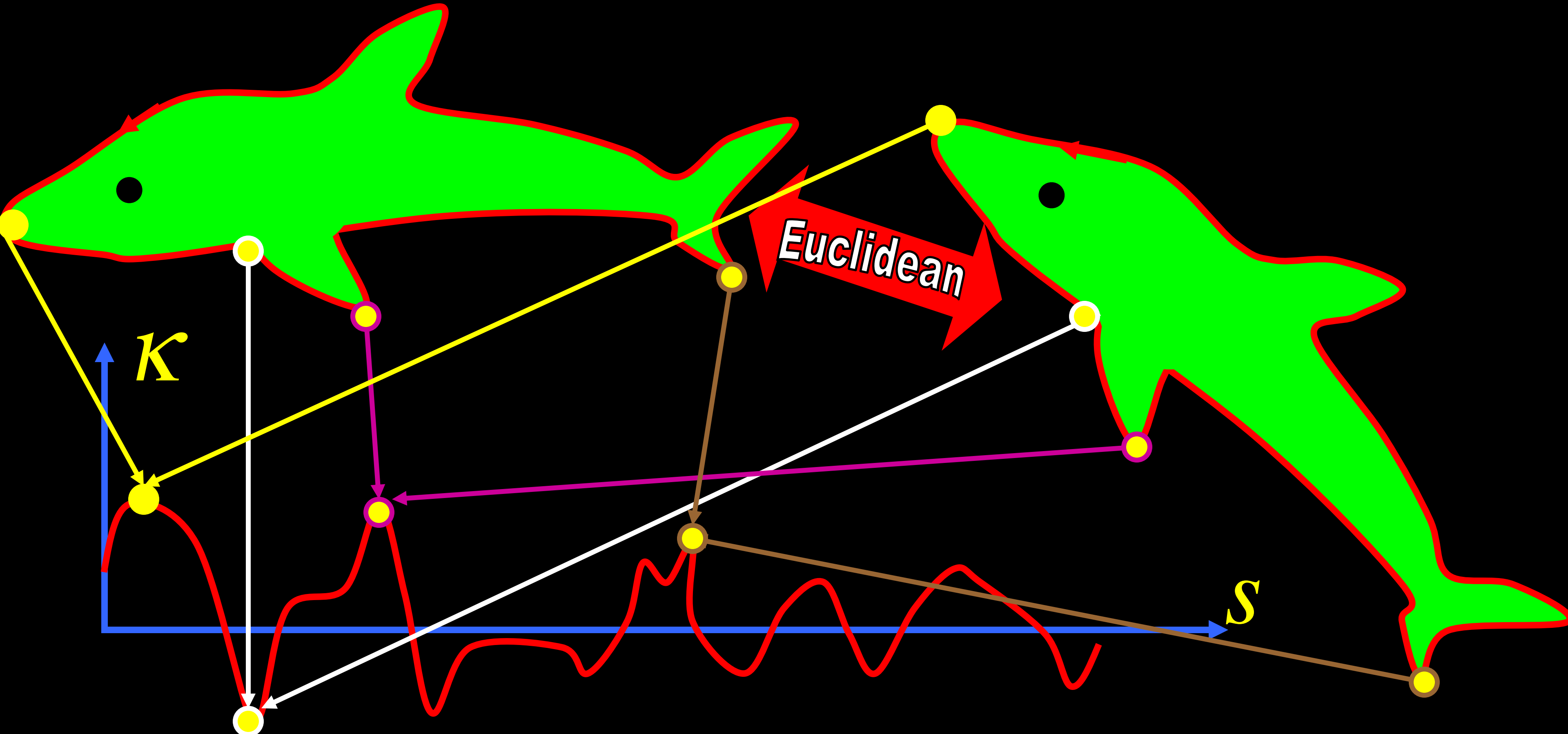
How could we
use algebra to
describe geometry?

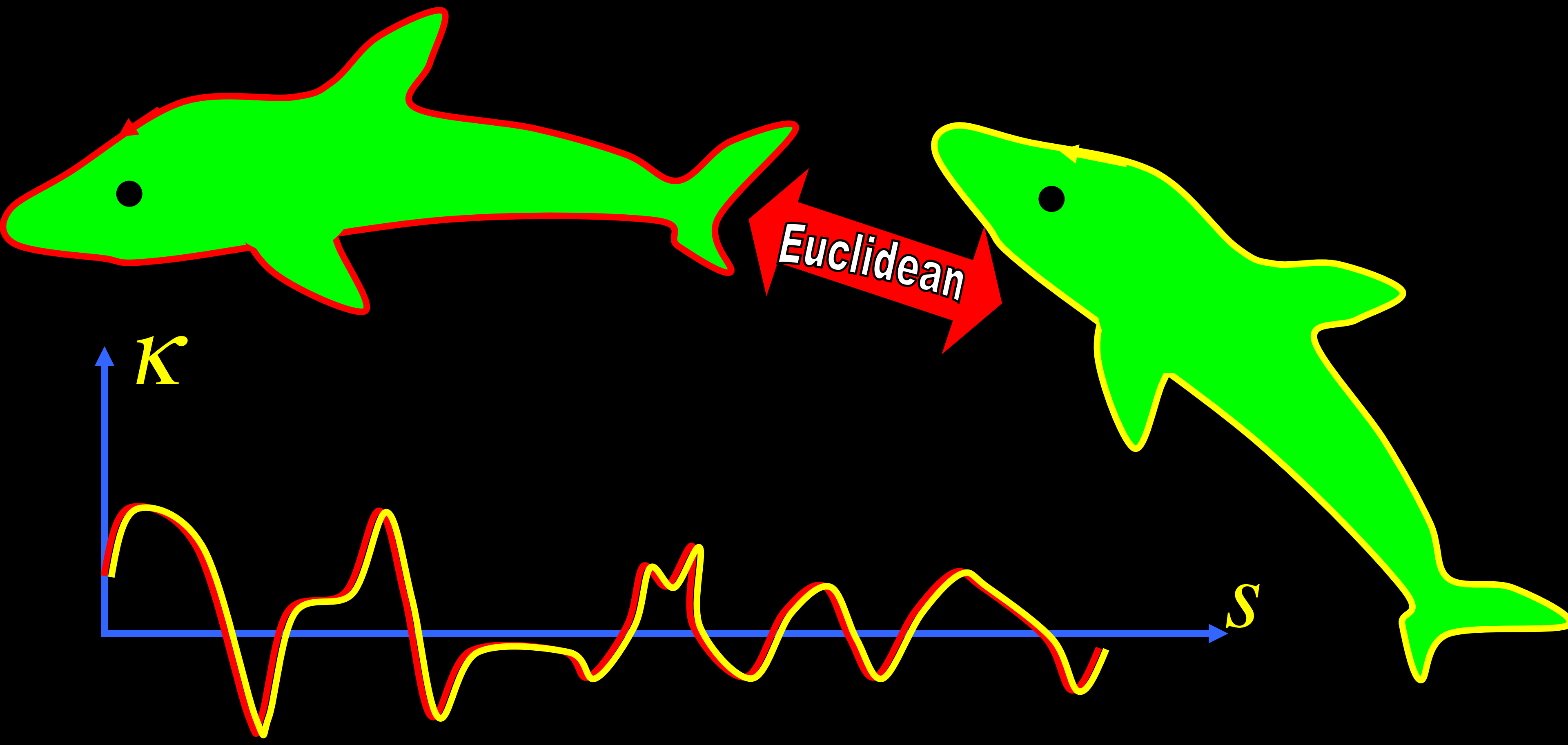
Differential Signatures

- Euclidean invariant signature $\{s, \kappa(s)\}$

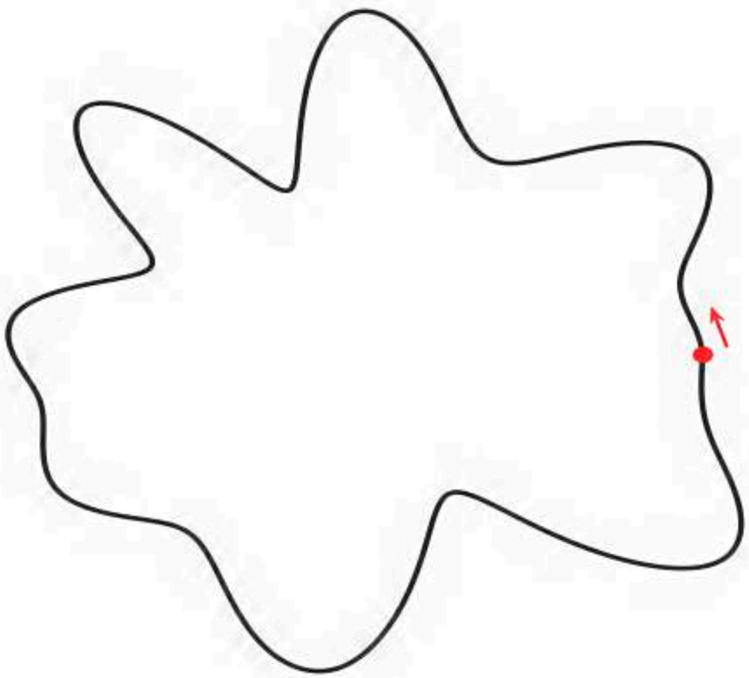


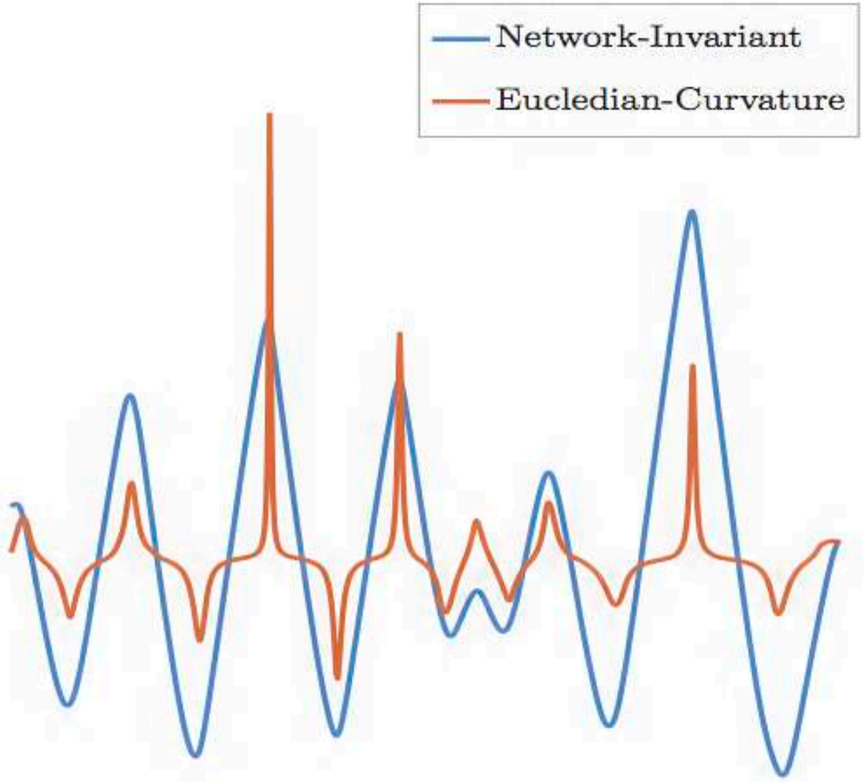
Differential Signatures

- Euclidean invariant signature $\{s, \kappa(s)\}$



Learning invariants





Pai Wetzler K. ICLR 2017

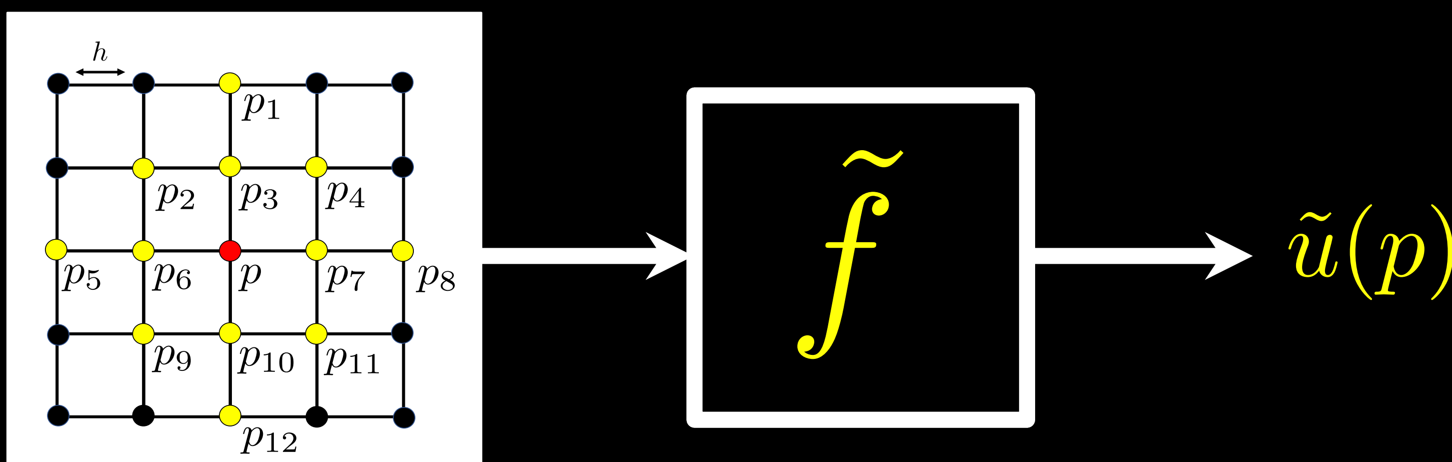
Deep Eikonal solvers

$$|\nabla u(x)| = 1, \quad x \in \Omega \setminus \Gamma$$

$$u(x) = 0, \quad x \in \Gamma$$

Local numerical approximations using neural networks

- Easy to extend the support
- Approximation is learned from analytically known solutions

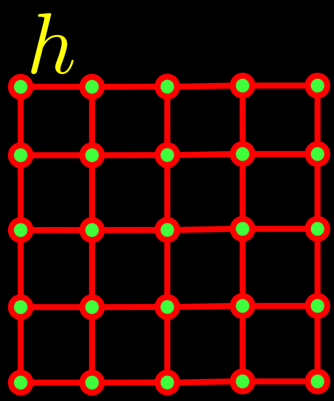


Lichtenstein, Pai, K., SSVM 2019 - Best Student Paper Award

On the evolution of accuracy/complexity

- Fast Marching (quasi-linear complexity) $\sim O(n)$

Regular grids



1995 $O(h)$

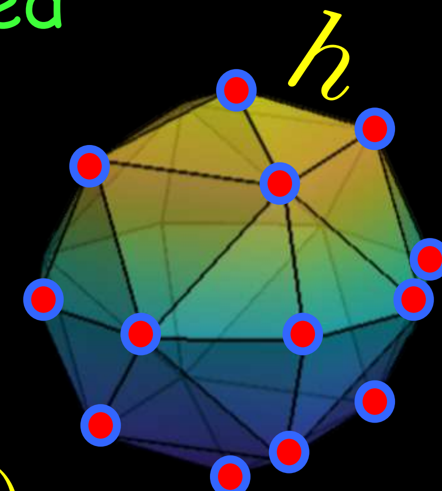
↓

1999 $O(h^2)$

↓

2011 $O(h^3)$

Triangulated domains

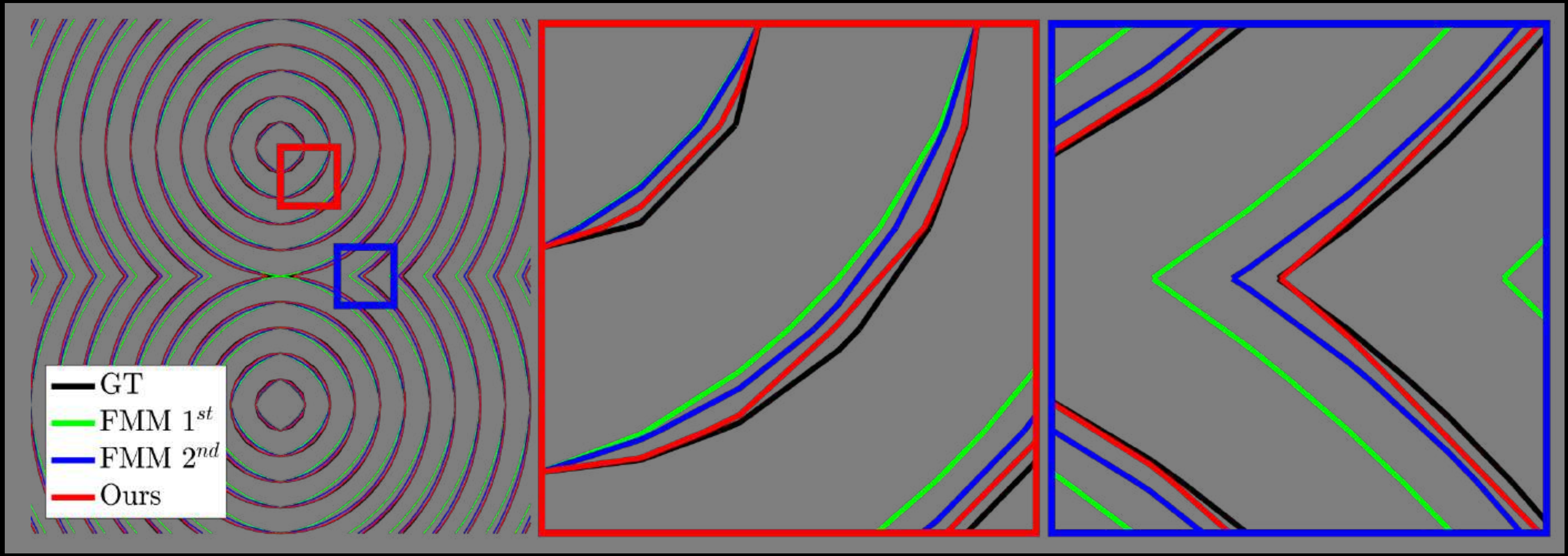


1998 $O(h)$

2005 $O(h^2)$ ← $O(n^2)$

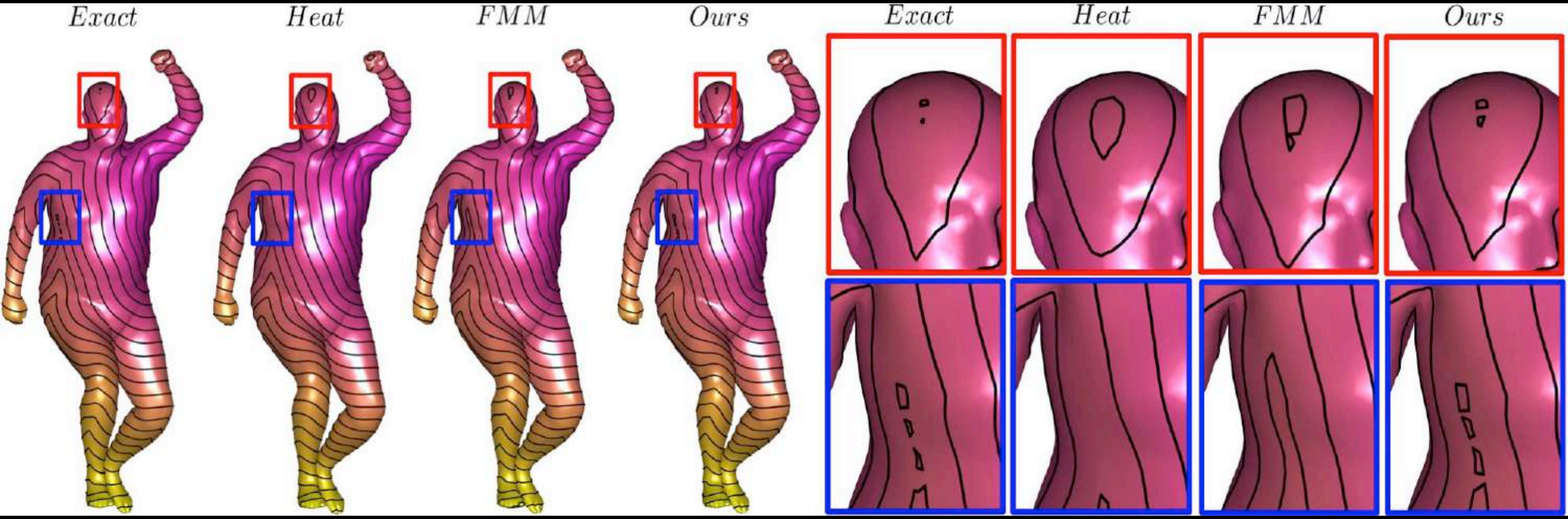
Surazhsky et al.
implementation of MMP's
`exact geodesic' 1987
(~2 decades!)

Results for Cartesian grids



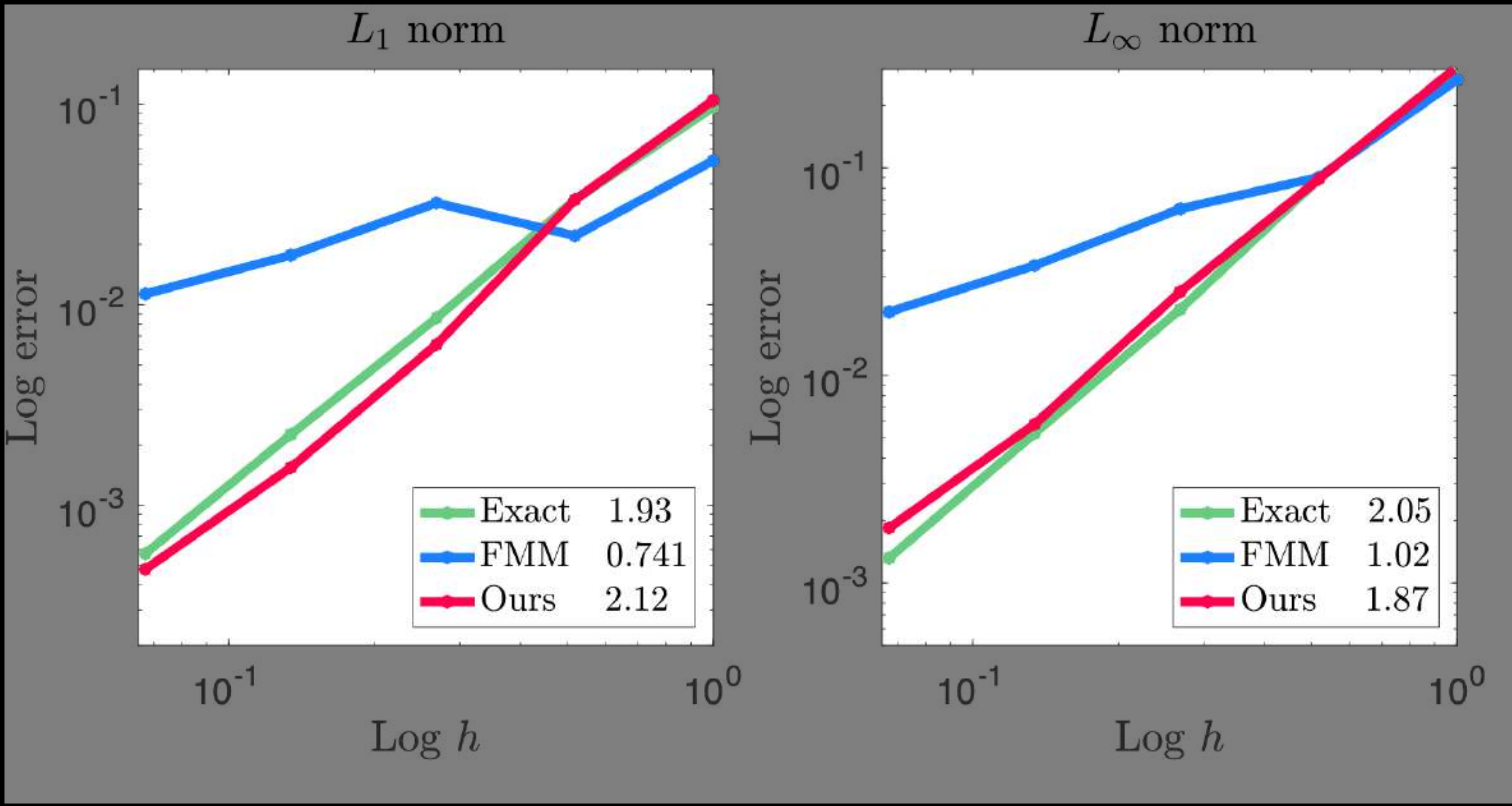
Lichtenstein, Pai, K., SSVM 2019 - Best Student Paper Award

Inter-dataset generalization



Lichtenstein, Pai, K., SSVM 2019 - Best Student Paper Award

Order of accuracy



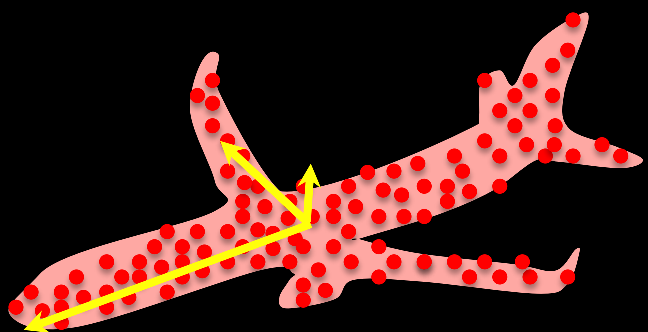
Lichtenstein, Pai, K., SSVM 2019 - Best Student Paper Award

Momenet

Algebraic invariants — 1903
3D moment as invariants — 1980
High order NN — 1987
Deep NN — 2010
PointNet — 2017
Momenet — 2019

Geometric moments

Given a set of points $P \subset \mathbb{R}^3$ and $p_i = (x_i, y_i, z_i)^T \in P$

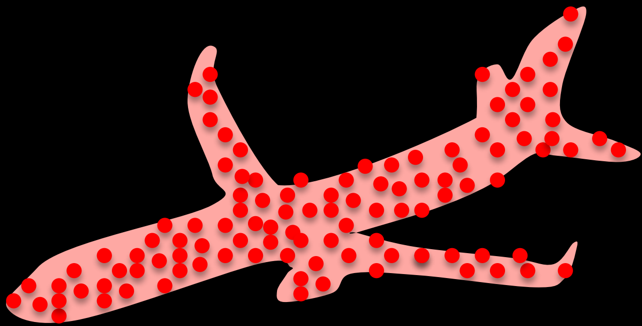


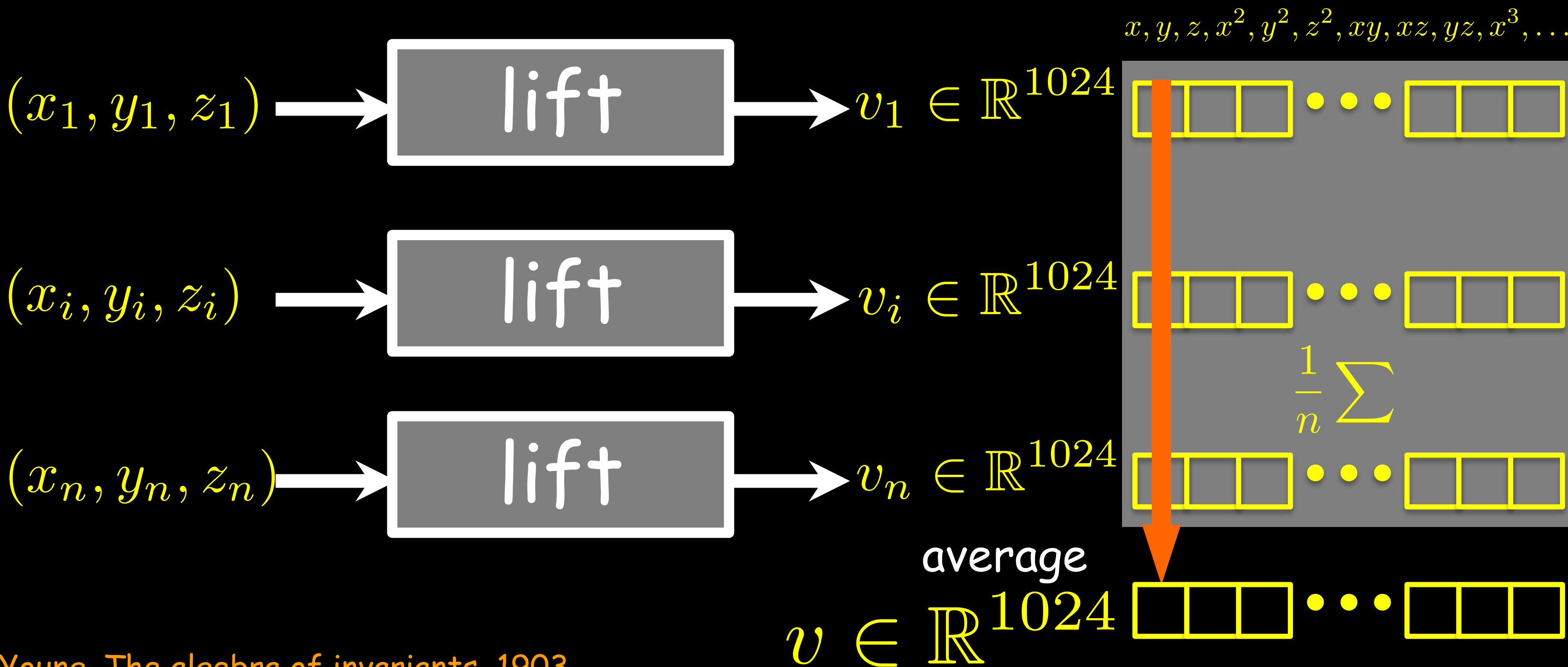
1st moments $\bar{p} = (\bar{x}, \bar{y}, \bar{z})^T = \frac{1}{n} \sum_{i=1}^{n} p_i$

2nd moments $\frac{1}{n} \sum_{i=1}^{n} p_i p_i^T$

Young. The algebra of invariants, 1903
Hall. Three-dimensional moment invariants. PAMI, 1980
Maxwell, Learning, invariance, and generalization in high-order neural networks. Applied optics, 1987
Su, Mo, Guibas. Pointnet: Deep learning on point sets. CVPR'17
Joseph-Rivlin, Zvirin, K., GMDL workshop, ICCV'19

Moments





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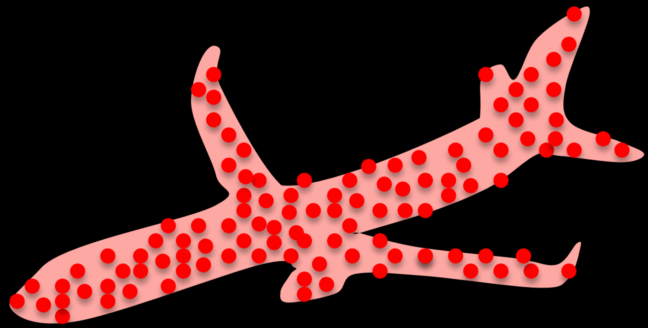
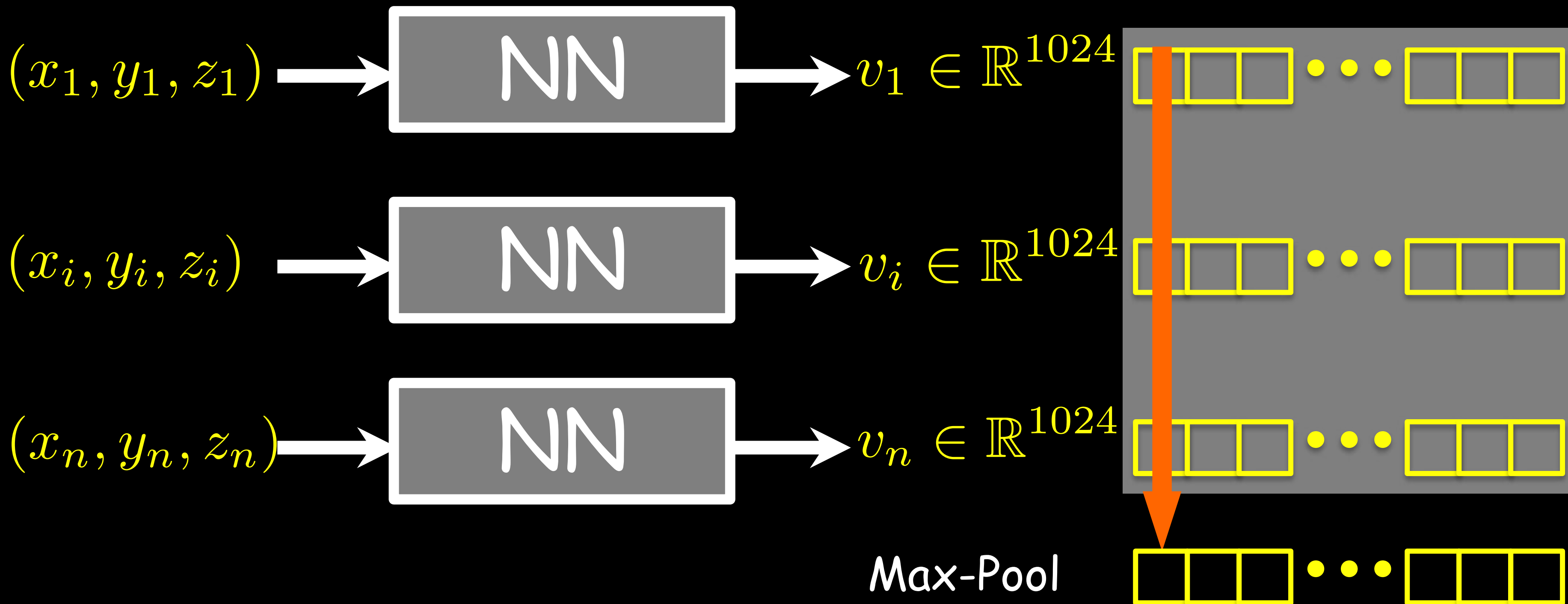
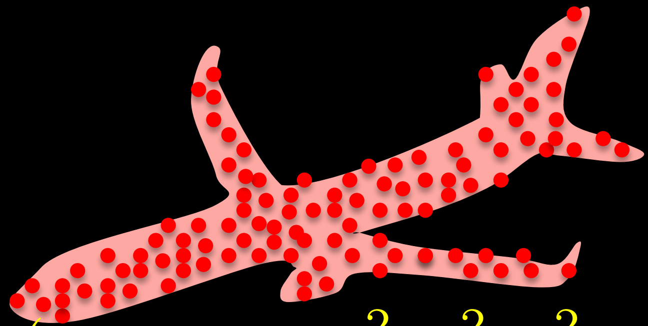


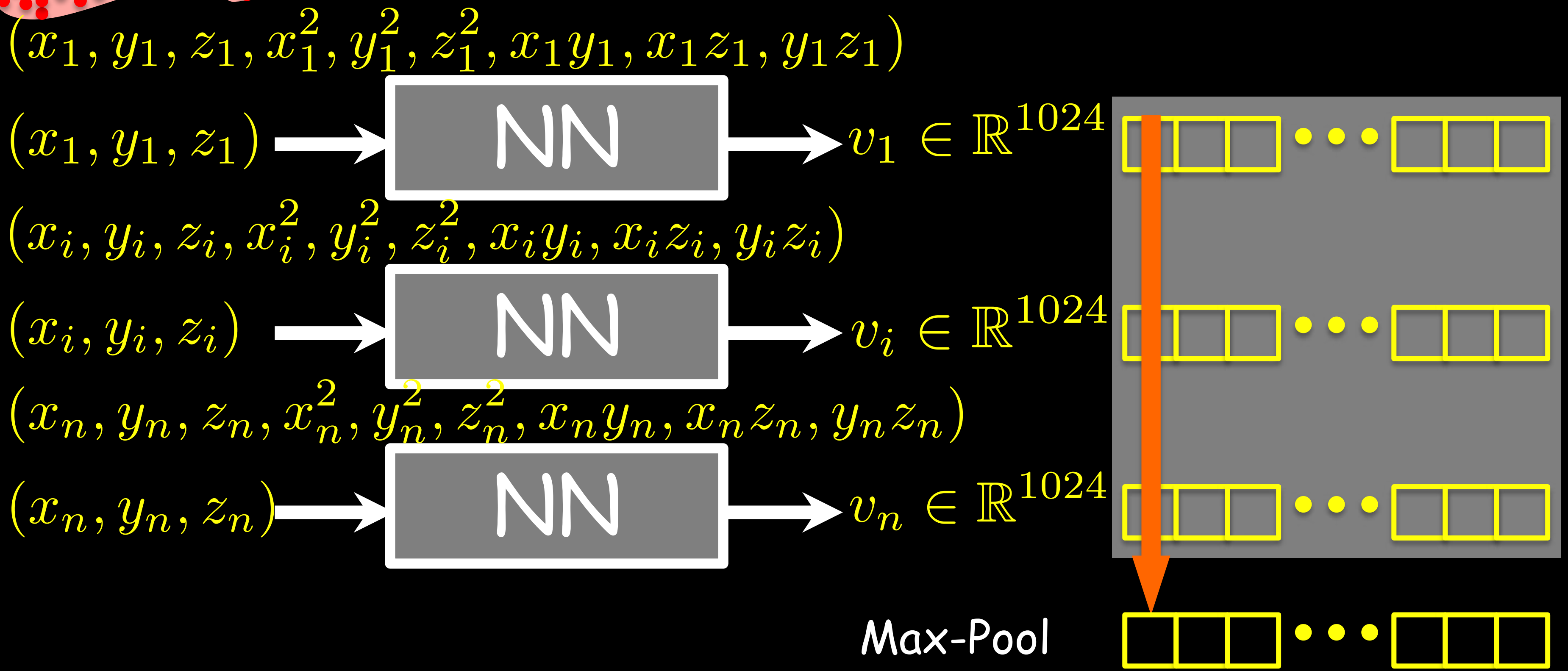
PointNet



Su, Mo, Guibas. CVPR'17

Momenet



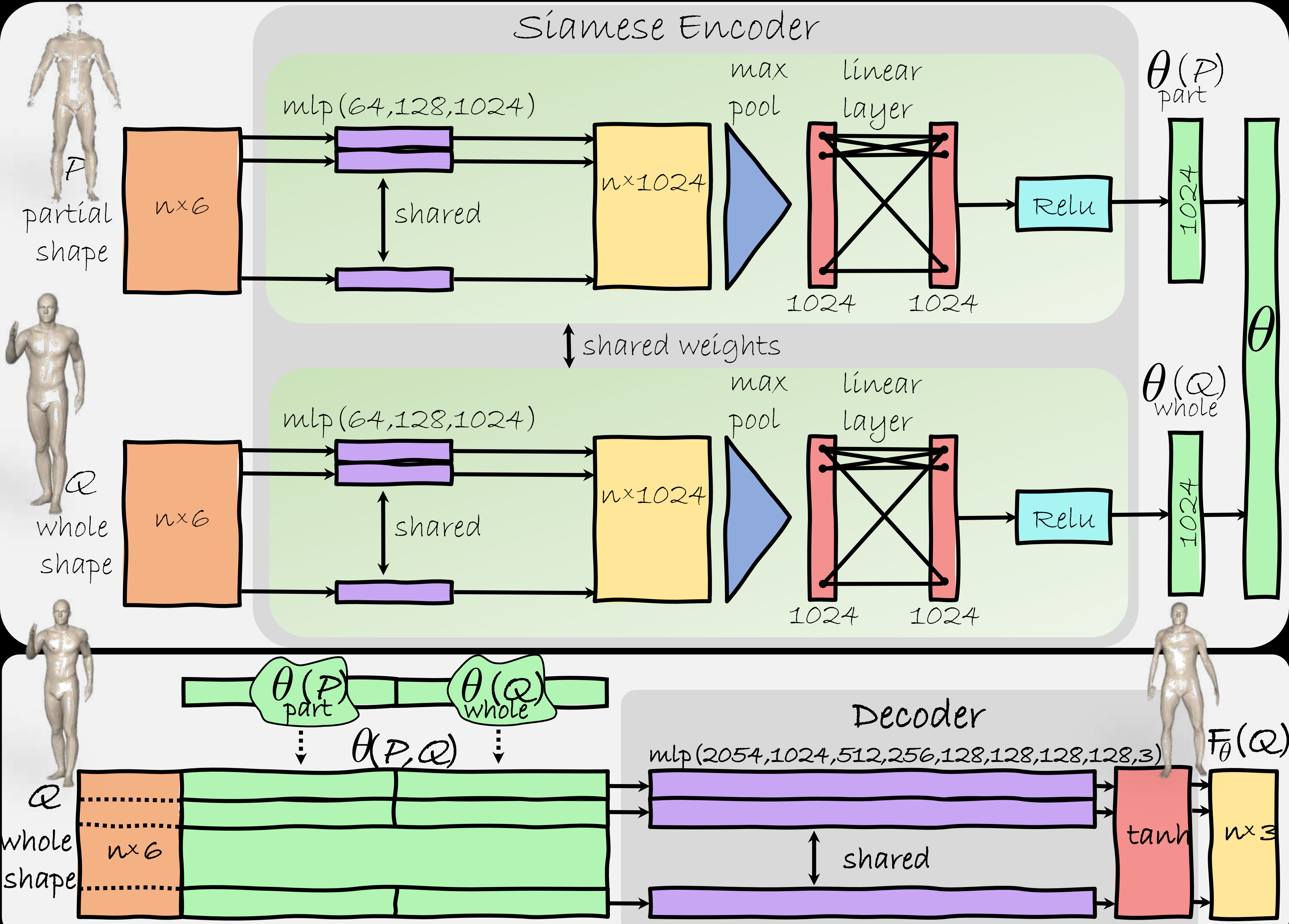


Joseph-Rivlin, Zvirin, K., Flavor the moments, GMDL workshop, ICCV'19

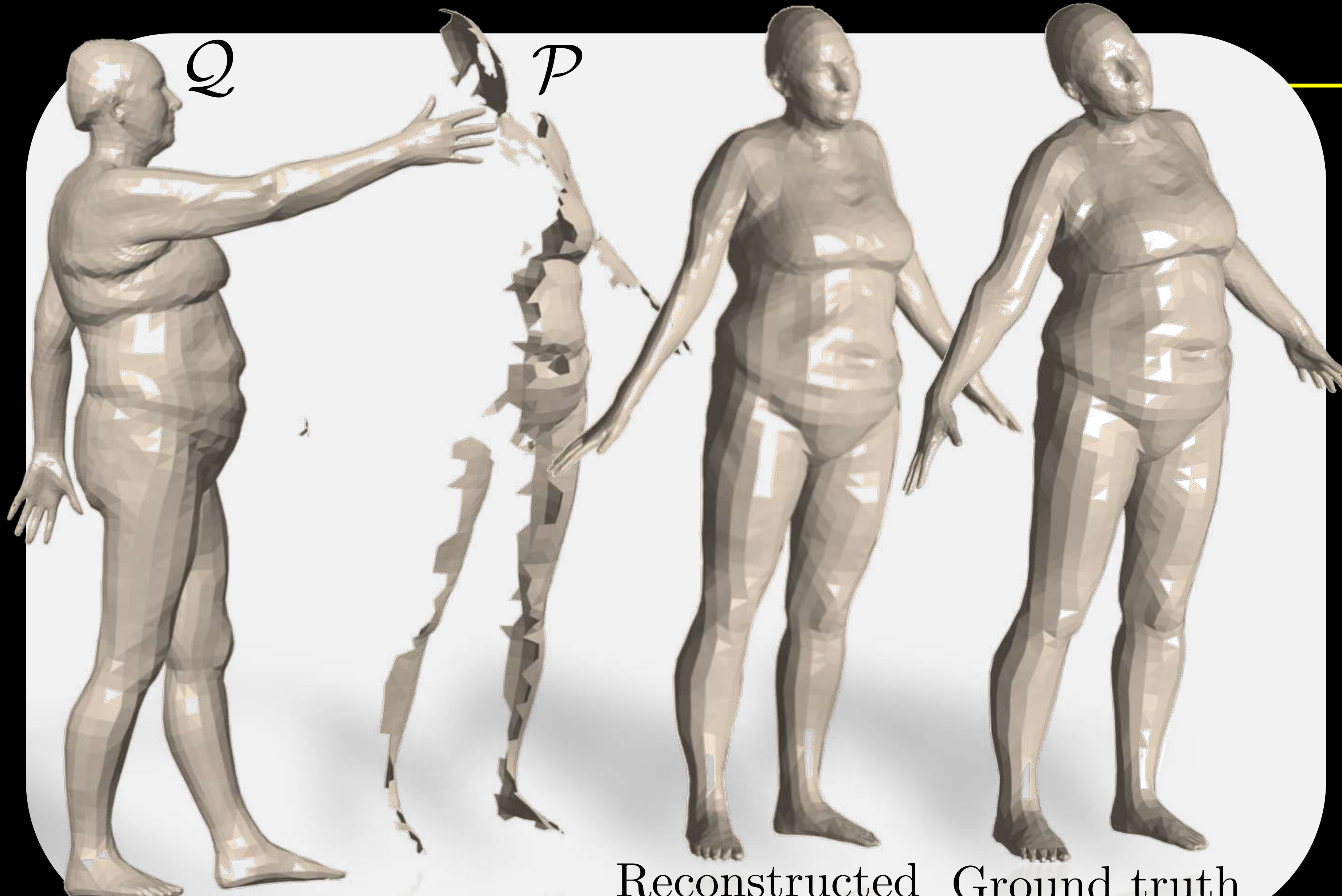
Results on ModelNet40

	Memory (MB)	Inference Time (msec)	Overall Accuracy (%)
PointNet	40	5.6	89.2
PointNet(baseline)	20	5.1	87.9
Momen^{e}t	**20**	**5.1**	**89.6**
PointNet++	12	10.4	90.7
DGCNN	21	17.3	92.2
PCNN	17	54.1	92.3
Momen^{e}t (+kNN)	**21**	**9.6**	**92.4**

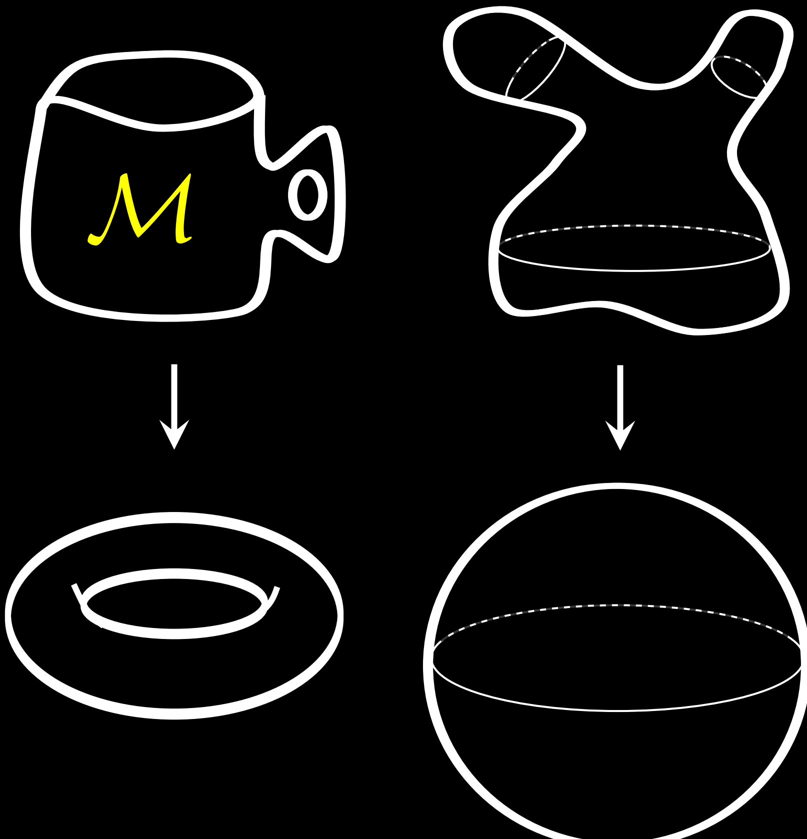
Wu, et al.. 3D shapenets. CVPR2015.
Wang et al. . Dynamic graph CNN. Arxiv 2018.
Atzmon, Maron, & Lipman. Point convolutional NN. ACM T Graph. 2018
Joseph-Rivlin, Zvirin, K., Flavor the moments, GMDL workshop, ICCV'19

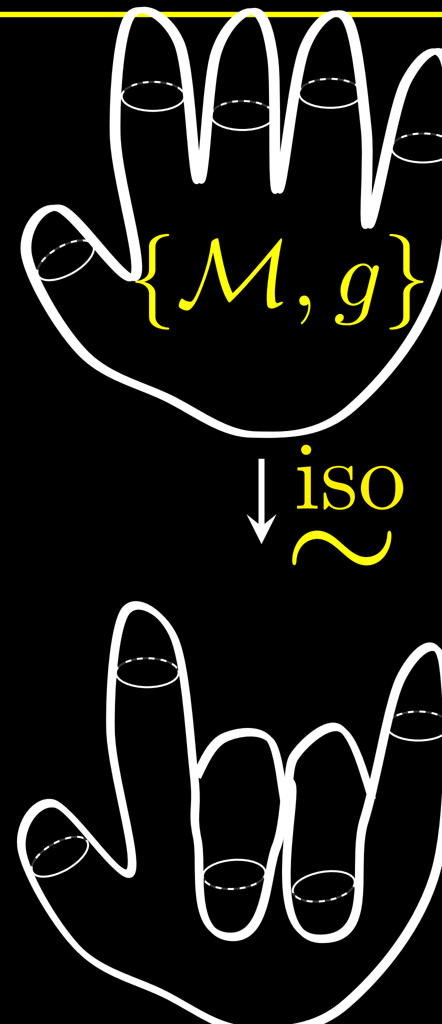


Halimi et al. ECCV 2020

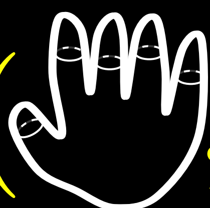
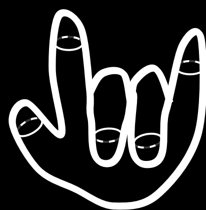
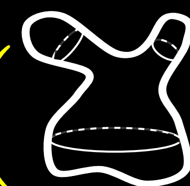

$\mathcal{Q}$
$\mathcal{P}$
Reconstructed
Ground truth

Manifold vs. Riemannian Manifold





Shapes as metric spaces

$d_{GH}($  ,  $) < d_{GH}($  ,  $)$

$MDS, GMDS, SGMDS, GDD, PCA, RPCA,$
$F\text{-}Maps, FM\text{-}Net, GMDS\text{-}Net, SF\text{-}Maps$

Functional Maps Ovsjanikov et al. 2012

$\{\psi_i\}$

$$f = \sum_i \langle f, \psi_i \rangle \psi_i = \sum_i \alpha_i \psi_i$$

$\{\phi_i\}$

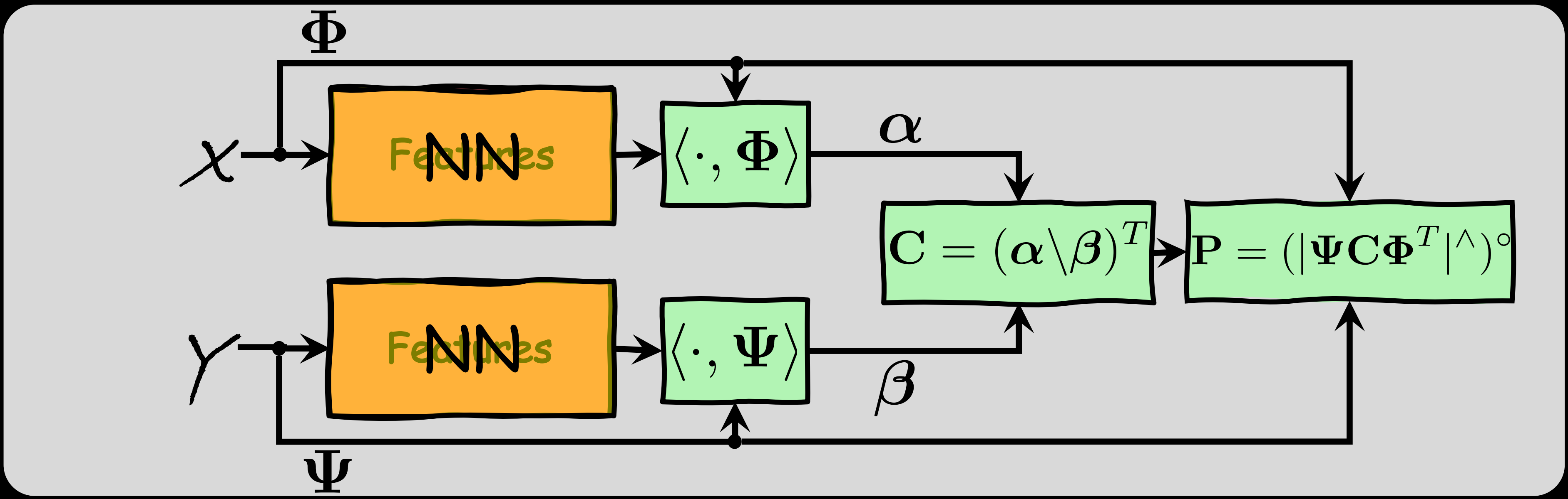
$$f = \sum_i \langle f, \phi_i \rangle \phi_i = \sum_i \beta_i \phi_i$$

$$\beta = C\alpha$$

$$C_{ij} = \langle T(\phi_i), \psi_j \rangle$$

Functional Maps

$$\mathcal{Y} \approx P\mathcal{X}$$



Ovsjanikov et al. 2012

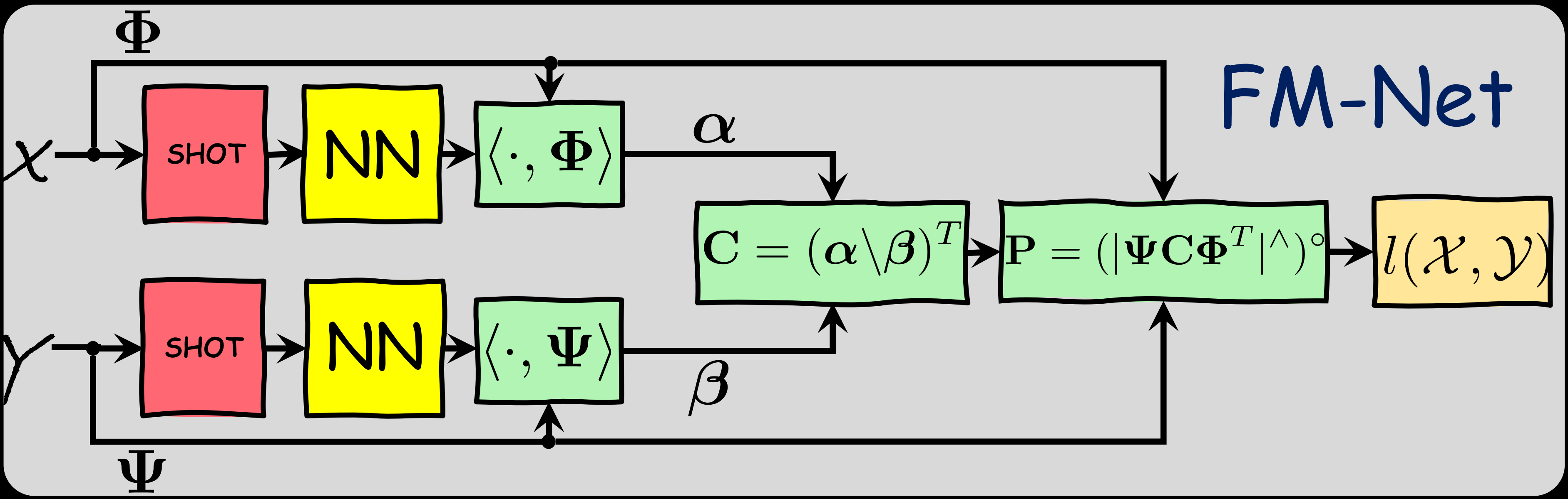
Litany, Remez, Rodola, A Bronstein, M Bronstein ICCV'17

Halimi, Litany, Rodolà, Bronstein, K. CVPR'19

Roufosse, Sharma, Ovsjanikov, ICCV'19

Functional Maps-Net

$$l_{sup}(\mathcal{X},\mathcal{Y}) = \sum_{i\in\mathcal{X}}\sum_{j\in\mathcal{Y}} p_{ij} d^2_{\mathcal{Y}}(j,\pi^*(i))$$



Ovsjanikov et al. 2012

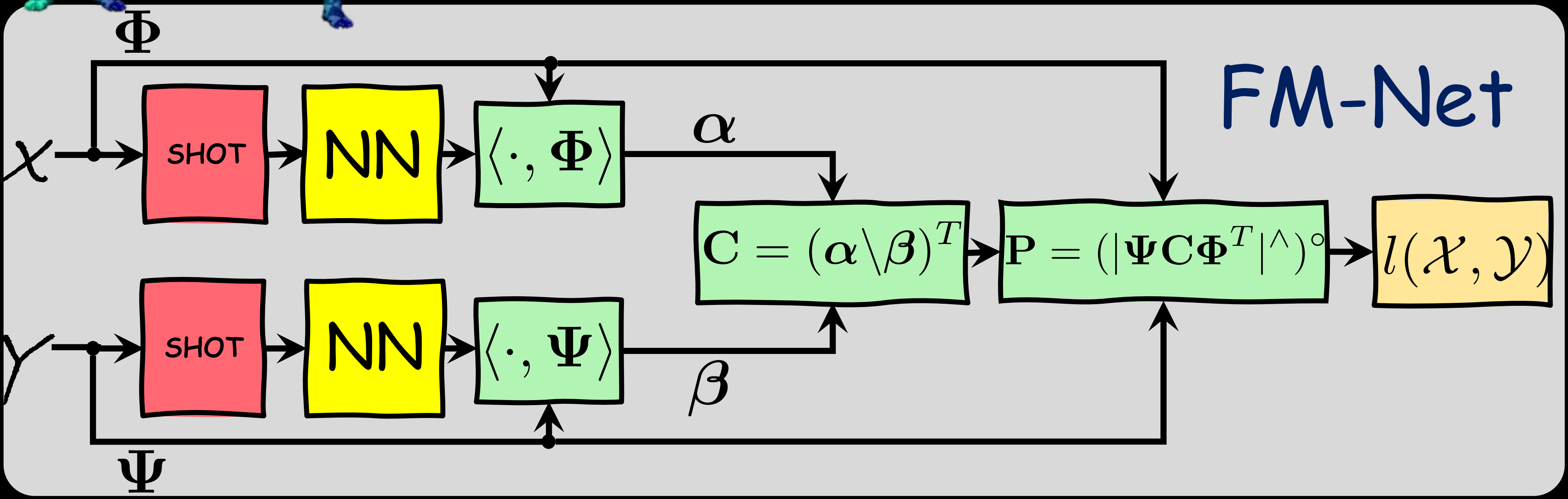
Litany, Remez, Rodola, A Bronstein, M Bronstein ICCV'17

Halimi, Litany, Rodolà, Bronstein, K. CVPR'19

Roufosse, Sharma, Ovsjanikov, ICCV'19

Unsupervised Functional Maps-Net

$$l_{uns}(\mathcal{X},\mathcal{Y}) = \|D_{\mathcal{X}} - P D_{\mathcal{Y}} P^T\|$$



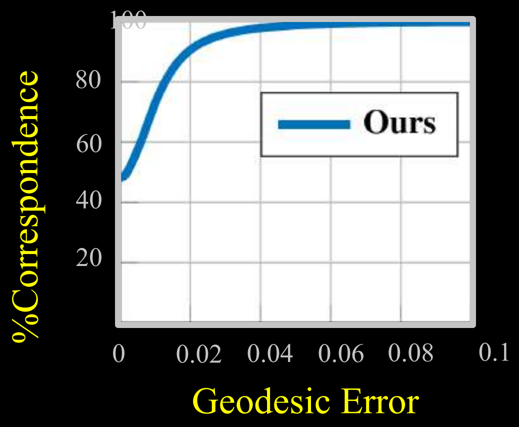
Ovsjanikov et al. 2012

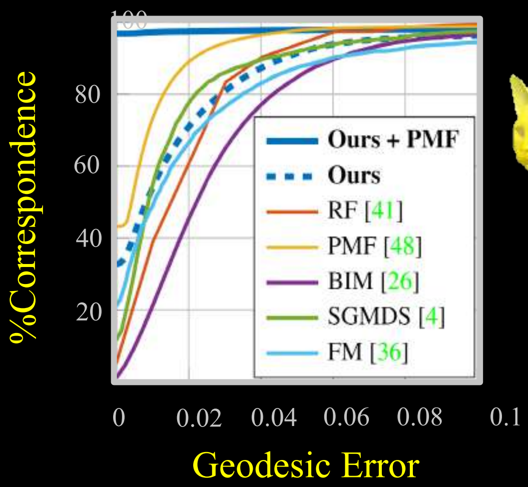
Litany, Remez, Rodola, A Bronstein, M Bronstein ICCV'17

Halimi, Litany, Rodolà, Bronstein, K. CVPR'19

Roufosse, Sharma, Ovsjanikov, ICCV'19

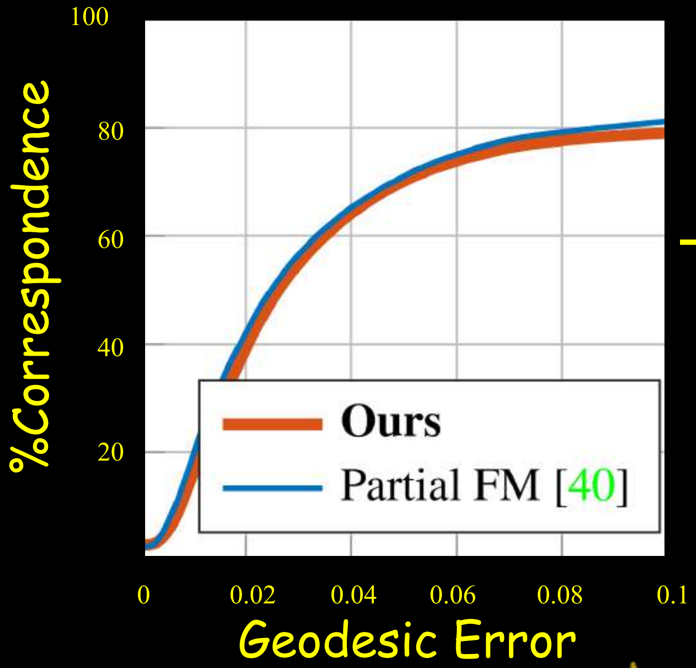
Generalization





Halimi, Litany, Rodolà, Bronstein, K. CVPR'19

Partial Correspondence



Halimi, Litany, Rodolà, Bronstein, K. CVPR'19

Single-pair correspondences

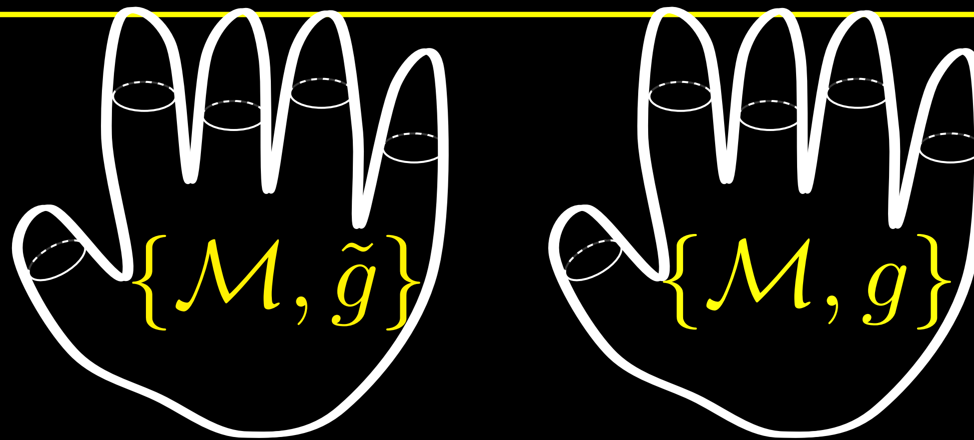
Halimi, Litany, Rodolà, Bronstein, K. CVPR'19

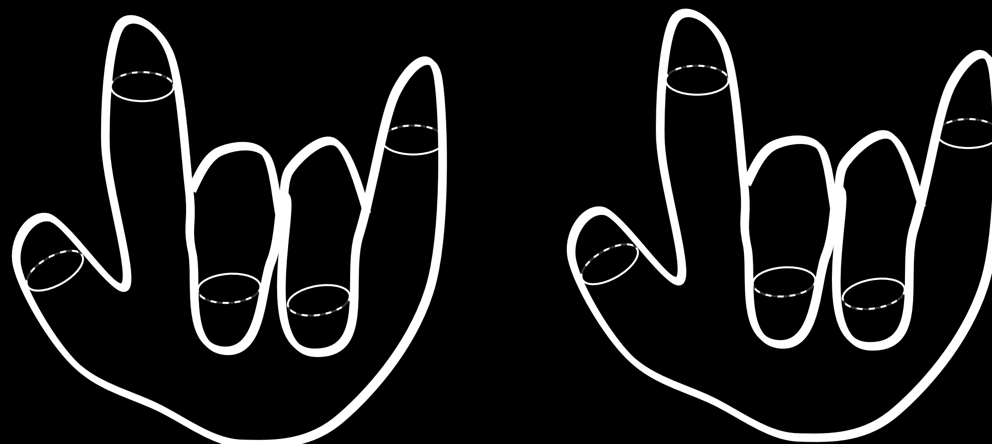
Manifold vs. Riemannian Manifolds

$$\Delta_g \psi_i = \lambda_i \psi_i$$

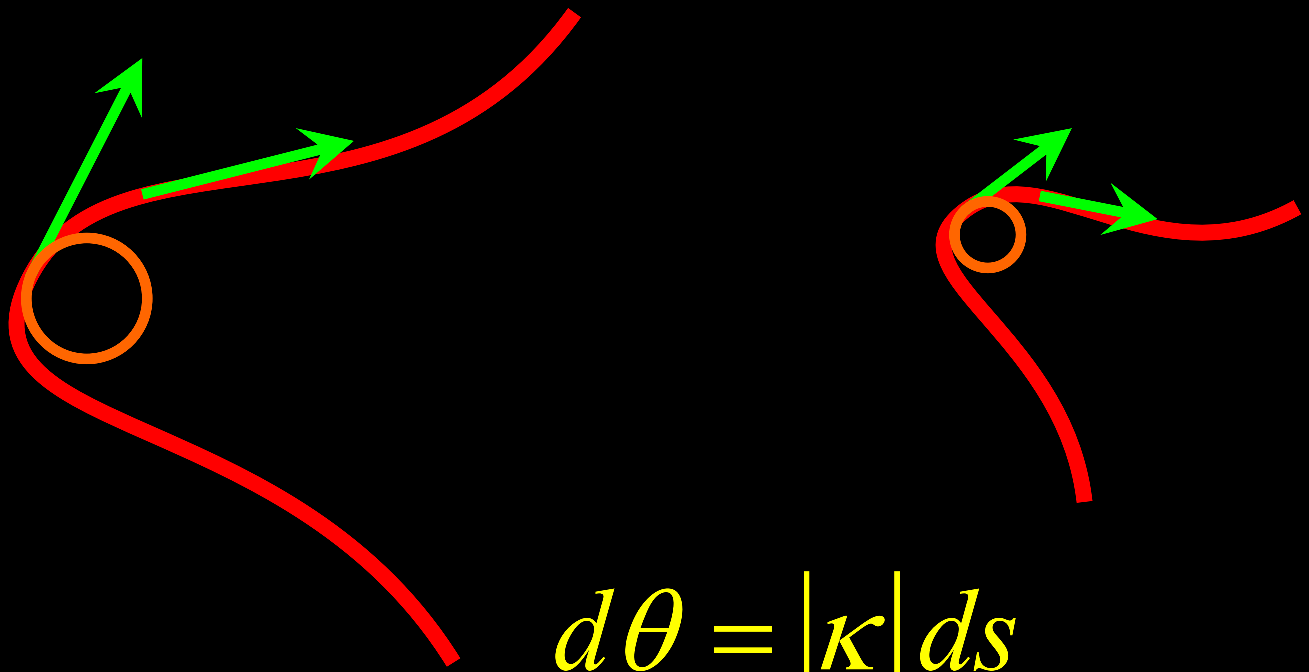
$$\Delta_{\tilde{g}} \tilde{\psi}_i = \tilde{\lambda}_i \tilde{\psi}_i$$

$$C_{ij} = \langle \psi_i, \tilde{\psi}_j \rangle_{\tilde{g}}$$





Scale invariance?



$$d\theta = |\kappa| ds$$

Surface Laplacian

$$\Delta_{\tilde{g}} \equiv -\frac{1}{\sqrt{\tilde{g}}}\partial_i \sqrt{\tilde{g}}\,\tilde{g}^{ij}\partial_j$$

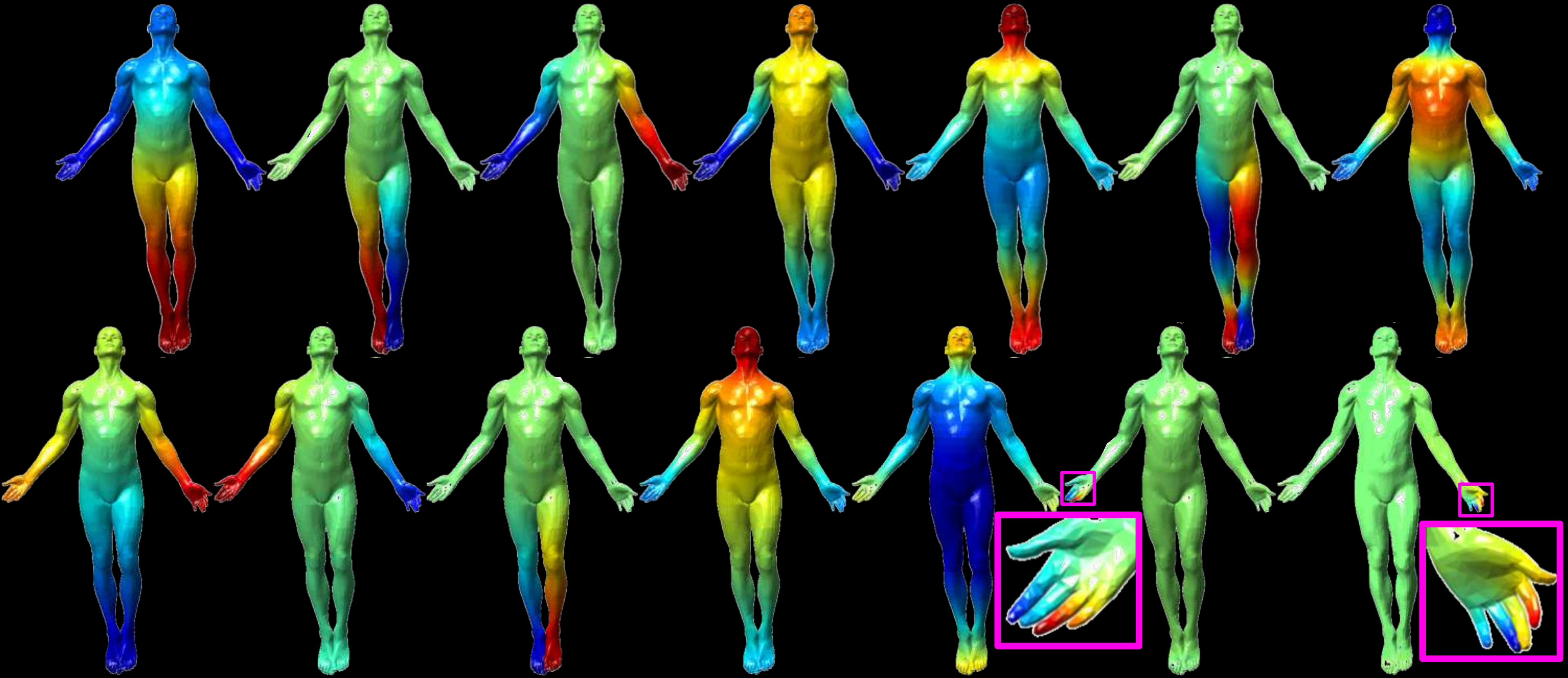

Δ_g

$\tilde{g}_{ij} =$  $= |\kappa_1 \kappa_2| \langle S_i, S_j \rangle$

Eigenfunctions

$$g_{ij} = \langle S_{\omega_i}, S_{\omega_j} \rangle$$

$$-\Delta_g \psi_i = \lambda_i \psi_i$$

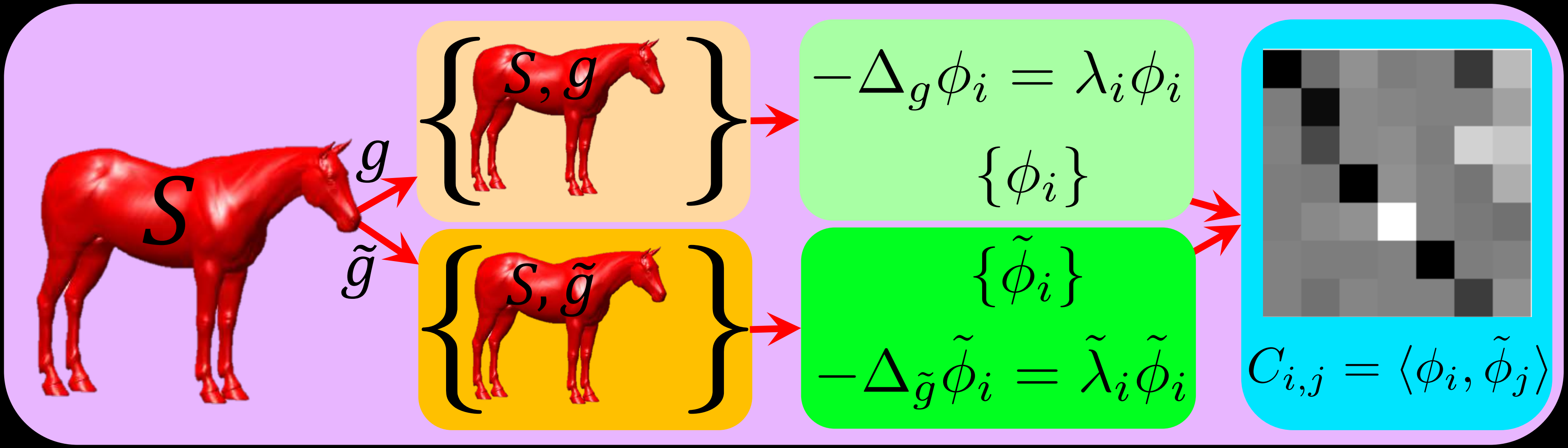


$$\tilde{g}_{ij} = |K| g_{ij}$$

$$-\Delta_{\tilde{g}} \tilde{\psi}_i = \tilde{\lambda}_i \tilde{\psi}_i$$

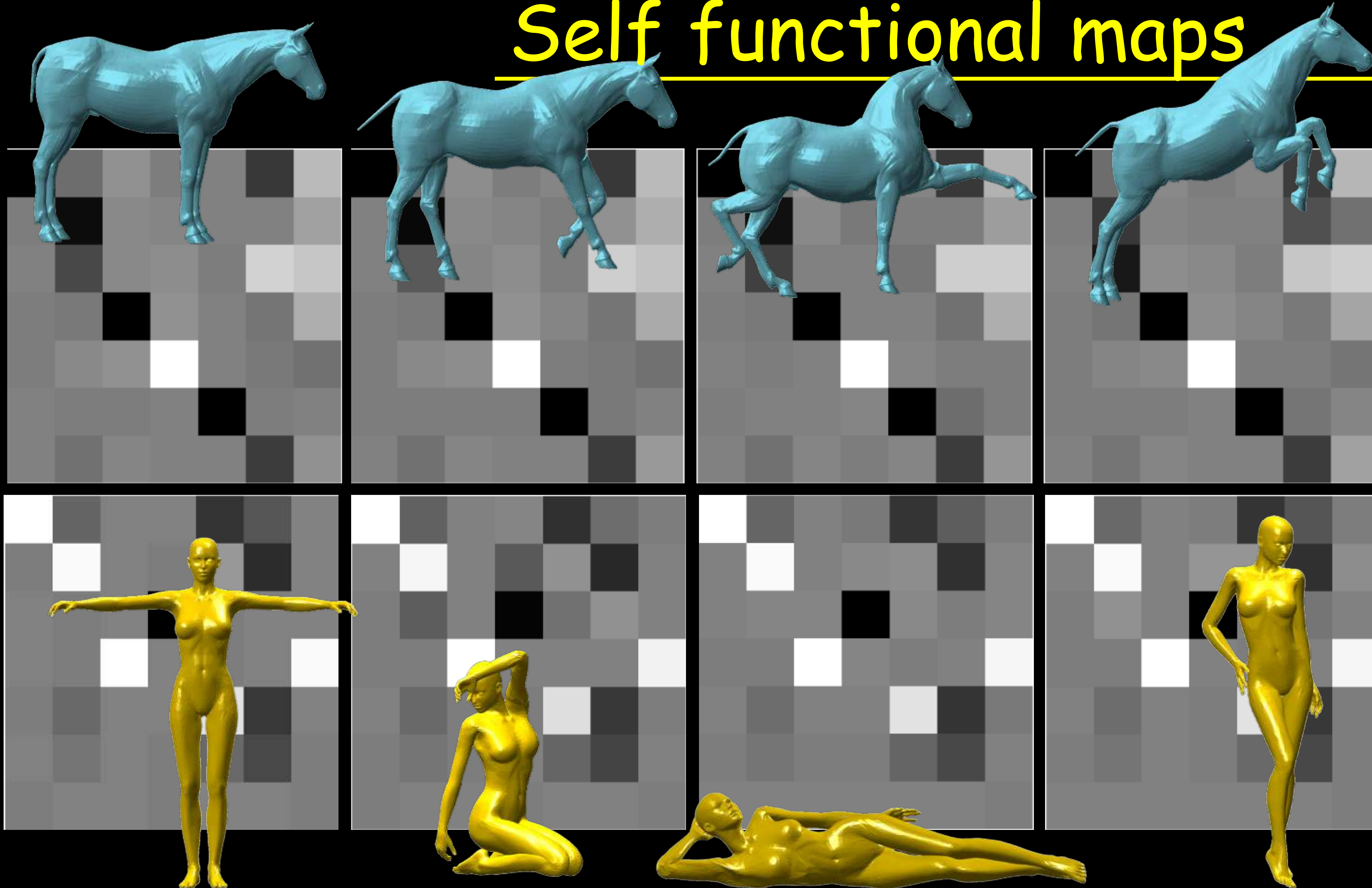
Halimi & K. 2018

Self functional maps

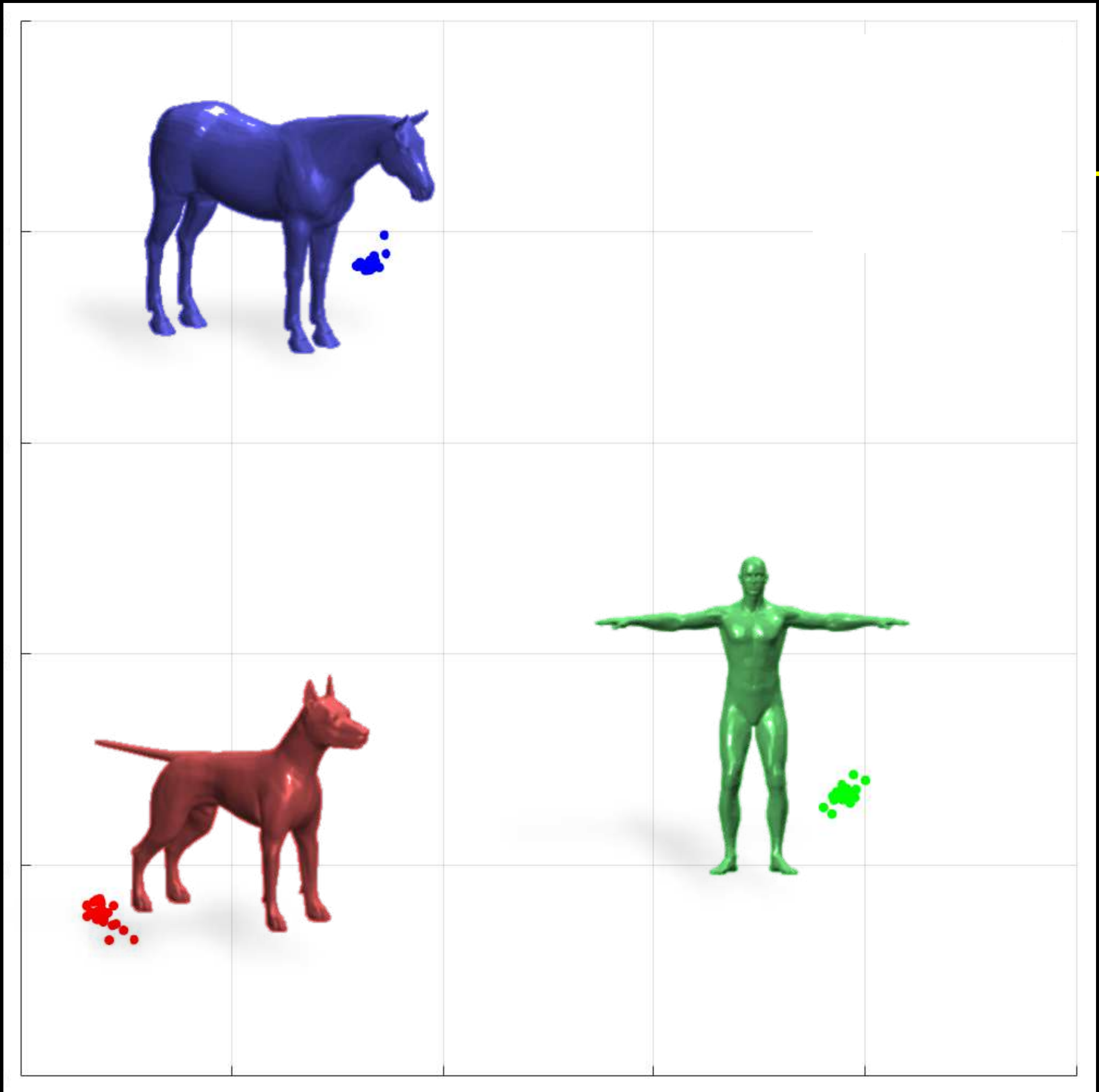


Halimi & K. 2018

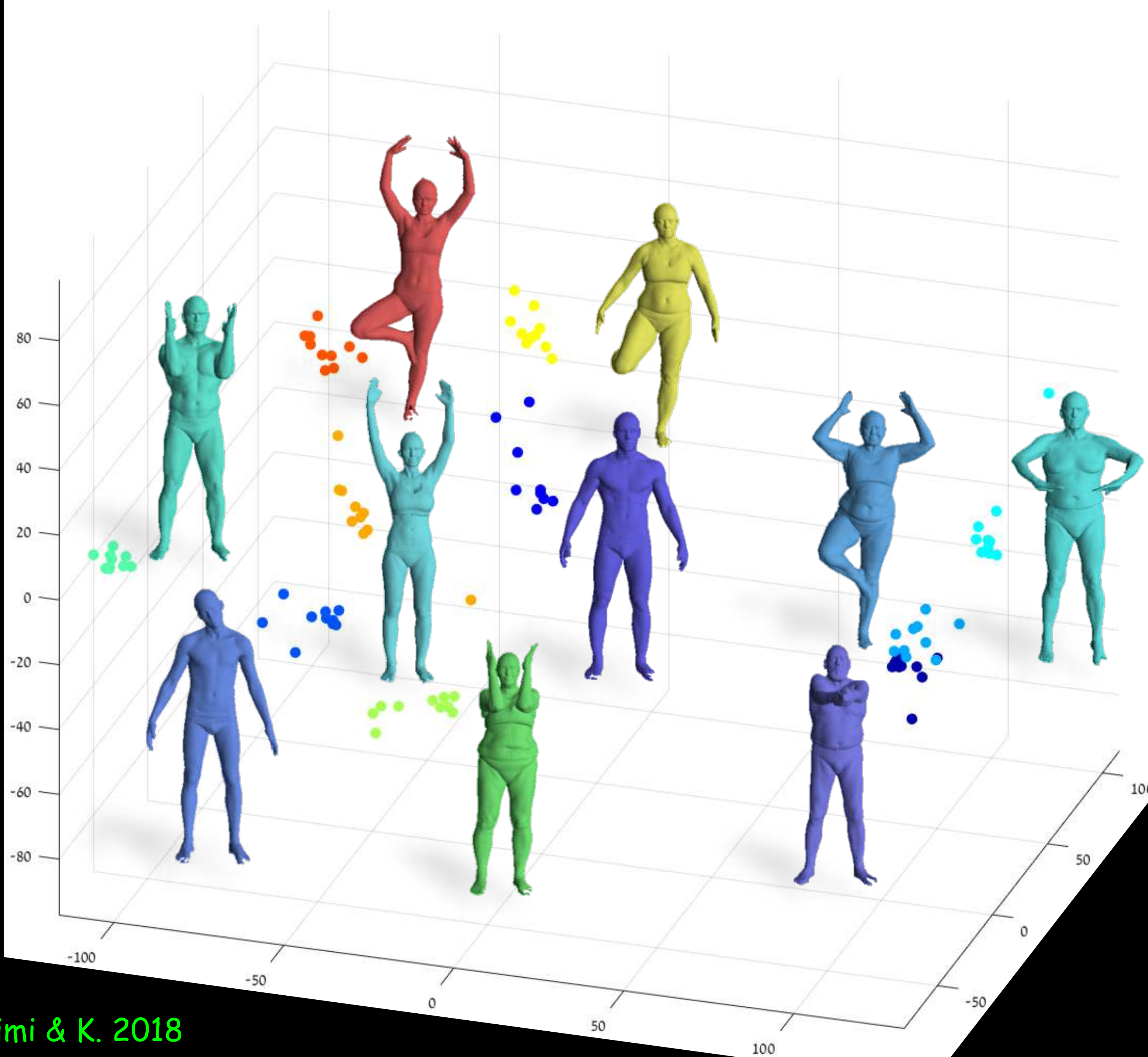
Self functional maps



Halimi & K. 2018



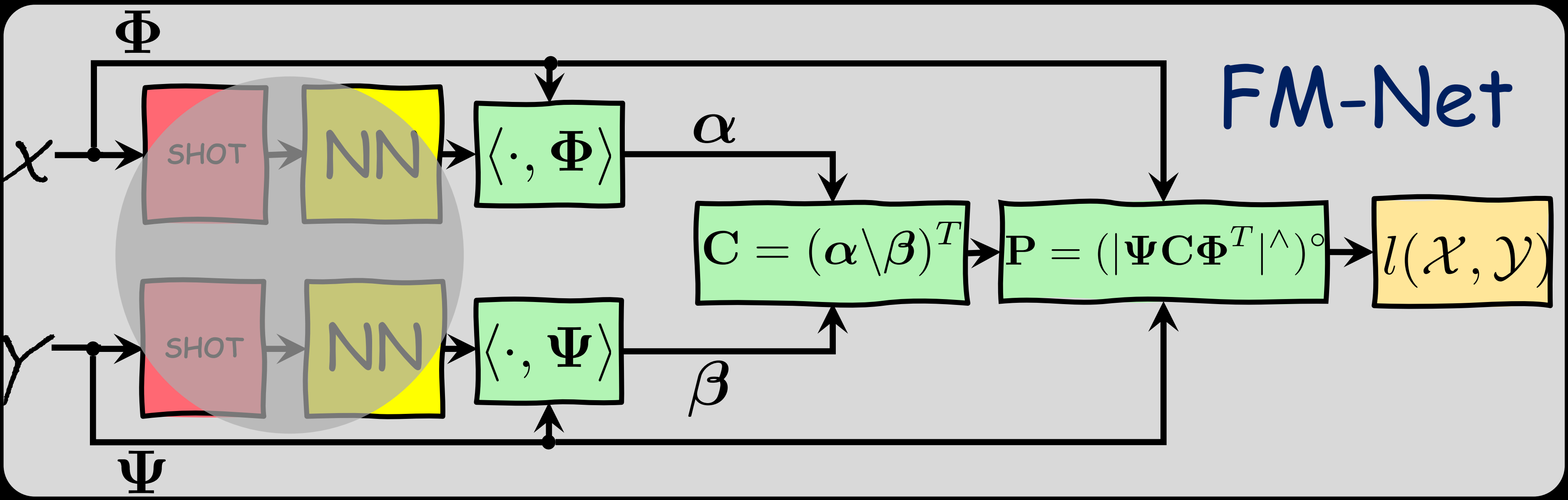
Halimi & K. 2018



Halimi & K. 2018

Unsupervised Functional Maps-Net

$$l_{uns}(\mathcal{X},\mathcal{Y}) = \|D_{\mathcal{X}} - P D_{\mathcal{Y}} P^T\|$$



Ovsjanikov et al. 2012

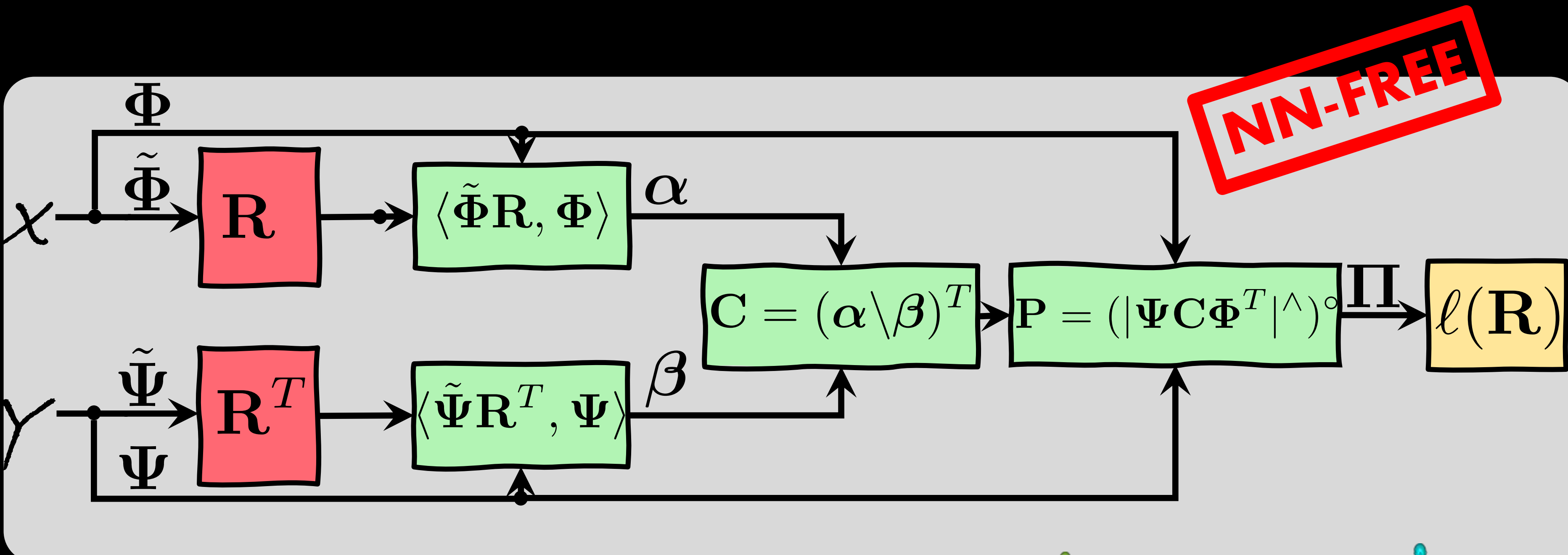
Litany, Remez, Rodola, A Bronstein, M Bronstein ICCV'17

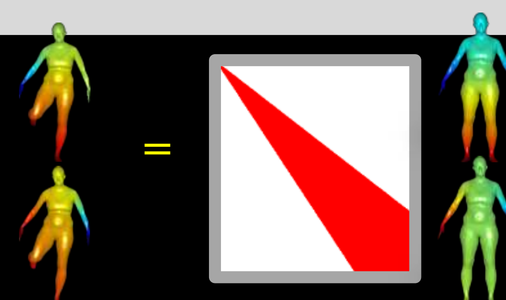
Halimi, Litany, Rodola, A Bronstein, K. CVPR'19

Roufosse, Sharma, Ovsjanikov, ICCV'19

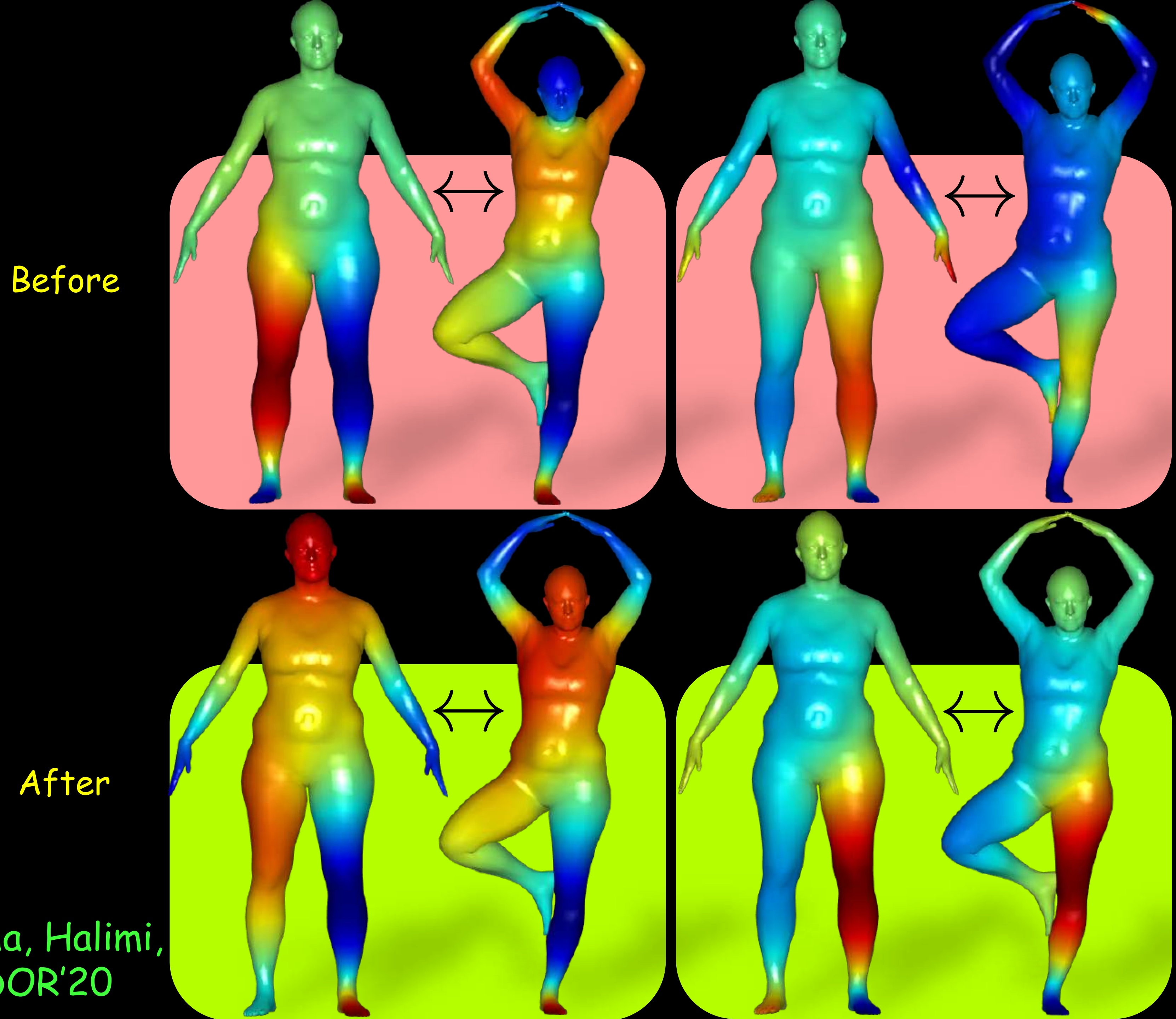
Aligning scale-invariant LBO eigenfunctions

$$l_{uns}(\mathcal{X},\mathcal{Y}) = \|D_{\mathcal{X}} - P D_{\mathcal{Y}} P^T\|$$





Bracha, Halimi, K. 3DOR'20



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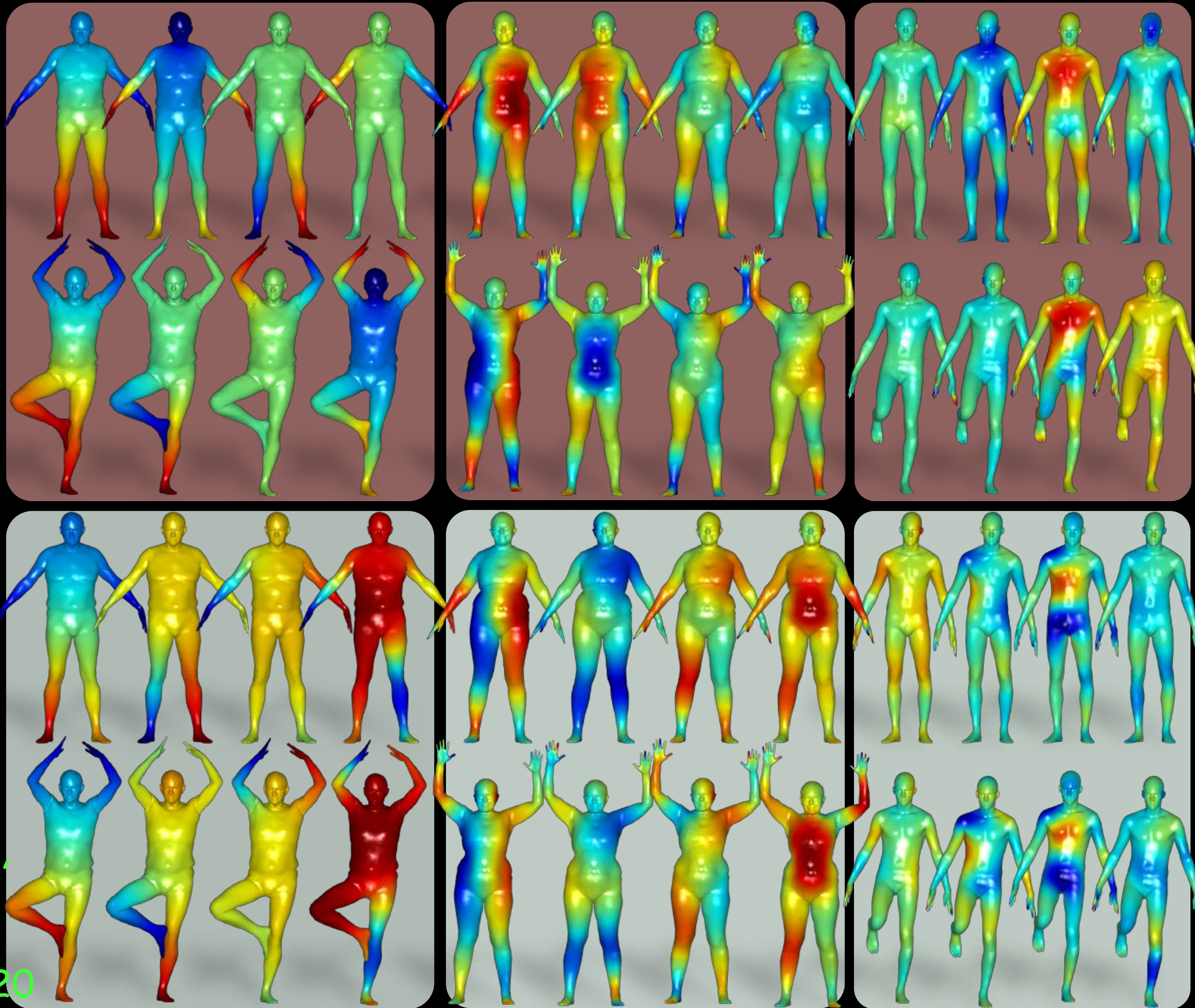
Before

After



Bracha, Halimi, K. 3DOR'20


Thank you for your attention