



# Book of Abstracts

Workshop INDAM  
"Mathematical Methods for Objects Reconstruction:  
from 3D Vision to 3D Printing"

February 10-12, 2021

## Invited talks

### **Additive Manufacturing: from object reconstruction to component production. A world full of geometrical and modeling challenges!!**

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Additive Manufacturing (AM) – also known as 3D printing – is taking off in many industrial processes. In particular, powder bed fusion for metal manufacturing has definitively changed the way of prototyping metal parts but also plastic 3D printing is changing many approaches in modern engineering. The presentation will focus on the whole process required in going from object reconstruction to component production. Particular attention will be devoted to the computational aspect, however without neglecting object reconstruction, computational grid generation, machine control and final production steps.

### **Reconstruction from 3D Point Clouds: Subtle Deformations and Tiny Features**

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Often 3D models are acquired using optical methods such as LiDAR, structured light, or automated photogrammetry. While these methods are generally effective, we often face challenges in the downstream pipeline, and in this talk we will consider two specific challenges. The first challenge pertains to registration. Almost invariably, we need to scan an object several times in order to get a 3D model which covers the entire object. While great strides have been made in the area of global registration, it is not always possible to perfectly align objects due to subtle non-rigid deformation due to e.g. lens distortion in the scanner. In many cases this leads to very high fre-

quency noise which can be removed by smoothing but at the expense of small details. Recently, we proposed a method which greatly improves preservation of features by projecting the vertex movement due to smoothing onto a nearly rigid deformation of the partial scans. The second challenge concerns reconstruction of 3D models with features at varying scales such as botanical objects. For instance, it is not a problem to reconstruct the trunk of a tree captured using LiDAR, but the smallest twigs might be impossible using common methods. In very recent work, we consider the use of graph theoretical methods as a path towards obtaining 3D surface reconstructions of such objects.

## Seeing in 3D from a single image with geometric priors

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It was established that 3D reconstruction from multiple images is solvable under some fairly general assumptions. This includes Structure-from-Motion (a moving camera filming a rigid object) and photometric stereo (a static camera filming a static object under a moving light source). From a single image however, 3D reconstruction seems at first sight to be a very ambiguous problem and indeed requires additional assumptions. Shape-from-Shading for instance makes strong assumptions about the light source and the surface albedo. In this presentation, I will discuss the 3D reconstruction of an object from a single image given a template, which is a deformable 3D object model, representing a strong but fairly general geometric prior. This framework is called Shape-from-Template and applies well to flexible objects: a simple example is a piece of paper, for which the template can be chosen as a plane, and the task will be to find the 3D shape of the piece of paper after it has been bent, from a single image. In Shape-from-Template, solving 3D reconstruction becomes equivalent to fitting the template to the input image. I will present the principle of Shape-from-Template, a differential model, the existing solutions and the open problems. I will eventually show how it can be used to facilitate augmented reality in laparoscopic surgery by using preoperative images.

## **Mathematics for Printing Metallic Structures**

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The talk shall illustrate some recent efforts in applying mathematical strategies for wire-arc additive manufacturing. One of the problems in this context is concerned with designing useful internal structures of printable shapes. To this end we consider first the construction of an underlying Voronoi tessellation. Furthermore we report on efforts to apply skeletonization schemes in a novel framework at the problem.

## **Geometric approach to matrix completion**

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We address the problem of reconstructing a matrix from a subset of its entries. Current methods, branded as geometric matrix completion, augment classical rank regularization techniques by incorporating geometric information into the solution. This information is usually provided as graphs encoding relations between rows/columns. In this work we propose a simple spectral approach for solving the matrix completion problem, via the framework of functional maps. We introduce the zoomout loss, a multiresolution spectral geometric loss inspired by recent advances in shape correspondence, whose minimization leads to state-of-the-art results on various recommender systems datasets. Surprisingly, for some datasets we were able to achieve comparable results even without incorporating geometric information. This puts into question both the quality of such information and current methods' ability to use it in a meaningful and efficient way.

## **Shape optimization and additive manufacturing: some new constraints and challenges**

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However they allow, in principle, to assemble arbitrarily complex structures - thereby arousing much enthusiasm within the engineering community - modern additive manufacturing technologies (also referred to as 3d printing) raise new difficulties which have to be taken into account from the early stages of the construction, and notably at the level of the design optimization. In this presentation, we shall deal with the modeling and the understanding of two such major challenges related to additive construction methodologies. The first one of these is to avoid the emergence of overhanging regions during the shape optimization process, that is, of large, nearly horizontal regions hanging over void, without sufficient support from the lower structure. The second difficulty addressed in this presentation is related to the fact that the use of an additive technique to realize a structure entails a significant alteration of the mechanical performance of the constituent material of the assembled shape: this material turns out to be inhomogeneous, and it presents anisotropic properties, possibly depending on the global shape itself. These works have been conducted together with Grégoire Allaire, Rafael Estevez, Alexis Faure and Georgios Michailidis.

### **Anisotropic mesh adaptation for 3D printing-oriented structural design**

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Mesh adaptation is a well-known technique to suitably modify the computational mesh employed for the discretization of a PDE. The purpose is to adapt the allocation of the elements of the grid by means of local operations so that the discretization error (or a functional of the error) is controlled [1].

Hence, the mesh is expected to be finer in correspondence with the source of errors (e.g., boundary layers, shocks) and coarser elsewhere. Unlike isotropic mesh adaptation techniques, which modify the size of the elements only, one can decide to tune at the same time the size, the shape, and the orientation of the elements, thus moving to an anisotropic setting. An anisotropic discretization is particularly useful for describing phenomena characterized by preferential directions, since a properly oriented tessellation can guarantee low computational burden while ensuring the desired accuracy.

We consider a recent algorithm, named SIMPATY (SIMP with AdaptiviTY), which suitably combines the mathematical formulation of a standard optimal design framework with an ad-hoc anisotropic grid adaptation procedure [2]. In particular, we employ the SIMP method for the design characterization via topology optimization [3]. SIMP belongs to the density-based methods, where the material distribution is modeled via an auxiliary scalar field (i.e., the density) taking values between zero (void) and one (material) in the design domain. The enrichment of SIMP with an adapted anisotropic mesh allows to deliver reliable mechanical configurations, whose geometries are intrinsically smooth thanks to the smart allocation of the mesh elements. The resulting highly detailed structure boundaries make the configurations optimized via SIMPATY essentially ready for a 3D printing phase, without the need of a massive post-processing step.

In this presentation, we will highlight the properties of SIMPATY, with a specific focus on additive layer manufacturing techniques. In this context, we will provide different applications of the algorithm for several choices of the objective functions and of the design constraints. A verification phase will be finally addressed, showing some examples of 3D printed structures.

## References

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3. BENDSØE, MARTIN P., *Topology optimization*. Springer US, 2009.

## **Shape-from-Polarisation**

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The polarisation of light can be used to simultaneously probe the shape, reflectivity and surface composition of 3D objects. Moreover, it is used in the natural vision systems of both aquatic and land animals to augment their visual capabilities. In this talk I will commence by introducing the physics of light polarisation and give examples of its use in natural vision systems. In particular, I will explain that even under unpolarized incident light certain types of material generate a spontaneously polarised reflection. This can be used to analyze their shape and refractive index. Moreover, in the case of a polarised or partially polarised illuminant, by decomposing the reflected light into specular and diffuse components, allows more complex surface analysis methods to be developed. I will cover the state-of-the-art in this area, and point to some potential practical uses of the resulting algorithms.

## **Deep Learning Geometry**

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Accurate Eikonal solvers, matching and classifying shapes by Gromov distance approximated via generalized multi-dimensional scaling, operating in dual spectral spaces, identifying hormonal receptors of breast cancer in digital pathology, classifying clouds of points, synthesizing geometric and photometric faces of imaginary identities - can all be addressed using deep learning methodologies. The question we will address in this talk would deal with interpreting these deep learning procedures in a geometric manner which goes beyond its classical deep hashing/general optimization properties.

## **Different ways to impose 3D printing overhang restrictions in topology optimization**

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This contribution focuses on three fundamentally different ways to incorporate a constraint on the inclination of downfacing surfaces of a 3D solid geometry, also known as an overhang restriction. Formulation of such restrictions has recently been the focus of intensive research in the field of topology optimization for additive manufacturing (AM), as in most AM processes the printability of parts is characterized by this overhang angle. Next to the correct identification of problematic overhangs, the computation of sensitivity information that drives the topology optimization is also an important point of attention. Here we will consider the following three methods: 1) A method based on a simplified representation of the AM process on a voxel grid; 2) A method utilizing a front propagation approach to identify printable and unprintable regions, and 3) An approach based on the physics of the printing process, in this case local heat accumulation. The different approaches will be illustrated by various numerical topology optimization examples.

## **Invariant shape priors and optimization on fiber bundles**

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This paper presents a construction to obtain similarity and affine invariant shape priors based on an idea of shape normalisation, and their use in variational segmentation with an a priori knowledge of shape distributions. We first show how this construction leads to optimisation on classical objects from topology and geometry: principal bundles, and their and pull-back via feature maps. Optimisation is performed along and across fibres of these principal bundles to obtain the segmentation. This is demonstrated on 3D medical volumes.



## Density derivative-based approaches for overhang control in topology optimization for 3D printing

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Topology optimization is an increasingly popular tool in the design of several objects. In this context, additive manufacturing is particularly attractive to produce the optimized designs, which are often characterized by complex shapes. Notwithstanding its versatility, however, also additive manufacturing has some limitations. In particular, the presence of unsupported parts (or “overhangs”) in the optimized design may lead to the collapse of the structure during manufacturing, unless costly sacrificial support structures are added. Thus, optimizing such supports or eliminating unsupported parts from the optimized design has become a critical research topic in topology optimization for additive manufacturing.

In this talk, we focus on approaches where overhangs are suppressed by adding, to the topology optimization problem, constraints that are based on the analysis of first- and second-order derivatives of the density function. More specifically, the density gradient is used to formulate a constraint that suppresses all surfaces where the overhang angle is more critical than a certain threshold. The second-order derivatives are instead needed to prevent that the first-order constraint is satisfied by boundary oscillations at the interface between solid and void regions. These approaches are simple and effective, as they are based on geometrical considerations and allow to express the self-support condition by integral constraints.

In order to analyze these approaches, first we review the projected undercut perimeter (PUP) constraint for gradient-based overhang control. We then discuss the formulation of a second-order measure of boundary oscillations, and detail how it can be used to formulate filters and constraints that prevent the PUP constraint from being satisfied by boundary oscillations. Throughout the discussion, several numerical examples are reported and analyzed, both in 2D and 3D settings.

This talk is based on joint work with X. Qian (University of Wisconsin-Madison, U.S.A.).

## **Fine-scale 3D-copying for Culturage Heritage**

Yvain Quéau

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This talk will discuss the combination of Reflectance Transform Imaging (RTI) image acquisition systems and 3D-reconstruction by Photometric stereo, in order to automatically create 3D-models of cultural artifacts. Once such a 3D-model is obtained, it can be printed in 3D, resulting in a replica containing all the fine-scale geometric details of the original artifact. An application of this methodology on the Bayeux tapestry, a medieval embroidery depicting the Norman conquest of England in 1066, will be presented.

## **A phase-field-based graded-material topology optimization with stress constraint**

Elisabetta Rocca

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In the talk we introduce a phase-field approach for structural topology optimization for a 3Dprinting process which includes stress constraints and potentially multiple materials or multiscales. First-order necessary optimality conditions are rigorously derived and a numerical algorithm which implements the method is presented. A sensitivity study with respect to some parameters is conducted for a two-dimensional cantilever beam problem. Finally, a possible work flow to obtain a 3D-printed object from the numerical solutions is described and the nal structure is printed using a fused deposition modeling (FDM) 3D printer.

This is a joint work with F. Auricchio, E. Bonetti, M. Carraturo, D. Hoemberg, A. Reali.

## Spectral Perturbations and Generative Models in Geometric Deep Learning

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Spectral geometry is at the heart of a variety of problems in computer vision, graphics, and pattern recognition. However, little is known on the practical utility of spectral methods within geometric deep learning pipelines, where the data has a non-Euclidean or combinatorial structure, as opposed to classical deep learning pipelines where the data lives on a “flat” domain (e.g. images). In this talk, I will present several new directions where spectral geometry not only leads to qualitative leaps in a range of challenging geometric deep learning problems; but also, I will show how it allows to tackle novel and unprecedented settings. Among these, I will demonstrate (1) how universal adversarial attacks can be concocted on geometric data via simple, yet purely spectral algorithms, and (2) how several tough tasks in 3D vision and graphics, including generative modeling, can be successfully addressed via data-driven approaches based on spectral quantities (eigenvalues). If time allows, I will also briefly present exciting results toward few-shot, generative, geometric deep learning.

### Self-supervised inverse rendering

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Inverse rendering is the task of decomposing an image into geometry, illumination and reflectance such that these quantities would recreate the original image when rendered. Deep learning has shown great promise for solving components of this task in unconstrained situations. However, the challenge is a lack of ground truth labels to use for supervision. I will describe two approaches that require no ground truth, extracting a self-supervision signal from the image data alone. The first uses unstructured image collections

for training so that multiview observations can be used to constrain photometric invariants across lighting and viewpoint changes. The second uses a parametric model of faces to simplify the problem but, rather than posing the problem as one of parameter regression, we propose a "backwards rasterisation" framework. This enables completely unsupervised training that avoids common problems in the state-of-the-art as well as bringing several other benefits. I will show an example application to single image relighting in which a neural renderer is used to achieve photorealistic results.

## Parametric shape optimization for combined additive-subtractive manufacturing

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Marco Attene<sup>5</sup>, Oliver Barrowclough<sup>6</sup>, Marco Livesu<sup>5</sup>,  
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In the industrial practice, Additive Manufacturing (AM) processes are often followed by post-processing operations such as Heat Treatment, subtractive machining, milling, etc. to achieve the desired surface quality and dimensional accuracy. Hence, a given part must be 3D-printed with extra material to enable such finishing phase. This combined additive/subtractive technique can be optimized to reduce manufacturing costs by saving printing time, reducing material and energy usage. In this work, a numerical methodology based on parametric shape optimization is proposed for optimizing the thickness of the extra material, allowing for minimal machining operations while ensuring the finishing requirements. The computational effort induced by classical constrained optimization methods is alleviated by replacing both the objective and constraint functions by their sparse-grid surrogates. Numerical results showcase the effectiveness of the proposed approach.

## Contributed talks

### **Thermal-Net: Convolutional neural network for 3D printing**

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Additive Manufacturing, also known as 3D printing, is the process of joining materials in a layer-by-layer manner to build an object from 3D model data, using a source of thermal energy to fuse the different layers. Numerical simulations have been carried out with the aim of understanding and optimizing the process by predicting the temperature fields within the object during manufacturing. The process is usually modeled by non-linear heat conduction partial differential equation (PDE) implemented within a finite element framework. We propose a neural network for solving the 2D heat conducting problem for selective laser melting additive manufacturing of metal powder in supervised fashion. The solution is approximated by a deep neural network which is the minimizer of a cost function, and satisfies the PDE, boundary conditions, and additional material conditions. The straightforward approach used to solving time-dependent PDE is to apply a semidiscretization in time by a finite difference approximation, which yields a sequence of stationary problems, and then to use the network to solve each stationary problem. The network has two primary components: an encoder, designed to map high-level input functions to low-dimensional latent features, and a decoder, used to map these latent features to approximate solutions. These two components consist of a series of convolutional layers. The neural network proposed is capable of accurately simulating the temperature evolution of a 2D component manufactured. The good performances of the proposed approach are addressed through several experiments on synthetic data.

## On the solution of the photometric stereo problem with unknown lighting

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The photometric stereo problem, typical of Computer Vision, consists of determining the shape of an object starting from a set of images taken from the same point of view, but under different lighting conditions. A well-known result shows that to solve the problem when the lights are located at an unknown position, ideally located at an infinite distance from the subject, at least 6 images are needed. We will discuss a technique to approximate the solution of the problem when the lights are at a finite distance from the subject, determining at the same time their position.