Fine-scale 3D-copying for culturage heritage

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Joint work with Matthieu Pizenberg
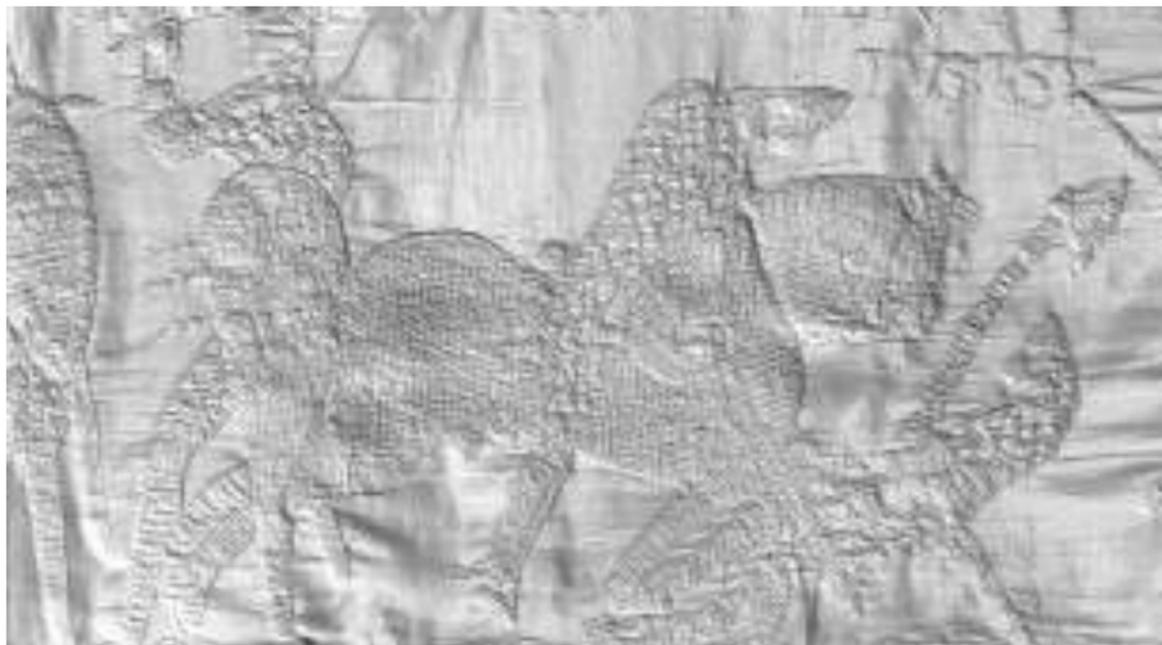
INDAM Workshop: from 3D Vision to 3D Printing
Feb 12th, 2021

Aim of the talk

Fine-scale 3D-reconstruction of Bayeux tapestry,
to create 3D-printed mockups to be touched
by visually impaired people

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Outline

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Context of the project

- Funding: Normandy region and ANR projects "Inclusive Museum Guide" (2019-2024)
- Partners: Caen univ., Rouen univ., CNRS, Bayeux museum, Royal Holloway univ., Westminster univ.

Bayeux tapestry

70m-long medieval wool and linen *embrodery* telling the
conquest of England by William, Duke of Normandy, in 1066

Bayeux tapestry: Edward

Story starts with the death of old king Edward...

Bayeux tapestry: William

Edward had chosen his cousin William for his
succession on the British throne

Bayeux tapestry: Harold

Yet Edward's brother-in-law Harold took the throne

Bayeux tapestry: the Northmen armada

William's armada crossed the Channel

Bayeux tapestry: the battle of Hastings

... and William defeated Harold during the battle of Hastings, hence becoming King of England

The IMG project

Question: how can we make visually-impaired people "feel" such artifacts, knowing that

- they cannot see it
- they cannot touch it (the tapestry is protected by glass)

$\rightarrow$ Re-think the way we access visual artworks: audio-descriptions, navigation tools, **tactile representations**, etc.

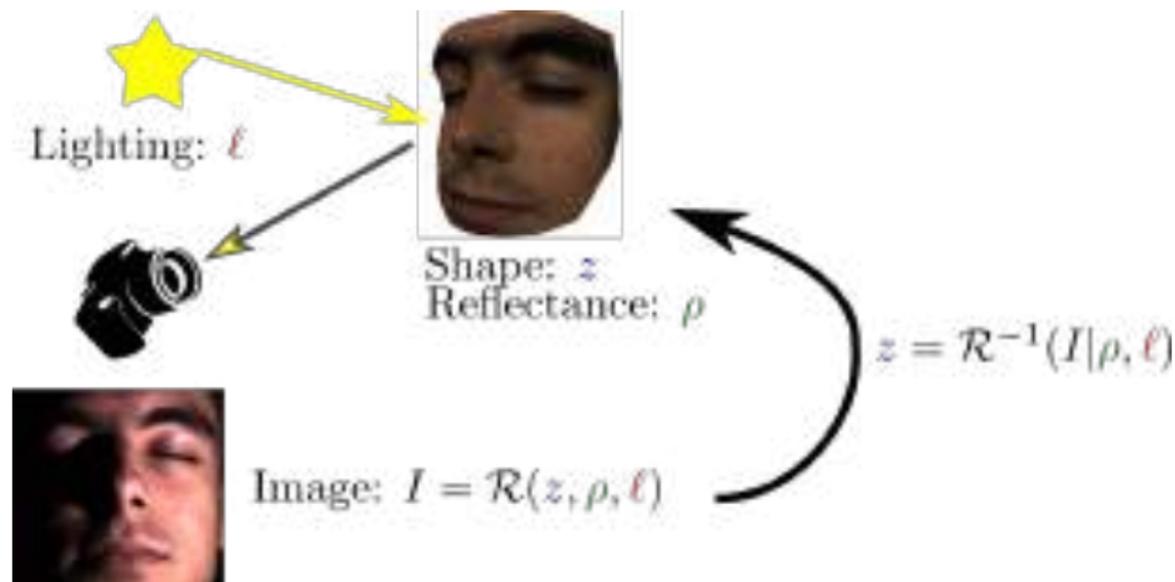# Towards a tactile representation of visual artworks

Goal: extract micro-geometry of the scene ("texture"),
in view of 3D-printing

Challenges: no direct access (behind glass), fragility to light
exposure, thinness of the wool strings, color changes...

Outline

3D-reconstruction as an inverse problem

Lighting: ℓ

Shape: z
Reflectance: ρ

$z = \mathcal{R}^{-1}(I | \rho, \ell)$

Image: $I = \mathcal{R}(z, \rho, \ell)$

Goal: invert the radiance function $\mathcal{R}$ to recover the shape z (and, possibly, lighting ℓ and reflectance ρ) of the pictured surface

Context: shape-from-X

	Geometric techniques	Photometric techniques
Single image	Structured light	**Shape-from-shading**
	Shape-from-shadows	
	Shape-from-contours	
	Shape-from-texture	
	Shape-from-template	
Multi-images	Structure-from-motion	**Photometric stereo**
	Stereopsis	Shape-from-polarisation
	Shape-from-silhouettes	
	Shape-from-focus	

Geometric techniques aim at identifying and analysing features. This presentation rather focuses on two photometric techniques (shape-from-shading and photometric stereo), which aim at inverting a physics-based image formation model.

Geometric techniques

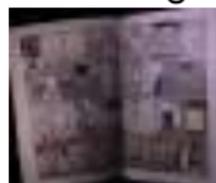Mature workflows based on structure-from-motion and multi-view stereo are available, e.g. Alice Vision's Meshroom:
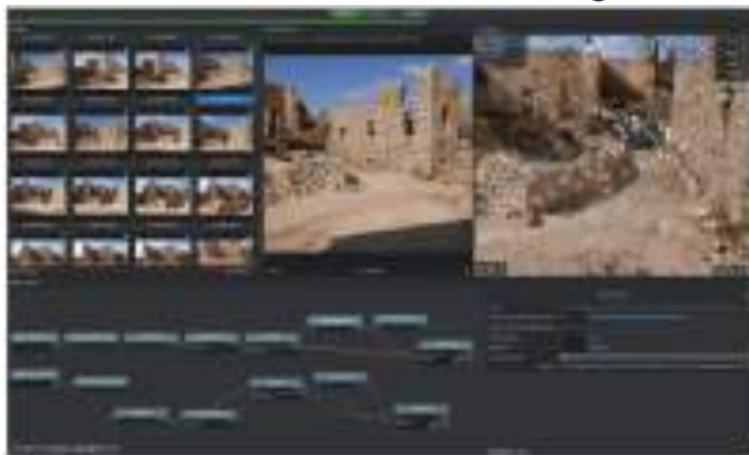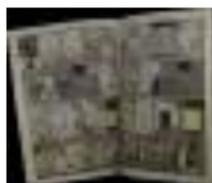

Limitations:
- May miss thin structures
- Only estimate shape, not reflectance

3D-scanning

3D-scanning $=$ estimation of shape *and* color

Image

Reflectance

3D-model

(a)   (b)   (c)

(d)   (e)

- Geometric techniques (e.g. SfM, MVS) only recover shape (no color estimation)
- Photometric techniques (e.g., shape-from-shading, photometric stereo), which are based on inverting the image formation model, recover both ($\approx$ *inverse rendering*)

Photometric techniques

Top-performing for the recovery of very thin structures

Example: close-up on a 10 euros banknote

Normals to the surface
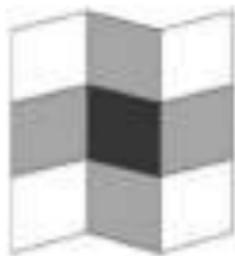
Shape

RGB camera

Albedo (reflectance)

Durou et al., *Advances in Photometric 3D-Reconstruction*

Shape-from-shading: a classic ill-posed problem

Given an image $\mathbf{I} : \Omega \subset \mathbb{R}^2 \to \mathbb{R}^m$, *Shape-from-Shading* (SfS) consists in inverting the image irradiance equation

$$\mathbf{I} = \mathcal{R}(z, \rho, \ell) \tag{1}$$

with $\mathcal{R}$ a *radiance* function depending on the unknown depth $z : \Omega \to \mathbb{R}$, surface reflectance $\rho : \Omega \to \mathbb{R}^m$, and incident lighting $\ell : \Omega \to \mathbb{S}^2$.

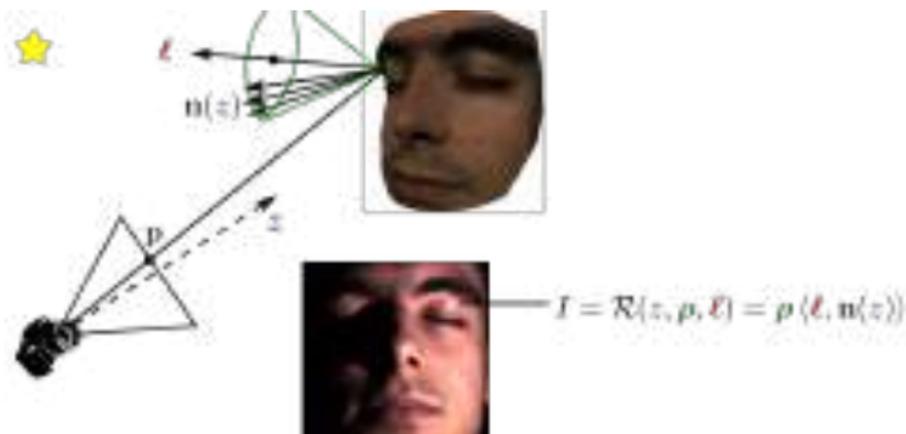

RGB image:
$\mathbf{I}$

Sculptor's explanation:
z

Painter's explanation:
ρ

Gaffer's explanation:
ℓ

Adelson and Pentland's workshop metaphor

Local ill-posedness in shape-from-shading

$$I = \mathcal{R}(z, \rho, \ell) = \rho \langle \ell, \mathbf{n}(z) \rangle$$

Even if ρ and ℓ are known, the normal cannot be estimated directly by inverting the forward model, because infinitely many normals, lying on a cone, are equally admittible

$\rightarrow$ A regularization mechanism is necessary. The most natural way to do so consists in considering a differential version of the forward model, assuming the surface is differentiable almost everywhere

Illustration of ill-posedness

Even with known surface reflectance ρ and incident lighting ℓ, shape estimation by SfS is an ill-posed inverse problem (Horn, 1970)

Example: two solutions of $\mathbf{I} = \mathcal{R}(z, \rho, \ell)$ with $\mathbf{I} :=$ Lena, white reflectance ($\rho \equiv 1$) and frontal lighting ($\ell \equiv [0, 0, -1]^\top$):

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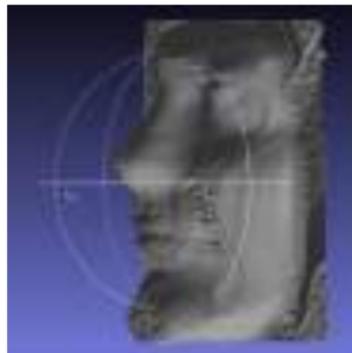
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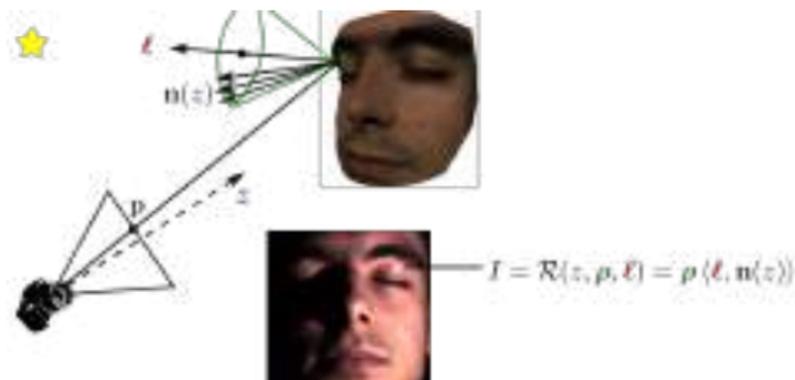
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Differential image formation model

$$I = \mathcal{R}(z, \rho, \ell) = \rho \langle \ell, \mathbf{n}(z) \rangle$$

Assuming orthographic projection, the scene's surface is characterized by a depth map $z : \Omega \subset \mathbb{R}^2 \to \mathbb{R}$ such that

$$\mathbf{n}(z) = \frac{1}{\sqrt{|\nabla z|^2 + 1}} \begin{bmatrix} \nabla z \\ -1 \end{bmatrix}$$

And thus we get the following nonlinear PDE in z:

$$I = \mathcal{R}(z, \rho, \ell) = \frac{\rho}{\sqrt{|\nabla z|^2 + 1}} \langle \ell, \begin{bmatrix} \nabla z \\ -1 \end{bmatrix} \rangle$$

Eikonal model

An important instance of the differential model

$$I = \mathcal{R}(z, \rho, \ell) = \frac{\rho}{\sqrt{|\nabla z|^2 + 1}} \langle \ell, \begin{bmatrix} \nabla z \\ -1 \end{bmatrix} \rangle$$

is that of a white surface ($\rho \equiv 1$) under frontal lighting
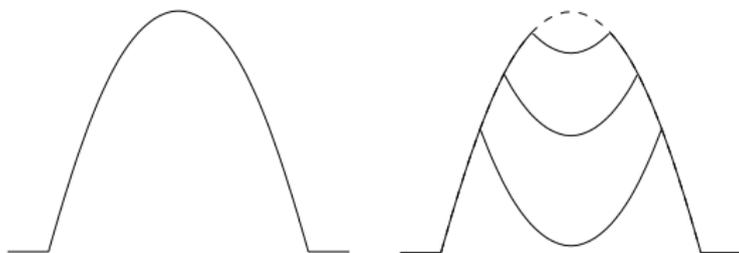($\ell = [0, 0, -1]^\top$). This yields the celebrated *eikonal equation*:

$$|\nabla z| = \sqrt{\frac{1}{I^2} - 1}$$

The concave/convex ambiguity

The eikonal equation

$$|\nabla z| = \sqrt{\frac{1}{I^2} - 1}$$

characterizes the gradient's magnitude, but not its sign: this is the concave/convex ambiguity:

Left: example of the 1D-surface $z = 1 - x^2$. Right: Under vertical lighting, three other solutions (amongst an infinity), which are differentiable almost everywhere, satisfy the same eikonal equation.

The concave/convex ambiguity

Humans solve the concave/convex ambiguity intuitively because we learnt that the world is mostly "convex"

A volcano in Hawaii (copyright Whitman Richards), looking like a cracter if the image is rotated by π.

Among all the solutions to the eikonal equation, it is thus reasonable to look for the "maximum" one

Solving the eikonal equation

The *maximum viscosity solution* of the discrete eikonal equation

$$\left|\nabla z_{i,j}\right| = f_{i,j} := \sqrt{\frac{1}{l_{i,j}^2} - 1}$$

can be numerically approximated using Fast Matching, semi-Lagrangian methods, or the following Lax-Friedrichs Hamiltonian scheme:
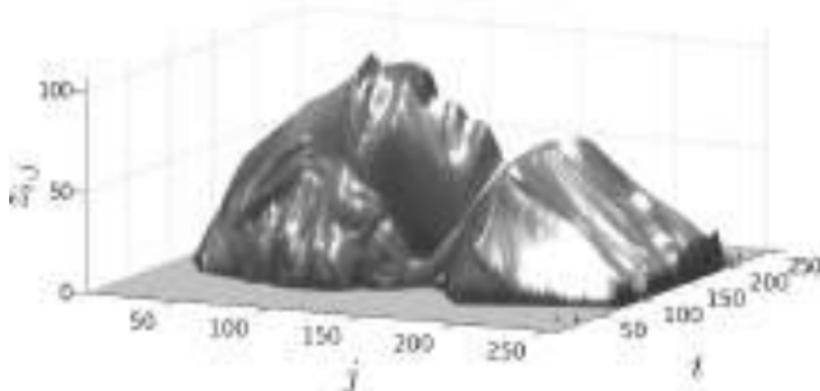
$$z_{i,j}^{(k+1)} = \frac{z_{i-1,j}^{(k)} + z_{i+1,j}^{(k)} + z_{i,j-1}^{(k)} + z_{i,j+1}^{(k)}}{4}$$
$$-\frac{1}{2}\left(\sqrt{\left(\frac{z_{i+1,j}^{(k)} - z_{i-1,j}^{(k)}}{2}\right)^2 + \left(\frac{z_{i,j+1}^{(k)} - z_{i,j-1}^{(k)}}{2}\right)^2} - f_{i,j}\right)$$

with Dirichlet boundary condition $z_{i,j} = g_{i,j}$ if $(i,j) \notin \Omega$

Example of SfS result

Example of result at convergence of the previous scheme, on a real-world dataset:


3D-reconstruction at iteration 512

Subtle geometric details are recovered, yet there is a clear low-frequency biass, and most real-world scenes do not have uniform reflectance.

Most natural solution: capture multiple images, under varying illumination.

Photometric stereo

When no prior knowledge is available, it is hopeless to achieve a reasonable 3D-reconstruction based on SfS

Photometric stereo (Woodham, 1978) is an extension of SfS which considers multiple images of the surface, taken from the **same viewing angle** but under **varying lighting**. It can unambiguously recover geometry, and it is the only shape-from-X technique which is able to **estimate reflectance**
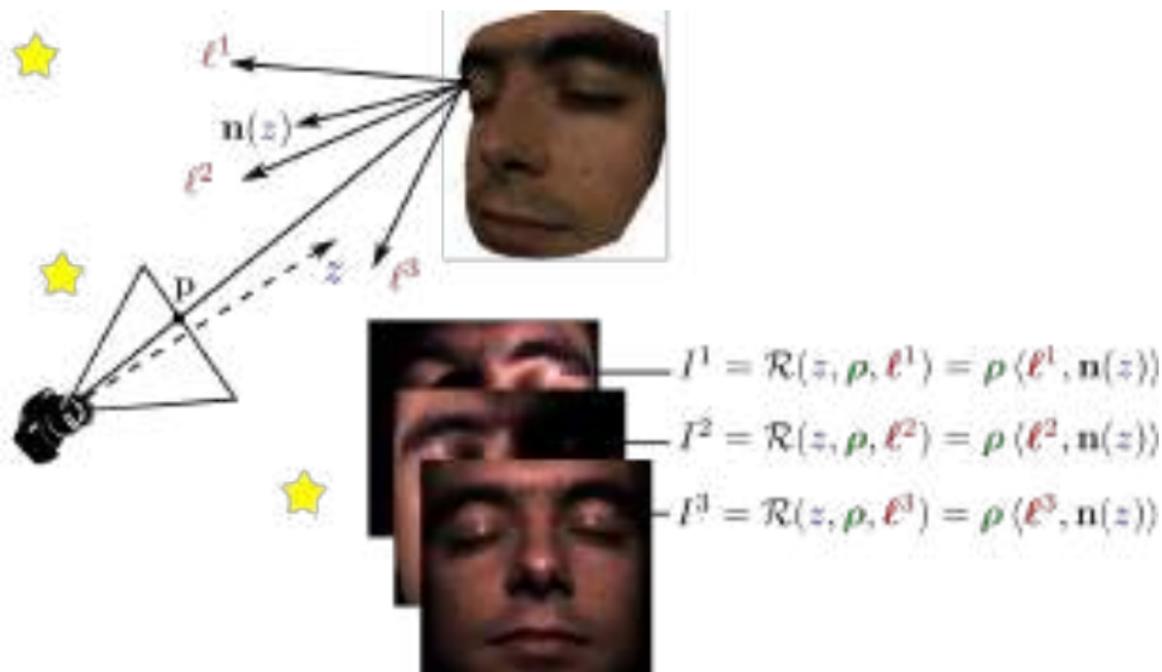

Normals to the surface

Shape

RGB camera + varying lighting
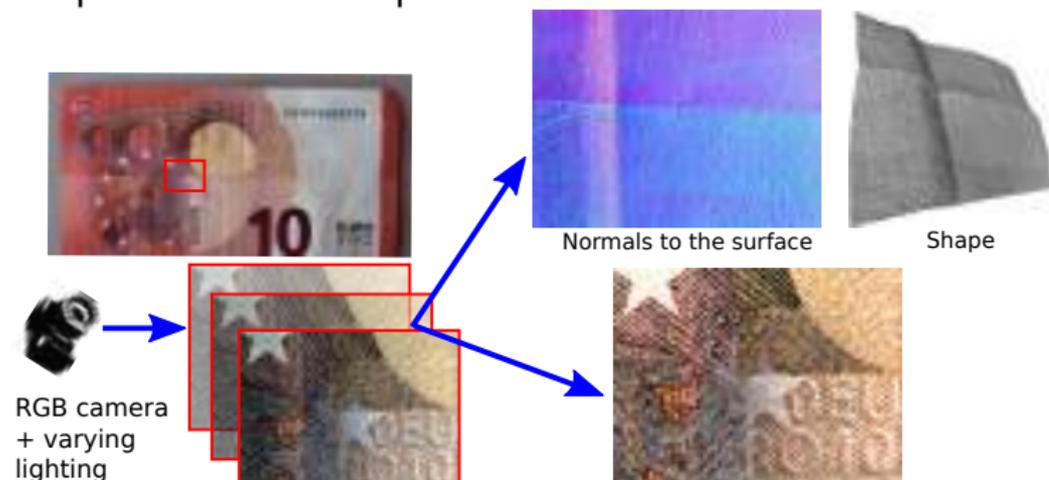
Well-posedness of photometric stereo

$$I^1 = \mathbf{l}^1 \cdot \mathbf{n} = \cos\theta^1$$
$$I^2 = \mathbf{l}^2 \cdot \mathbf{n} = \cos\theta^2$$
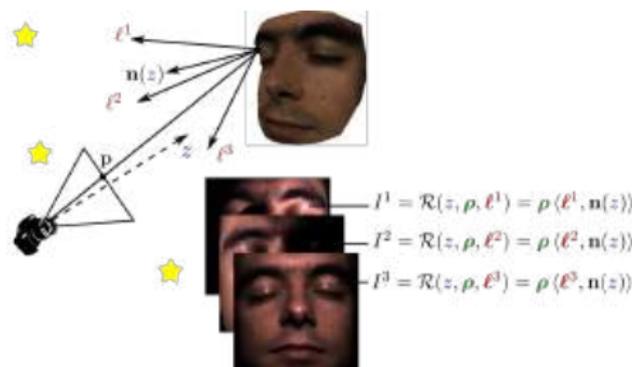$$I^3 = \mathbf{l}^3 \cdot \mathbf{n} = \cos\theta^3$$
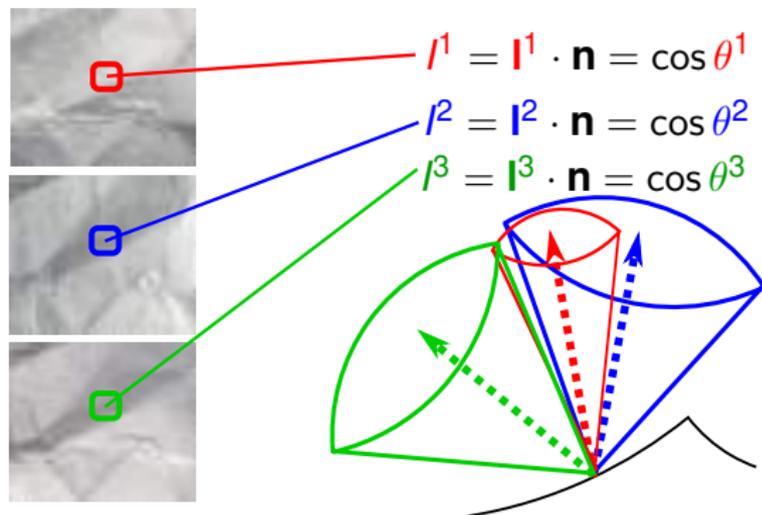
- $m = 1$ image (SfS), known albedo: infinitely many possible unit-length normals in each point
- $m = 2$ images, known albedo: up to two possible normals
- $m \geq 3$: unique approximate solution - problem is over-constrained (thus albedo can be estimated !)

Linear photometric stereo model

$$I^1 = \mathcal{R}(z, \rho, \ell^1) = \rho \langle \ell^1, \mathbf{n}(z) \rangle$$

$$I^2 = \mathcal{R}(z, \rho, \ell^2) = \rho \langle \ell^2, \mathbf{n}(z) \rangle$$

$$I^3 = \mathcal{R}(z, \rho, \ell^3) = \rho \langle \ell^3, \mathbf{n}(z) \rangle$$

Local estimation of shape and reflectance

Linear system in $\mathbf{m}(\rho, z) := \rho\, \mathbf{n}(z) \in \mathbb{R}^3$:

$$\begin{bmatrix} I^1 \\ I^2 \\ \vdots \\ I^m \end{bmatrix} = \begin{bmatrix} \boldsymbol{\ell}^{1\top} \\ \boldsymbol{\ell}^{2\top} \\ \vdots \\ \boldsymbol{\ell}^{m\top} \end{bmatrix} \mathbf{m}(\rho, z)$$

- If: a) $m \geq 3$, b) lighting vectors $\boldsymbol{\ell}^i$ are known, and c) they are non-coplanar,
- Vector $\mathbf{m}(\rho, z)$ can be estimated in each pixel,
- Then we deduce the normal **and albedo** by

$$\rho = |\mathbf{m}(\rho, z)|$$

$$\mathbf{n}(z) = \frac{\mathbf{m}(\rho, z)}{|\mathbf{m}(\rho, z)|}$$

- Eventually, normals $\mathbf{n}(z)$ are *integrated* into depth z

Integrated variational approach
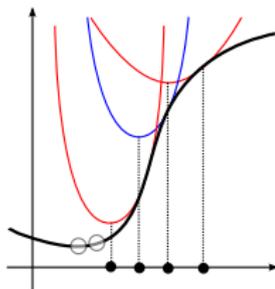
For robustness: direct estimation of depth $z : \Omega \to \mathbb{R}$ and albedo $\rho : \Omega \to \mathbb{R}$ solutions of

$$I^i = \rho \, \boldsymbol{\ell}^i \cdot \frac{\left[\nabla z^\top, -1 \right]^\top}{\sqrt{|\nabla z|^2 + 1}}, \; i \in \{1, \dots, m\}$$

Linearisation $\rho := \frac{\rho}{\sqrt{|\nabla z|^2 + 1}}$, then

$$\min_{z, \rho} \sum_{i=1}^m \iint_{x \in \Omega} \Phi \left(\left| \rho(x) \, \boldsymbol{\ell}^i \cdot \left[\nabla z(x)^\top, -1 \right]^\top - I^i(x) \right| \right) \mathrm{d}x$$

Non-convex problem: optimisation by alternating reweighted least-squares (alternating MM)

Quéau, Wu, et al., "A Non-Convex Variational Approach to Photometric Stereo under Inaccurate Lighting", CVPR 17

3D-reconstruction of metallic coins

1 euro (Italy)   50 cents (Spain)   1 yuan (China)

3D-reconstructions

Quéau, Mecca, and Durou, "Unbiased Photometric Stereo for Colored Surfaces: A Variational Approach", CVPR 16

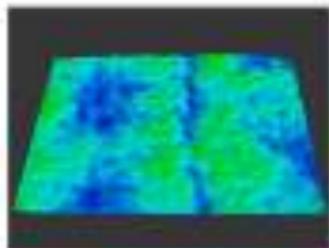# 3D-reconstructions of the human skin

Image 3D de ride   Image 3D de cicatrice   Image 3D d'acné

Carte d'élévation de ride   Carte d'élévation de cicatrice   Carte d'élévation d'acné
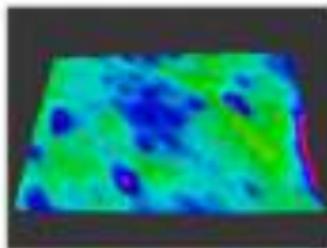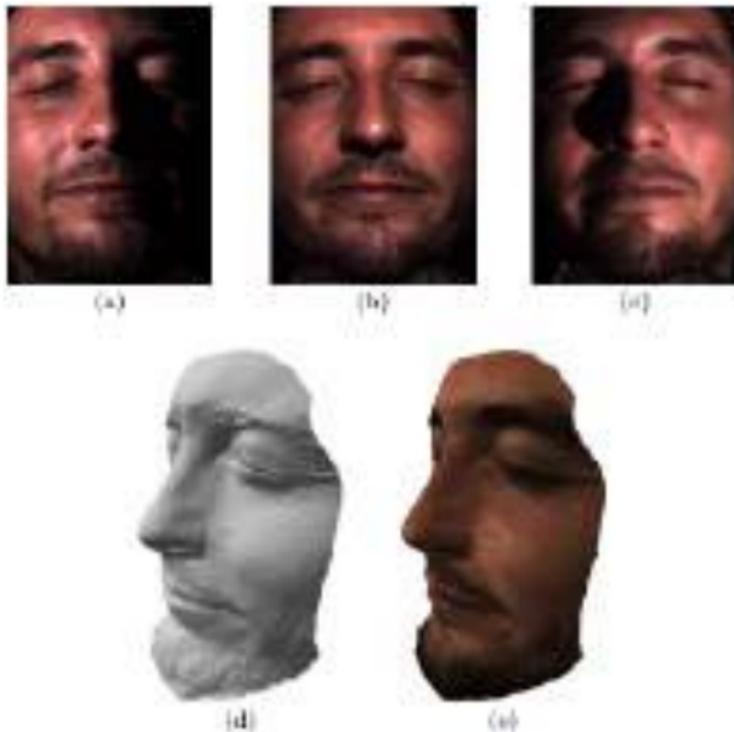
Source : `http://www.pixience.com/produits-2/c-cube-recherche-clinique/module-3d/`
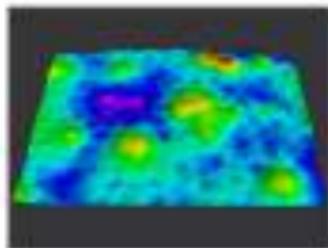
3D-reconstruction of a human face

Quéau, Durix, et al., "LED-Based Photometric Stereo: Modeling, Calibration and Numerical Solution", JMIV 18

Outline

Recall our challenges

1) Thinness of the wool strings
2) Color changes
3) Glass creating reflections
4) Fragility to light exposure

Photometric stereo already provides an answer to 1)-2)
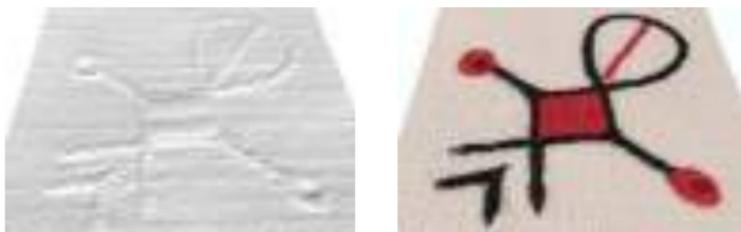
Campaign preparation: handling the glass

Proof of concept experiment by M. Pizenberg during
lockdown # 1: self-made embrodery behind a glass, with
sticked spheres for calibrating a moving flash light source

Campaign preparation: handling the glass

Captured images

3D-reconstruction

Fine-scale structures are revealed. Glass does not perturbate
3D-reconstruction, as long as 1) the camera is fronto-parallel,
2) light directions deviate enough from the viewing direction,
and 3) robust algorithm is used

Campaign preparation: limiting light

The tapestry is fragile, so we need to limit the luminous flux we will project onto it → very short camera exposure time of $1/200s$, and use of a synchronized flash light source

Campaign preparation: measuring light

To get access to the tapestry, we had to obtain agreement from authorities, announcing how much light would be projected $\rightarrow$ need to measure the luminous flux emitted by the flash, by calibrating the camera response and using a lux-meter
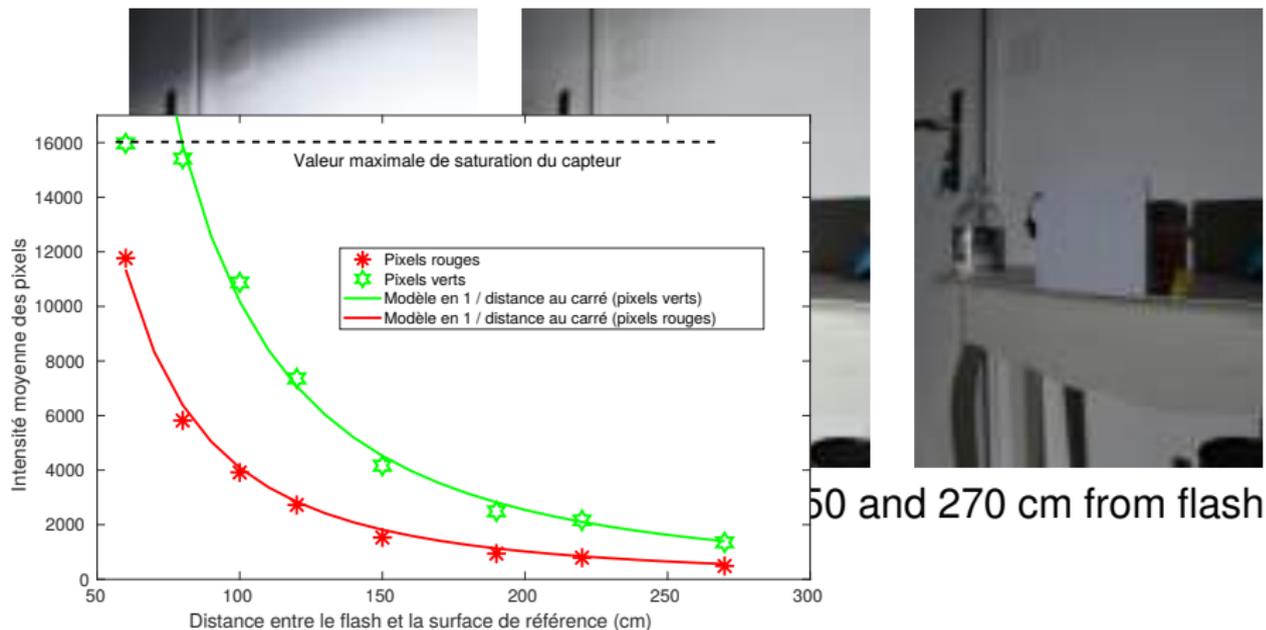

Instant light intensity measured by a lux-meter, to verify the sensor linearity

Campaign preparation: measuring light

Images of a Lambertian plane at 60, 150 and 270 cm from flash

Campaign preparation: measuring light

50 and 270 cm from flash

→ total surfacic energy at $1m$, for 12 shots per scene: 312 lux.s
($\approx$ 6 sec. of the already in-place lighting system)
⇒ acceptable for authorities

Data acquisition

Selection of 12 parts of the tapestry for digitization, and in-situ image acquisition in January 2021:

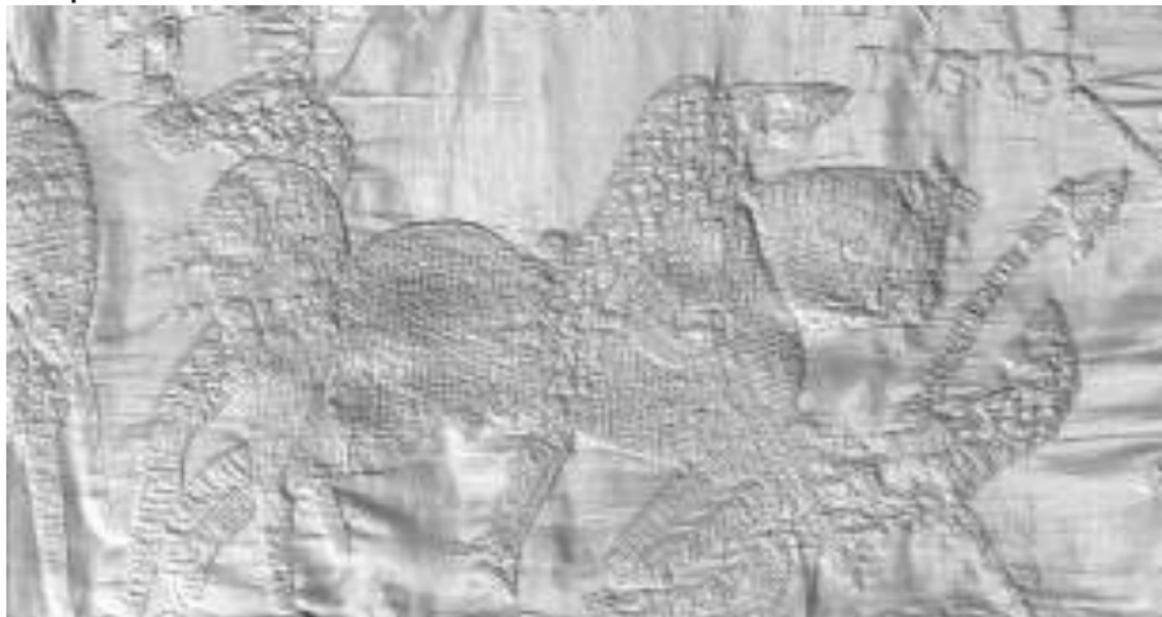Acquisition of the death of Harold sequence

The death of Harold sequence

Acquired data and 3D-reconstruction:

The death of Harold sequence

Acquired data and 3D-reconstruction:

The death of Harold sequence
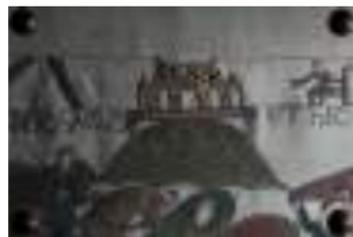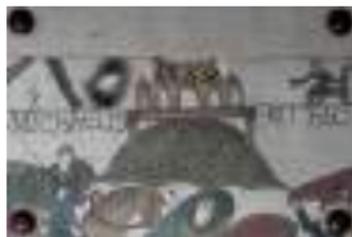
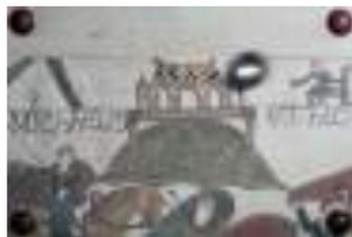Acquired data and 3D-reconstruction:

The death of Harold sequence

Acquired data and 3D-reconstruction:

The Mont St-Michel sequence

The Hastings sequence

From left to right: images (close-up), 3D-reconstruction, and estimated albedo (not used for the project)

What's next ?

We're now ready for 3D-printing :) !

Thank you for your attention !

References

Durou, J.-D., Falcone, M., Quéau, Y., and Tozza, S. *Advances in Photometric 3D-Reconstruction*. Springer, 2020.

Quéau, Y., Durix, B., Wu, T., Cremers, D., Lauze, F., and Durou, J.-D. "LED-Based Photometric Stereo: Modeling, Calibration and Numerical Solution". In: *Journal of Mathematical Imaging and Vision* (2018).

Quéau, Y., Mecca, R., and Durou, J.-D. "Unbiased Photometric Stereo for Colored Surfaces: A Variational Approach". In: *IEEE International Conference on Computer Vision and Pattern Recognition (CVPR)*. 2016.

Quéau, Y., Wu, T., Durou, J.-D., Lauze, F., and Cremers, D. "A Non-Convex Variational Approach to Photometric Stereo under Inaccurate Lighting". In: *The IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*. 2017.