Parametric shape optimization for combined additive-subtractive manufacturing

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Joint work with:

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Within the project **CAxMan, Computer-Aided technologies for Additive Manufacturing** (2015-2018), European Union's Horizon 2020 research and innovation program, Grant Agreement 680,448

CIMNE Fraunhofer IGD imati SINTEF caxman computer aided technologies for additive manufacturing

***Printer-aware* shape optimization**

- 3D printers **don't produce exactly** the shape requested **(nominal geometry)**, due to
  - Surface roughness
  - Imprecisions in the printing path
  - Support structures to be removed a-posteriori
  - Thermal deformations
  - Residual stresses
- Many of these defects can be fixed by post processing (**subtractive machining**)
- We need therefore an extra layer of material (***coating***)
- What is the **optimal coating thickness** for machining operations?
  - Enough to meet machining equipment tolerances
  - Not too much, to avoid wastes of time and material
  - Different surfaces can have different coating thicknesses

Printer-aware shape optimization

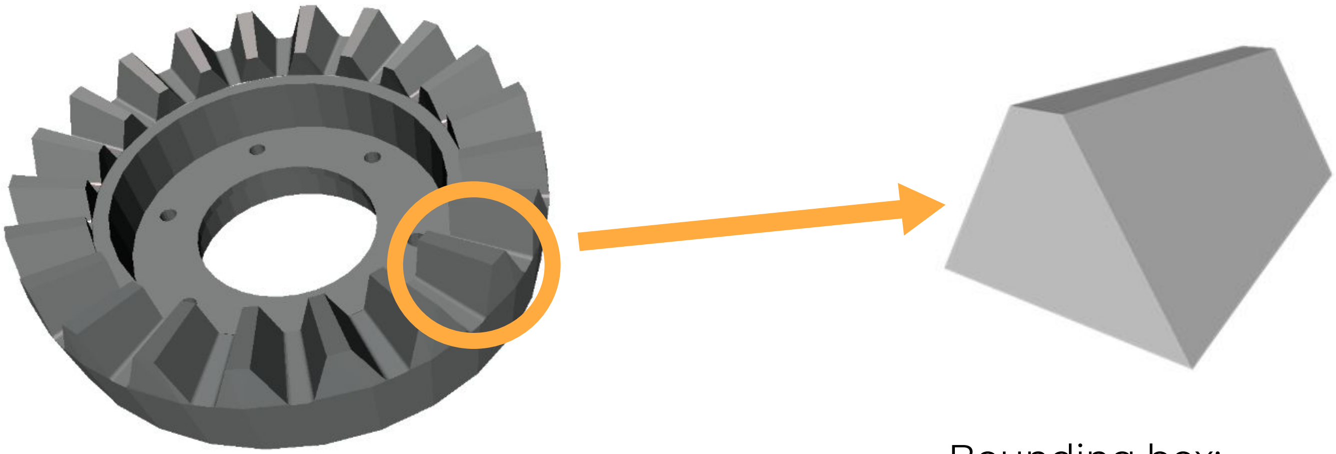
- **nominal geometry + coating = stock part**
- It's a constrained **optimization problem:**

$$\begin{cases} \min_{\text{coating}>0} \, Extra\,Volume[Printed\,stock\,part] \\ \text{s.t.} \\ \text{dist}(Printed\,stock\,part, Nominal\,geometry) > Tol \end{cases}$$

- Assessing the shape of the printed stock part for a given choice of coating is **expensive**: time-dep. elasto-plastic PDE to **simulate the printing process**. More on this later on
- Shortcoming: **we do not check mechanical compliance** to nominal loading (one extra elastic PDE constraining the optimization)
- More general question: what's the best shape to give to the printer as input given the printer limitations? (*inverse problem*)

Test case

- Printing one tooth of the NuGear by STAM (http://www.stamtech.com), one of CAxMan project use cases
- Material: Ti64
- Reference printer: EOS M280 (selective laser melting printer)
- Machining tolerance: 0.04 mm

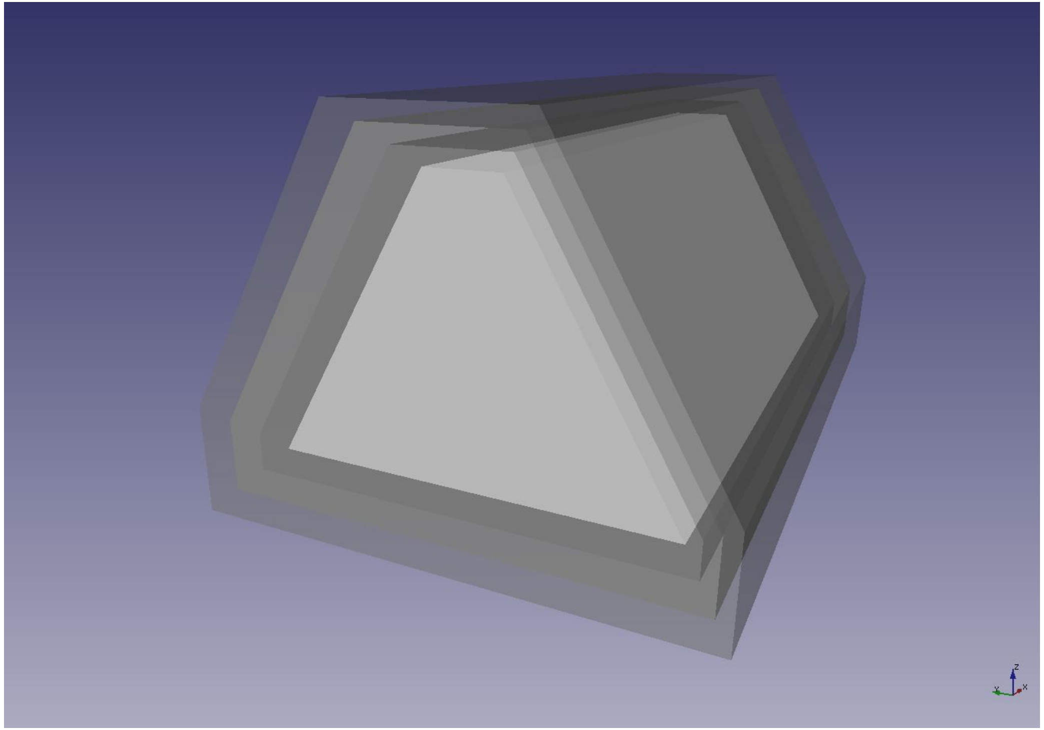


Bounding box:
6 mm x 10 mm x 4 mm

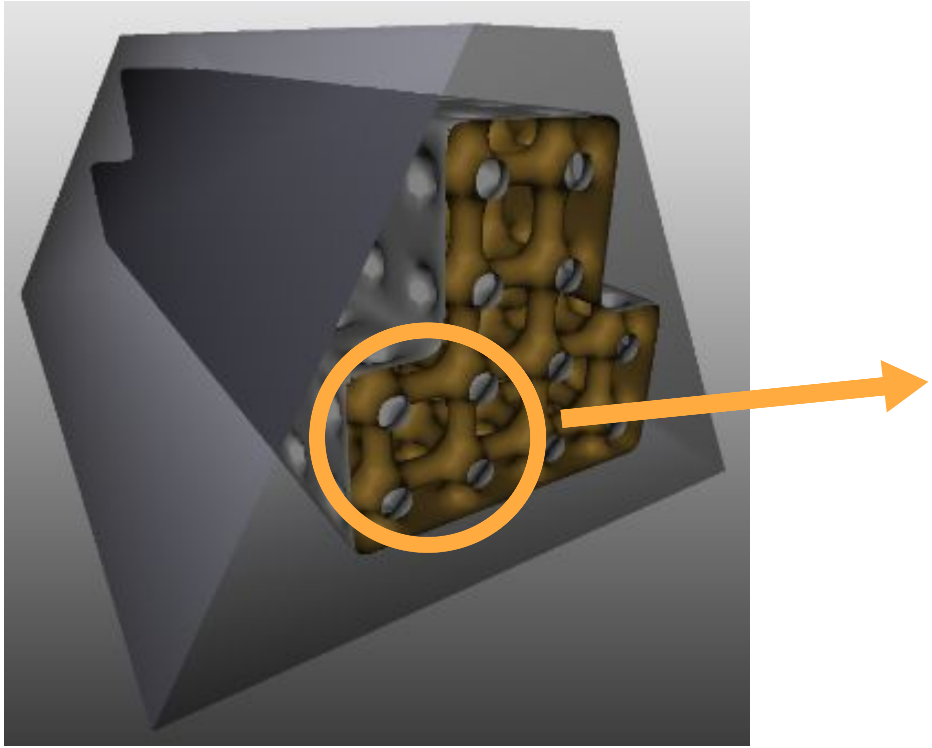
Parametric shape optimization

1. Instead of free-form shape optimization (i.e., each node of the mesh can be moved independently), we work in a **parametric optimization** setting.
2. Each stock part can be obtained by fixing $\mathbf{p} = [\mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3}]$
   - $\mathbf{p_1}$ = Coating thickness / offset (one constant value for the whole part)
   - $\mathbf{p_2}$ = size of internal voids
   - $\mathbf{p_3}$ = wall-thickness (distance of voids from surface)
3. $\mathbf{p_2}$, $\mathbf{p_3}$ are unrelated to coating, but having voids helps reducing the overall warping of the printed stock part, so hopefully they can help reducing the thickness of the coating layer
4. $\mathbf{p} \in \Gamma = [{p_1}_{min}, {p_1}_{max}] \times [{p_2}_{min}, {p_2}_{max}] \times [{p_3}_{min}, {p_3}_{max}]$

p_1: offset

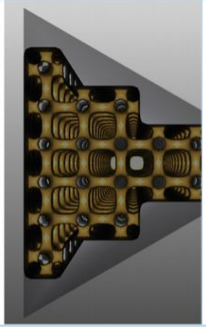
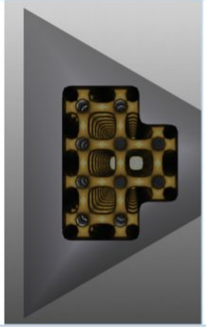
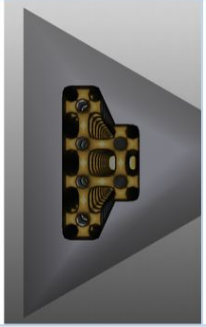
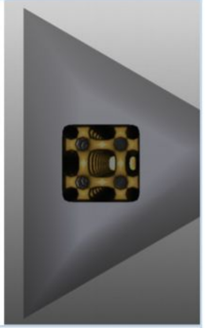


p_2: size of internal voids



p_2 controls the size of this basic pattern

p_3: wall thickness

Wall thickness	**0.0 mm**	**0.1 mm**	**0.2 mm**	**0.3 mm**	**0.4 mm**	**0.5 mm**
Resulting cavities						
Remaining material	50.89 %	74.82 %	80.02 %	81.57 %	86.27 %	90.88 %

Surrogate-model-based optimization

$$\begin{cases} \min_{\mathbf{p}\in\Gamma} \mathit{Extra\,Volume}[\mathit{Printed\,stock\,part}(\mathbf{p})] \\ \text{s.t.} \\ \text{dist}(\mathit{Printed\,stock\,part}(\mathbf{p}), \mathit{Nominal\,geometry}) > \mathit{Tol} \end{cases}$$

Problem: Assessing the shape of the printed stock part for a given choice of **p** is **expensive**: time-dependent elasto-plastic PDE to **simulate the printing process**.

Approach:

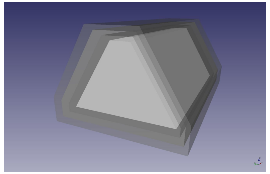
- **Step 1:** Build a **surrogate model** of the map: **p** → **Printed stock part(p).**
  Still requires solving the PDE for some values of p
- **Step 2: Replace surrogate** model in the optimization problem. **Very fast**, since evaluating the surrogate model is real-time
- **Rationale:** the **number of PDEs** to solve to build the surrogate is (hopefully) substantially **smaller** than the number of PDEs required by the optimization routine

Summary and sneak-peak of results

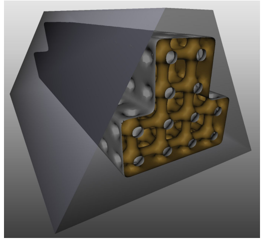
STEP 1

For values of **p** to be tested

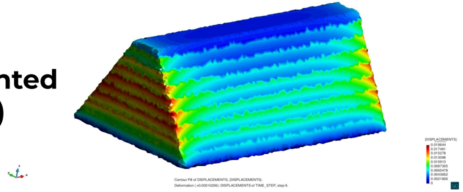
1. Coating



2. Voids



3. solve PDE, compute **printed stock part(p)**

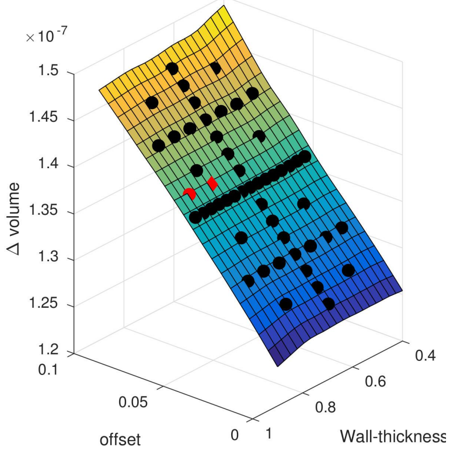


end

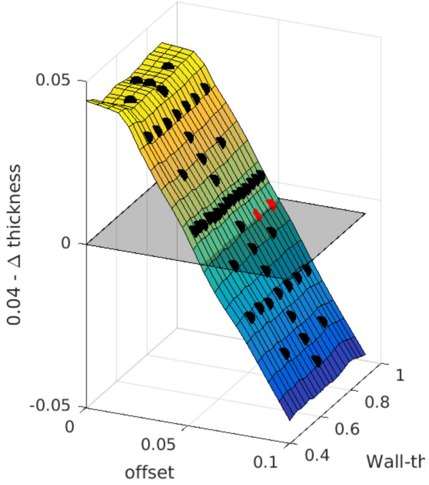
STEP 2

● Tested values

● Optimal values



Objective function



Constraint

And now, details

1. Given **p**, how do we
    a. Generate coating
    b. Generate voids
    c. Simulate the printing process
    d. Compute
        i. Extra volume of the printed stock part
        ii. distance between printed stock part and nominal geometry
2. How do we build the surrogate model
3. How do we solve the optimization problem
4. Results

And now, details

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Generate coating

- **The exact offset** of a polyhedron may have parts of its surface that are not piecewise-linear, i.e. some surface parts may be locally cylindrical or spherical. **Does not suit our needs**
- Generalization: **Minkowski sum S+M,** obtained rolling M on the surface of S. If M is a sphere, you get exact offset.

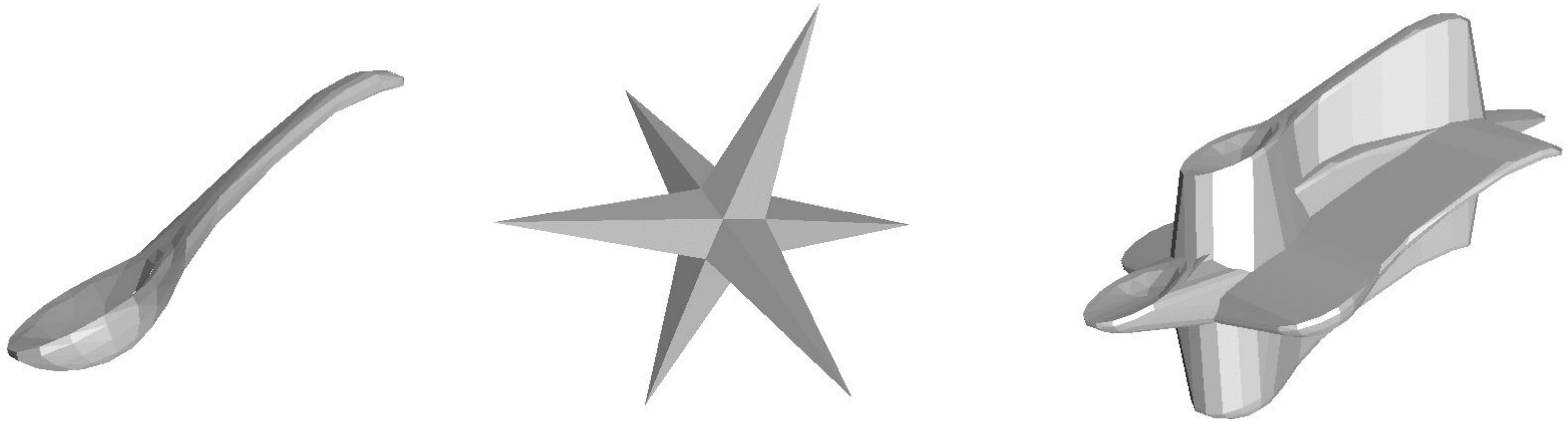
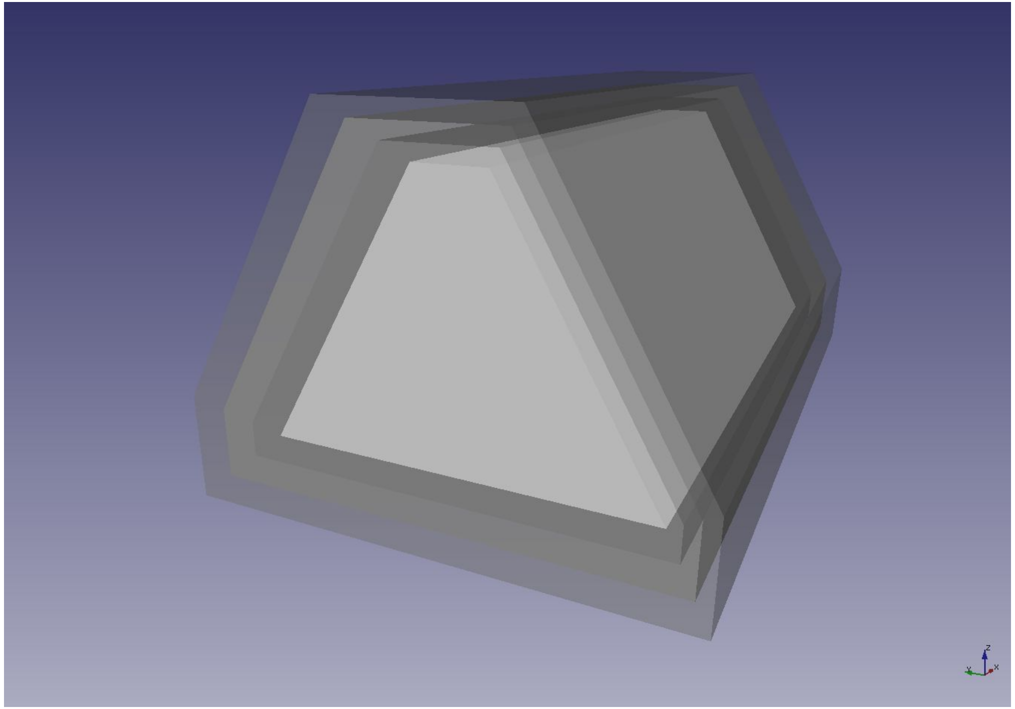


Image from:
https://doc.cgal.org/Manual/3.5/doc_html/cgal_manual/Minkowski_sum_3/Chapter_main.html

Generate coating

We take **M** to be a **cube.** The offset is the size of the cube.



Details

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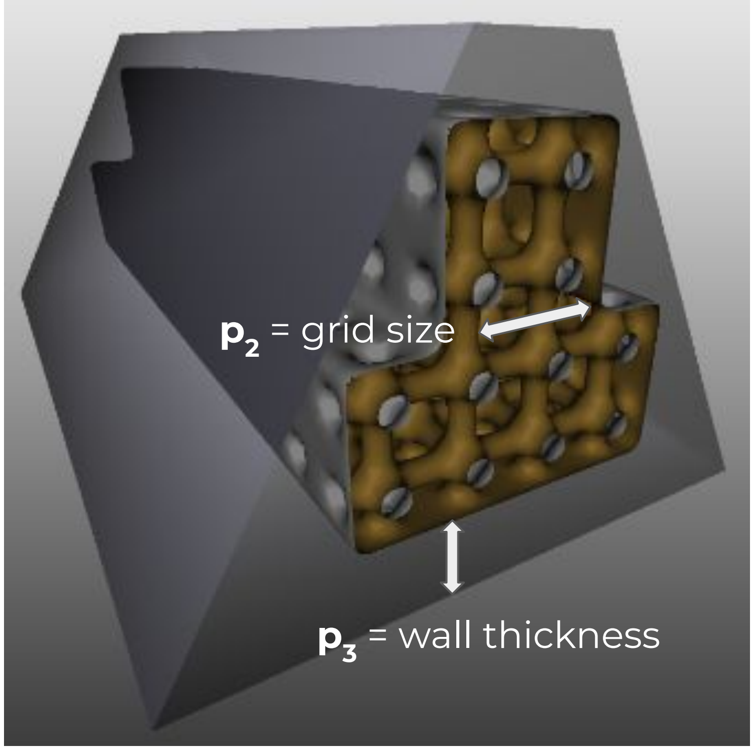
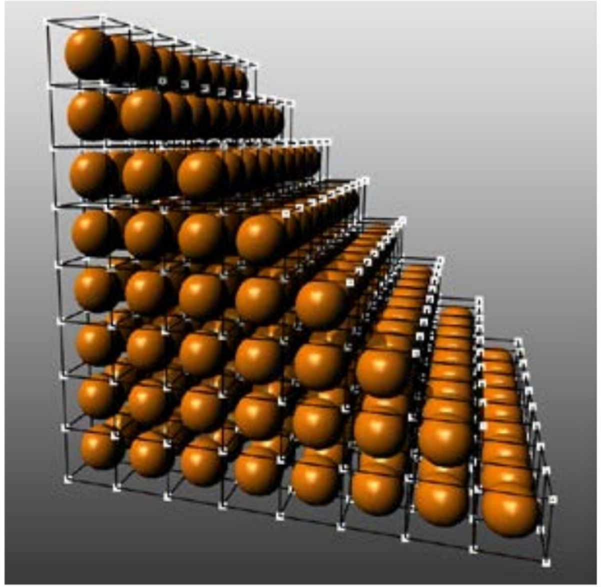
Generate voids

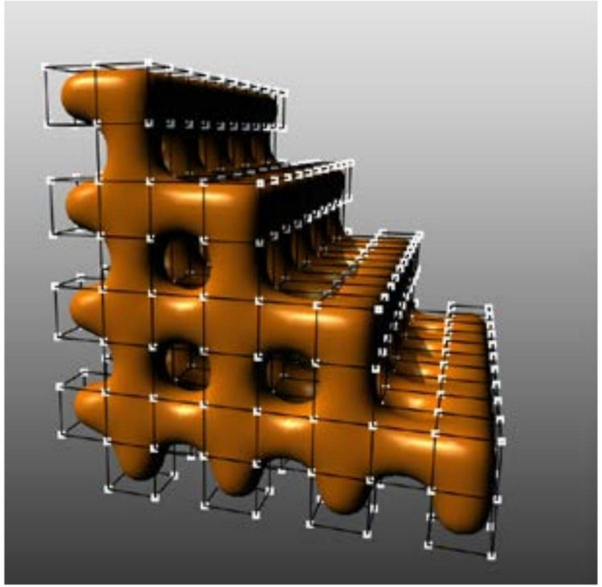


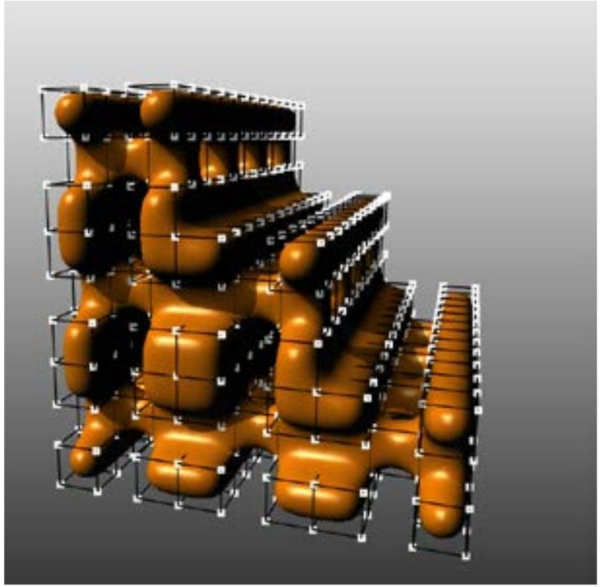
- **Rationale:**
  - reducing the volume of the part mitigates its thermal deformation
  - saves material and printing time.
- Based on **Catmull–Clark (CC) subdivision surfaces** (computer graphics, animation)
- **smooth** surfaces (C^2-cont. in regular areas, C^1-cont. around extraordinary points)
- Regular CC subdivisions correspond to bi-cubic B-spline surfaces (**CAD-compatible**)
- **Assumption:** inner cavities do not alter the mechanical performance of the tooth. Add elastic PDE as a constraint otherwise.

Generate voids

Several patterns can be used







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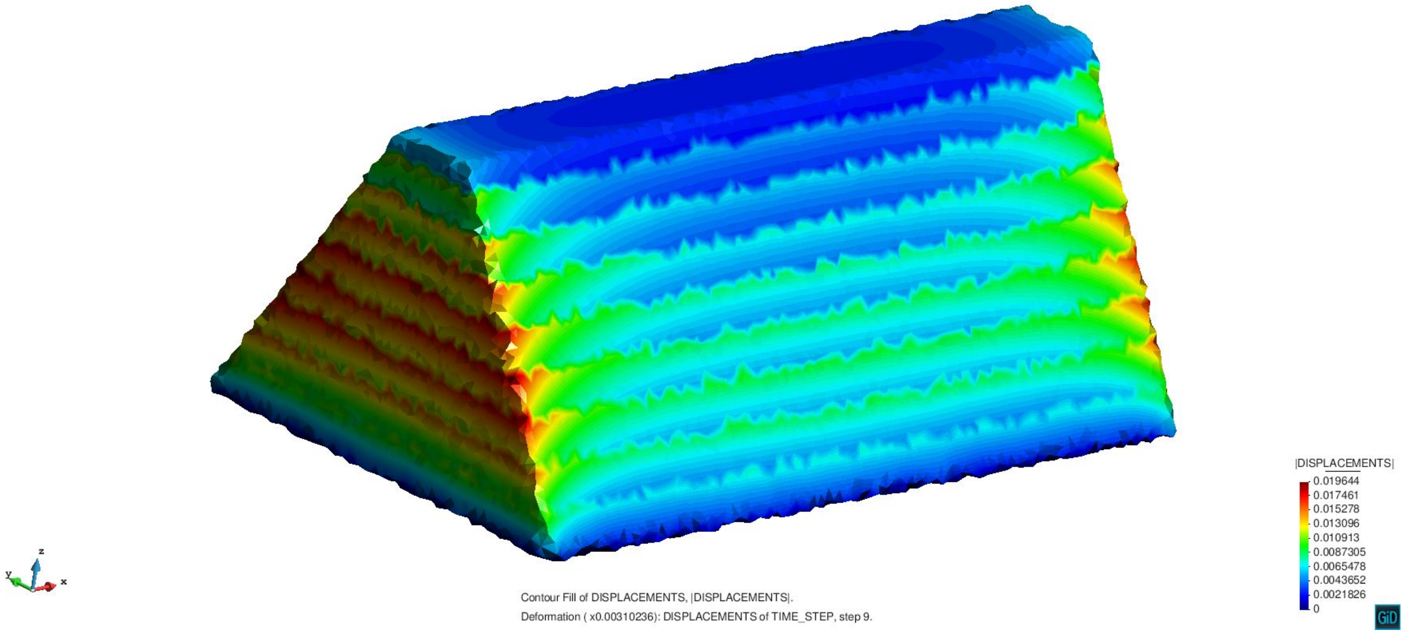
Simulate the printing process

- Goal: computing distortions of the component
- Full simulation is too expensive. Use a simplified method: the **inherent strain method**
- **main hypothesis**: the heat-affected zone is small, does not affect the rest of the domain. **Replace** coupled non-linear thermo-mechanical analysis **by a sequence of mechanical computations**
- layer-by-layer, skip simulation of scanning sequence.
- **inherent strains = thermal strains + plastic strains,** calibrated offline according to:
    - melt-pool temperature and thermal expansion coefficient (thermal strains)
    - scanning speed and the power heat (plastic strains)
- **multi-layer activation process:** pack up to **10 layers** (≈200 μm)
- back plate = clamping boundary conditions
- loose powder plays no role
- supporting structures: replaced by an equivalent stiffness

Simulate the printing process



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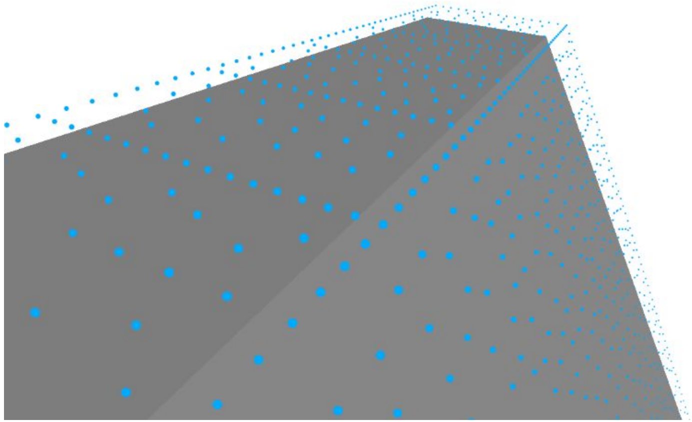
Compute extra volume and distance

- Computing the **extra volume is easy**; just remember that the nominal geometry has no voids. We assume negligible distortion between nominal and printed voids. Thus:

$$Extra\,volume = printed\,stock\,part\,volume - (Nominal\,geometry\,volume - nominal\,voids\,volume)$$

- Computing the **distance** between the printed stock part and the nominal geometry requires more work.

- We start from the **cloud of points** obtained by applying the deformation to the stock part mesh and use an iterative **closest point projection method** (*3d shapes registration*)



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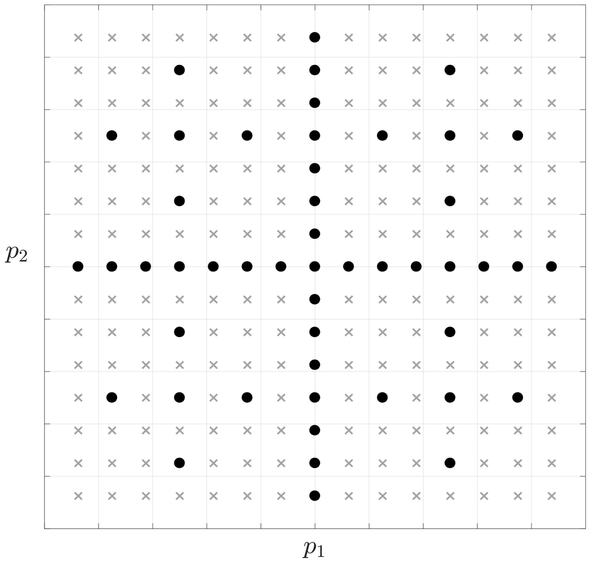
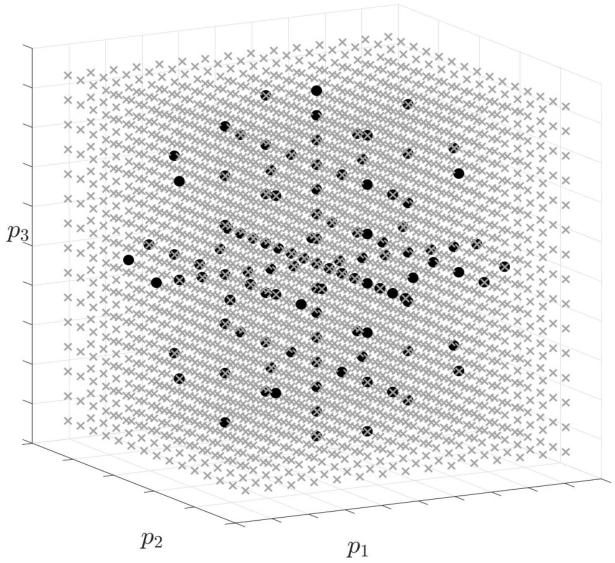
Surrogate-model construction

- It's actually two surrogate models:
    - one for the extra volume (objective function)
    - one for the distance (constraint function)
- Rough idea:
    - sample parametric space Γ
    - For each sample, compute the full-model deformations, and derive extra volume and distance
    - Compute an interpolant / regression polynomial for both
- What sampling strategy and polynomial?
    - cartesian sampling of the parametric space Γ + Lagrange interpolant is expensive, cost = $O(M^3)$
    - Monte Carlo sampling + Least Squares regression is also inaccurate if not enough points

Surrogate-model construction

- We consider sparse grids sampling with splines interpolant: good compromise between accuracy and cost
- Other choices in literature: reduced basis, PGD, radial basis functions, gaussian processes…





● = sparse grid point, **x** = cartesian grid point

Details

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How do we solve the optimization problem

- use penalization methods to convert constrained optimization into unconstrained

$$\begin{cases} \min_{\mathbf{p}\in\Gamma} f(\mathbf{p}) \\ \text{s. t.} \\ g(\mathbf{p}) \leq 0 \end{cases} \Rightarrow \min_{\mathbf{p}\in\Gamma} f(\mathbf{p}) + G(\mathbf{p})$$

  where G(**p**) is large if g(**p**)>0 (unfeasible point). Bounding box left but easy to deal with.

- Three penalization methods considered:
  - **Log-barrier**
  - **Squared penalty**
  - **Augmented Lagrangian**

- Solve unconstrained method by either
  - **Gradient** method: fast but can get stuck close to boundary of Γ
  - **Nelder-mead** (simplex) method, gradient-free: slower but more robust

- For each combination of penalization + unconstrained solver, consider **5 initial guesses** (Latin Hypercube or MC sampling), choose best result out of these 5 runs

- Total: 3 x 2 x 5 = 30 optimization runs. Massive costs without surrogate model!

Details

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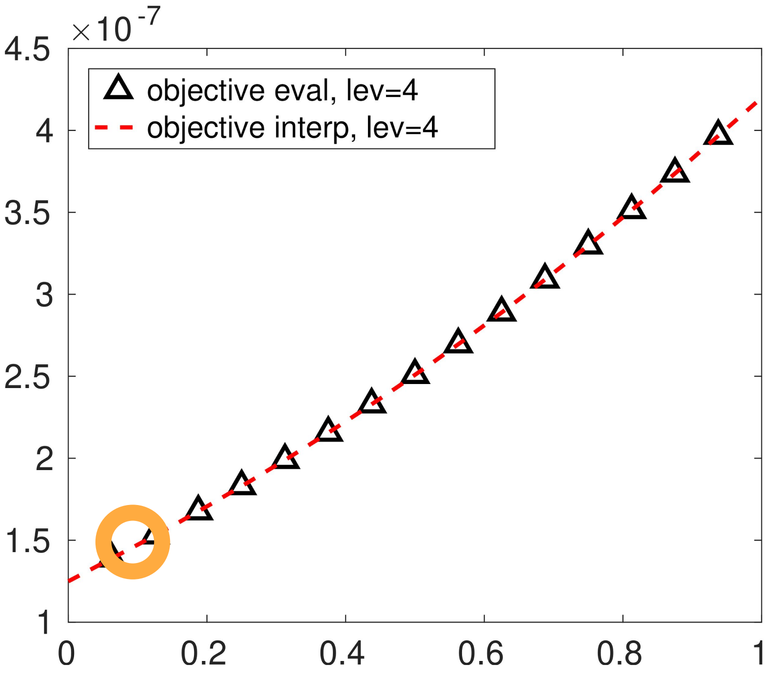
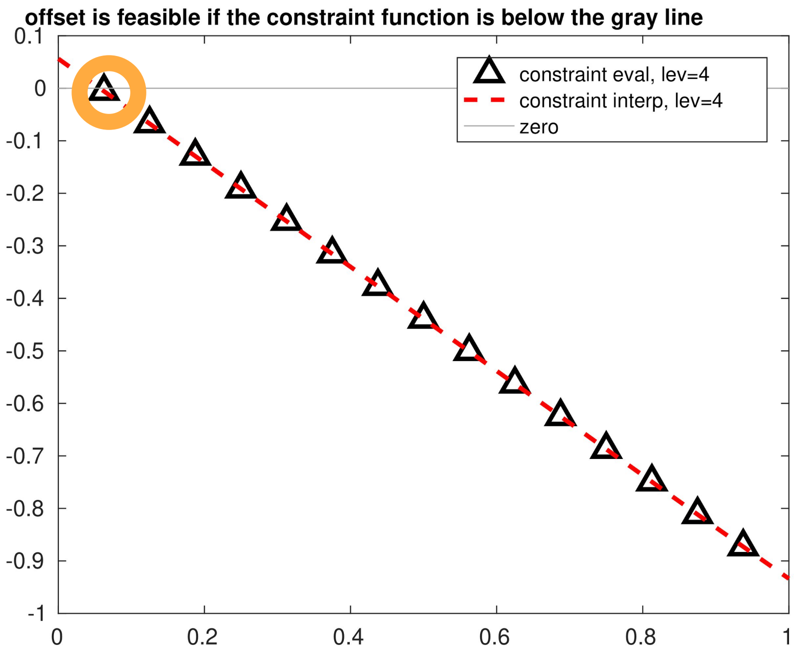
Results

- 4 cases
  - Offset
  - Offset + grid size
  - Offset + wall thickness
  - Offset + grid size + wall thickness
- For every case, report
  - Plot of extra volume (objective function)
  - Plot of distance (constraint function)
  - best of 30 optimization runs for increasingly refined surrogate models

Results, case 1: offset

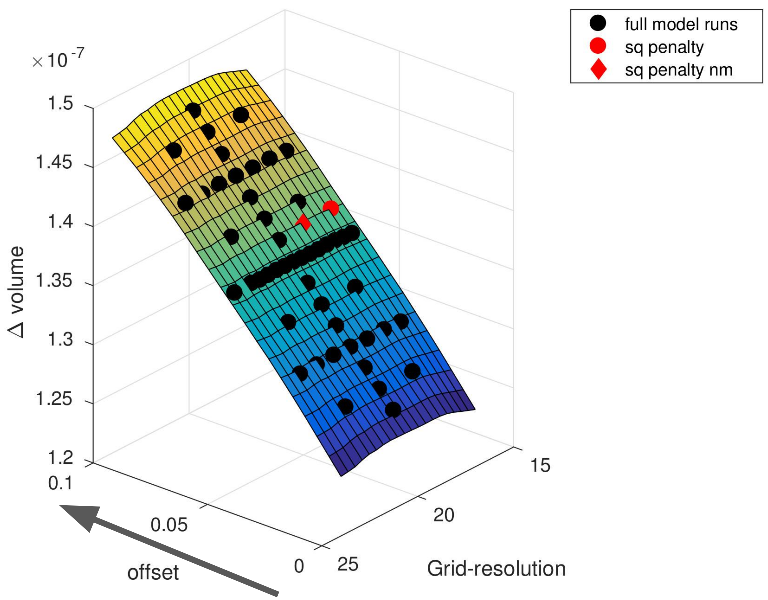


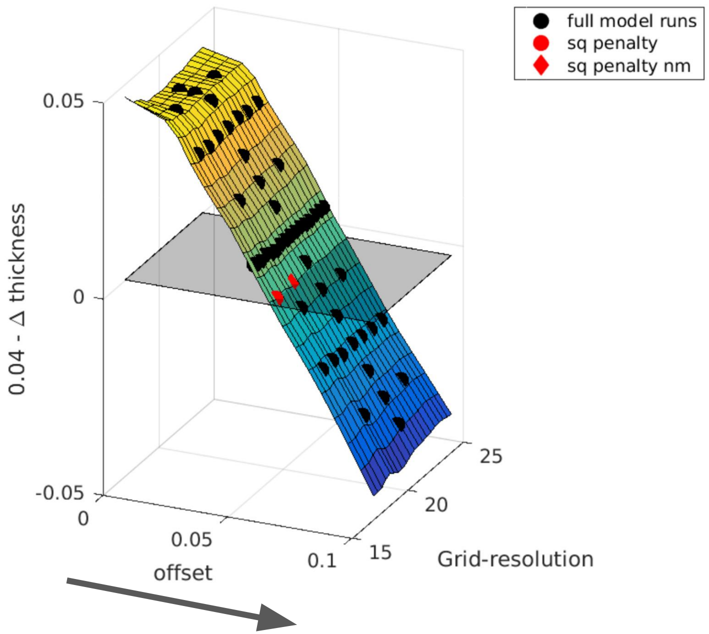


Results, case 1: offset

w	Design points	Optimal offset (mm)	Optimal volume (mm^3)	Method	Interpolant evaluations	Computational time (hh:mm:ss)
2	3	0.0569	132.67	Squared penalty gradient	52	00:12:01
3	7	0.0571	136.93	Squared penalty gradient	50	00:25:41
4	15	0.0567	137.59	Squared penalty gradient	49	00:56:35
5	31	0.0567	137.62	Squared penalty gradient	48	01:57:14

Results, case 2: offset + grid size

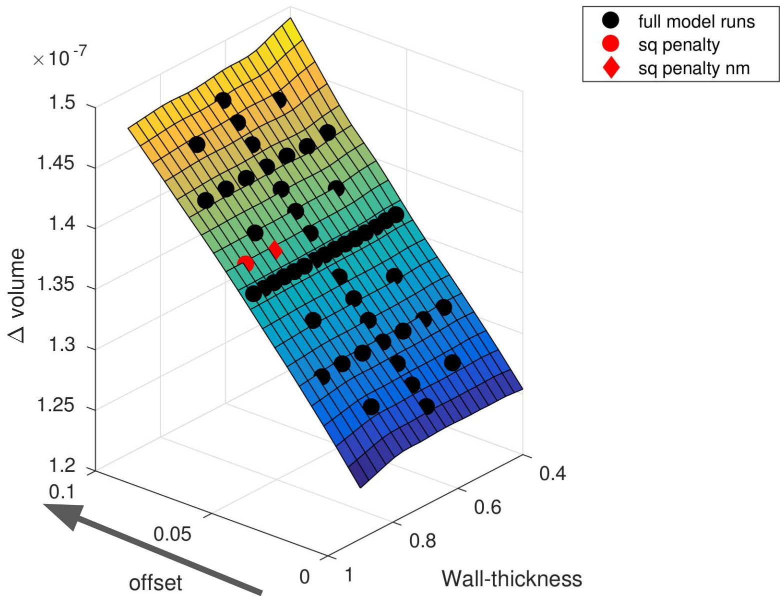


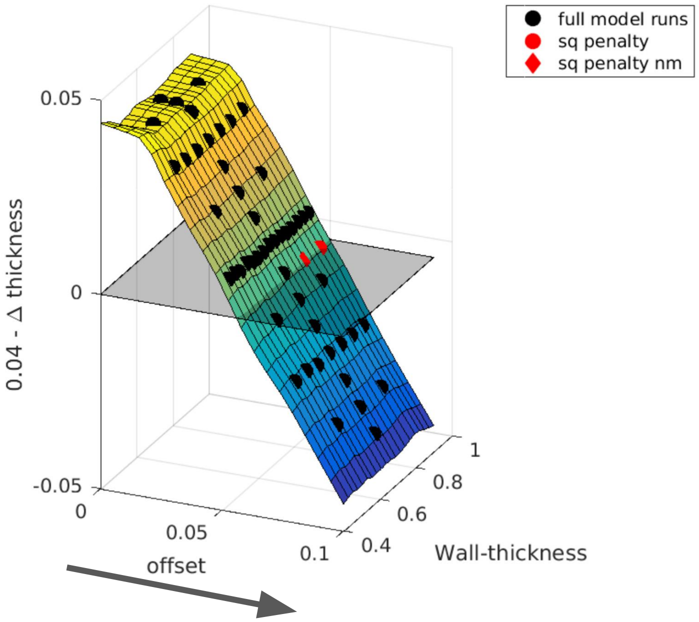


Results, case 2: offset + grid size

w	Design points	Optimal offset (mm)	Optimal volume (mm2)	Optimal grid resolution	Method	interpolant evaluations	Computational time (hh:mm:ss)
2	5	0.0568	137.96	24	Squared penalty Nelder Mead	1715	00:32:43
3	17	0.0576	138.31	22.4	Squared penalty Nelder Mead	2184	01:50:18
4	49	0.0576	138.25	17.6	Squared penalty gradient	58	05:15:11

Results, case 3: offset + wall thickness

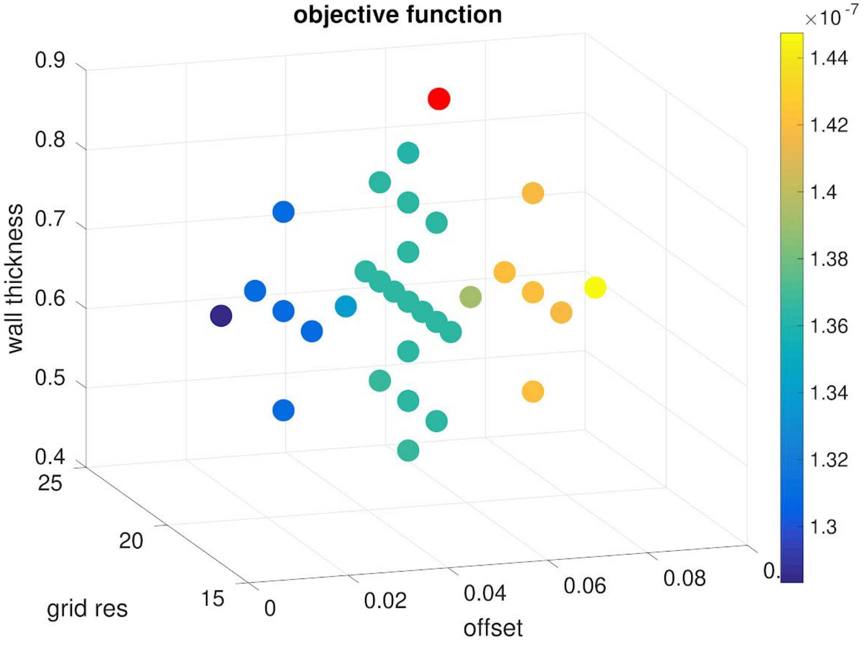


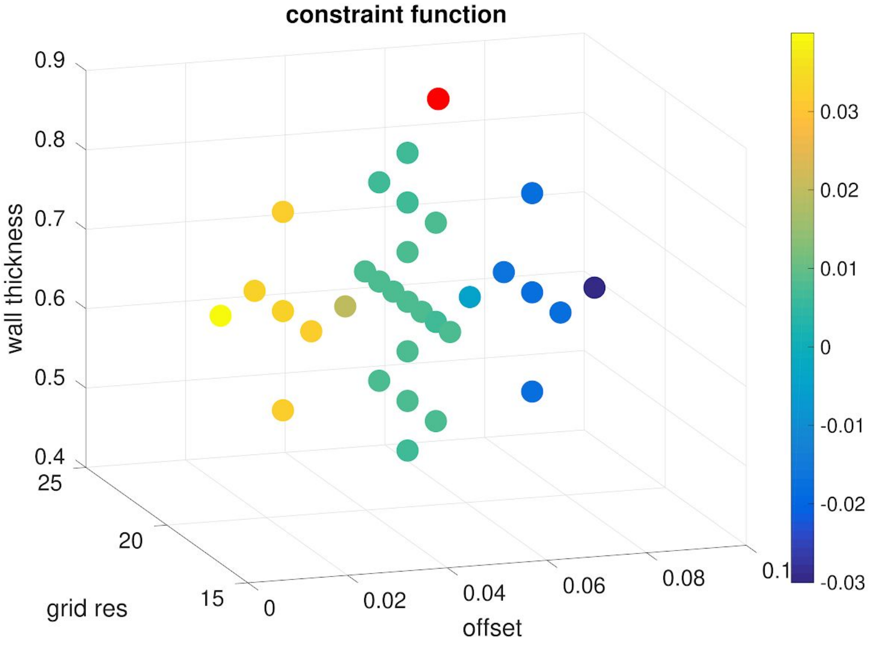


Results, case 3: offset + wall thickness

w	Design points	Optimal offset (mm)	Optimal volume (mm^3)	Optimal wall-thickness	Method	interpolant evaluations	Computational time (hh:mm:ss)
2	5	0.0562	137.45	0.9	Squared penalty Nelder Mead	1731	00:22:34
3	17	0.0576	137.73	0.9	Squared penalty Nelder Mead	1733	01:18:51
4	49	0.0567	137.82	0.75	Squared penalty Nelder Mead	2200	03:47:49

Results, case 4: offset + void parameters





Results, case 4: offset + void parameters

w	Design points	Optimal offset (mm)	Optimal volume (mm^3)	Optimal grid resolution	Optimal wall-thickness	Method	Interpolant evaluations	Computational time (hh:mm:ss)
2	7	0.0551	137.24	17	0.9	Aug. Lagrangian Nelder-M.	2008	00:34:35
3	31	0.0566	137.54	20.65	0.9	Squared penalty Nelder-M.	2057	02:27:23

Conclusions

- Printer-aware shape optimization to reduce post-print machining
- Ingredients:
  - Parametrization of shapes: offset, and two parameters for void generation
  - Constrained optimization
  - PDE solver to compute distortions due to printing
  - Surrogate model to reduce costs
- Results can be obtained within a few hours. Voids seem to play little role
- Possible extensions
  - More parameters (different offsets for different surfaces)
  - More printer-aware constraints (overhangs, etc)
  - Add elastic PDE as a further constraint
  - Monitor effects of uncertainties (Uncertainty Quantification)

Bibliography

L. Tamellini, M. Chiumenti, C. Altenhofen, M. Attene, O. Barrowclough, M. Livesu, F. Marini, M. Martinelli, V. Skytt.
“Parametric Shape Optimization for Combined Additive-Subtractive Manufacturing”
JOM, Vol. 72, No. 1, 2020

A version with more details is available as arXiv preprint 1907.01370

Thanks for your attention