Seeing in 3D from a Single Image with Geometric Priors

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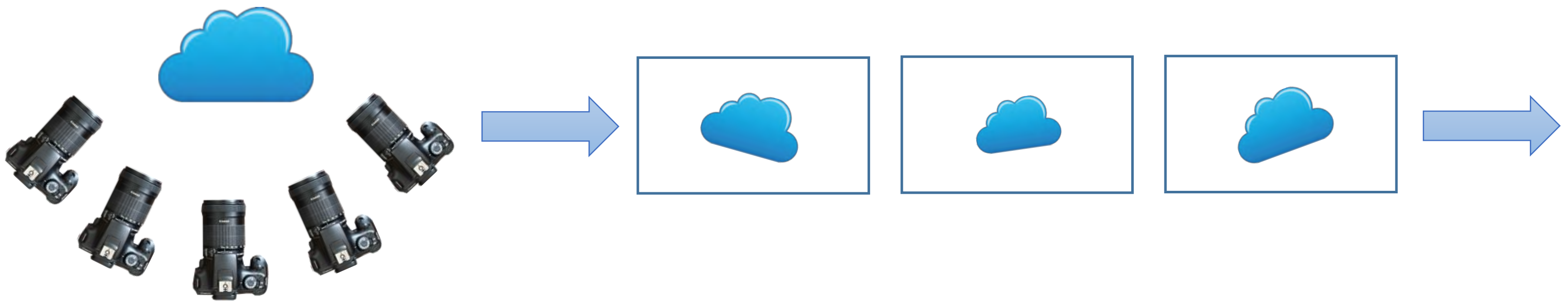
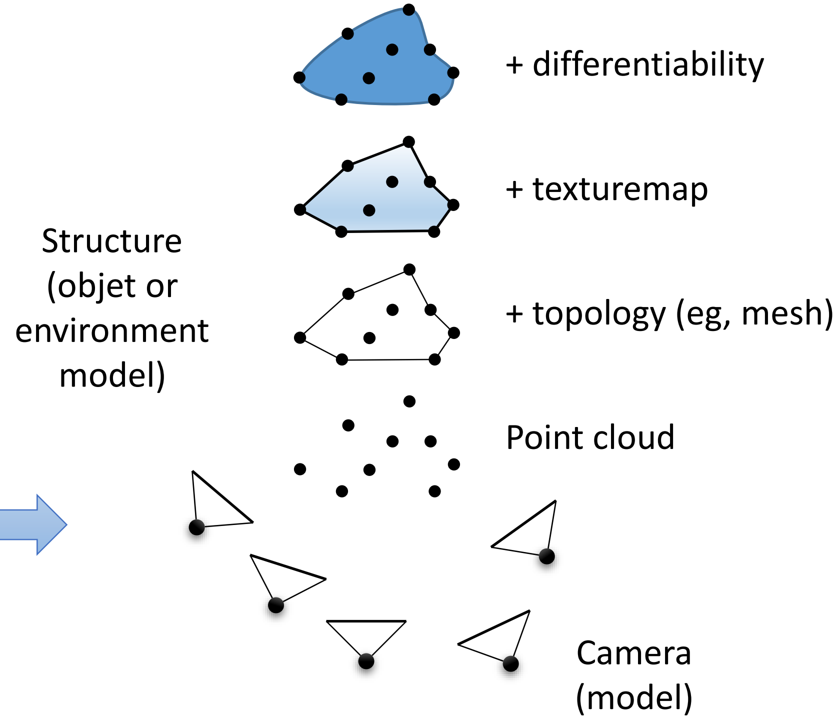
Toby Collins, Daniel Pizarro, Nicolas Bourdel, Michel Canis, Emmanuel Buc, Bertrand Le Roy et al

3D Reconstruction a.k.a. Inverse Imaging

To convert one or multiple 2D images to a 3D model of the observed object or environment





Filming or photographing

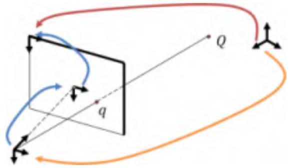
Digital images

3D reconstruction

3D Reconstruction Approaches

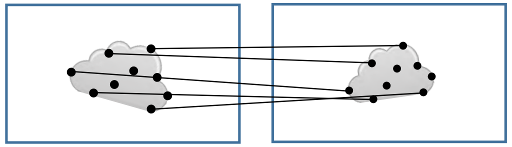
Physics-based

Analysis and **modelling** of the imaging process



Hand-crafted explicit use of visual cues

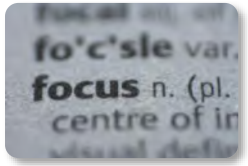
Visual motion



Optic flow or correspondences

Shading

Occlusions



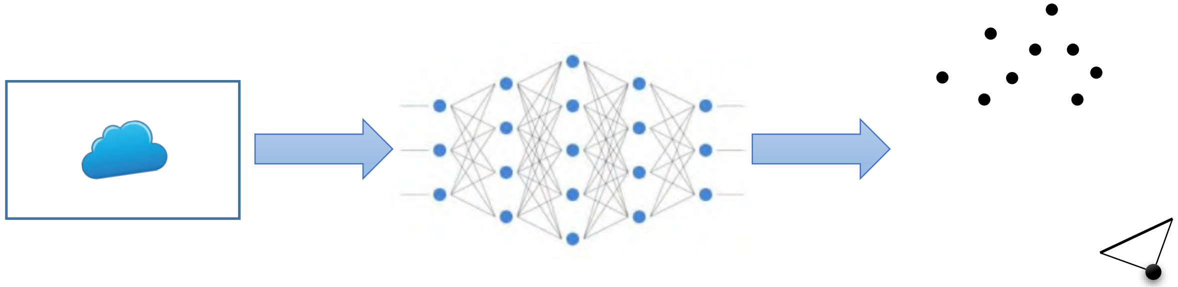
Blur

Silhouette

Shadow

Learning-based

Learning of the **reconstruction function** from data

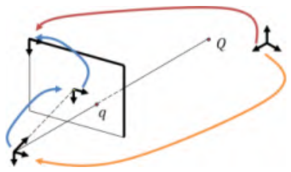


Automatic implicit combination of visual cues

3D Reconstruction Approaches

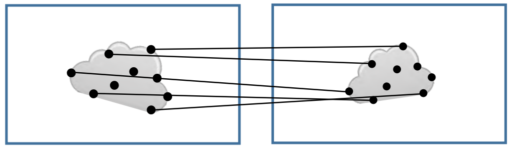
Physics-based

Analysis and **modelling** of the imaging process



Hand-crafted explicit use of visual cues

Visual motion



Optic flow or correspondences

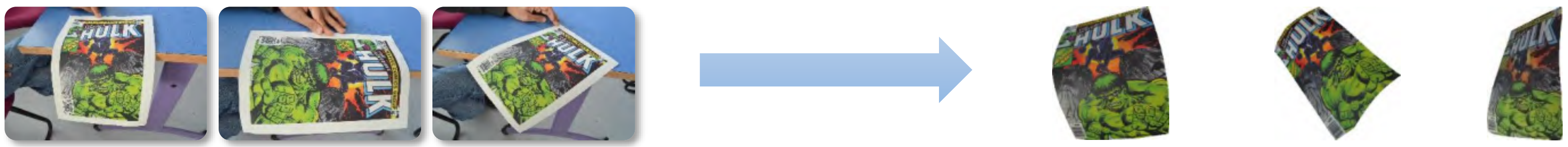
- **Structure-from-Motion** / SLAM
- Visual geometry theory
- Multiple images + **rigidity**

The result is **quantitative** and **uncertainty** can be analysed

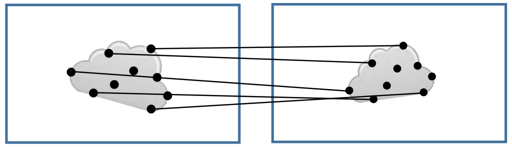
Agarwal et al, University of Washington

Quantitative Monocular Non-Rigid 3D Reconstruction

- To turn N images of a **deformable** object into N 3D shapes



- Proposal of a **physics-based** approach based on **visual motion**, as SfM



Which priors to **replace rigidity**?

- Priors on the **shape**
  - **Smoothness**
  - Low curvature
  - Subspace

- Priors on the **deformation**
  - **Smoothness**
  - **Isometry**, elasticity, etc
  - Low curvature
  - Subspace
  - Small baseline (video frames)

- The **template** prior

Template-based Case: Shape-from-Template (SfT)

Template

SfT

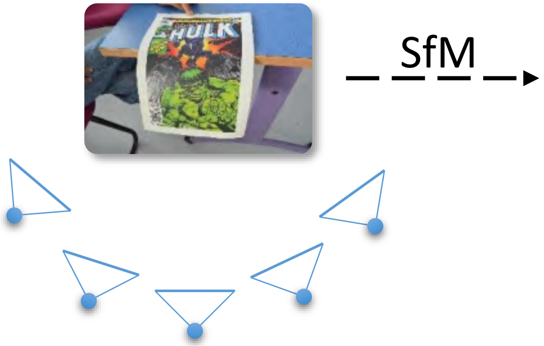
Possible solution from a **single image**

Template Computation

Using a **digital object model**

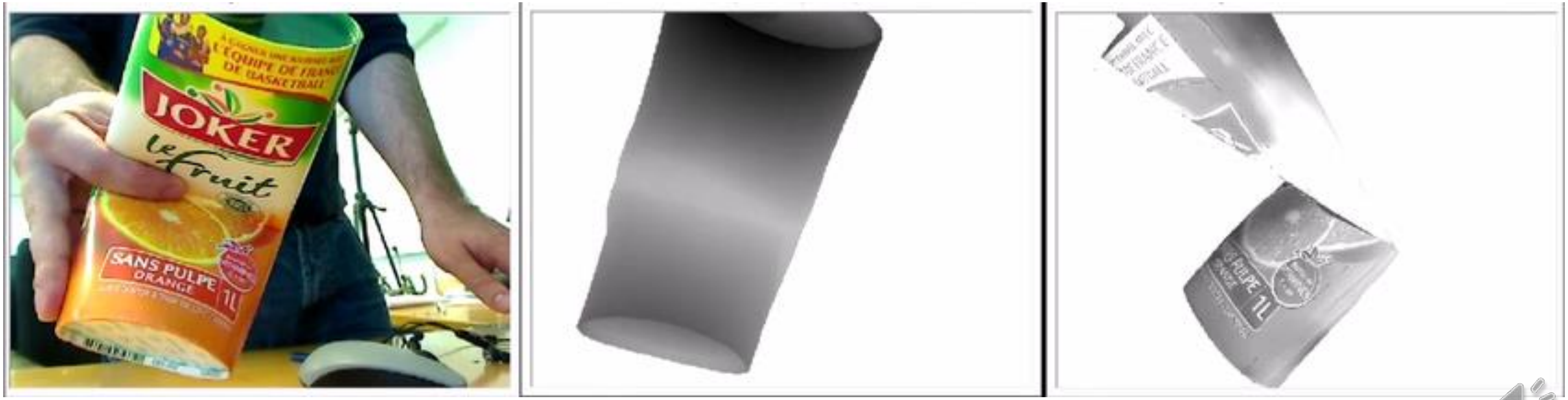
Template

Using **SfM** or **SLAM**



Template

Real-Time Isometric Shape-from-Template



Collins et al, ISMAR 2015

Template-free Case: Non-Rigid Structure-from-Motion (NRSfM)

Possible solution from **at least two images**

Presentation Layout

- Local depth computation in SfT
- Local depth computation in NRSfM
- Application to laparoscopy

Can we Reconstruct from **One** Point Correspondence?

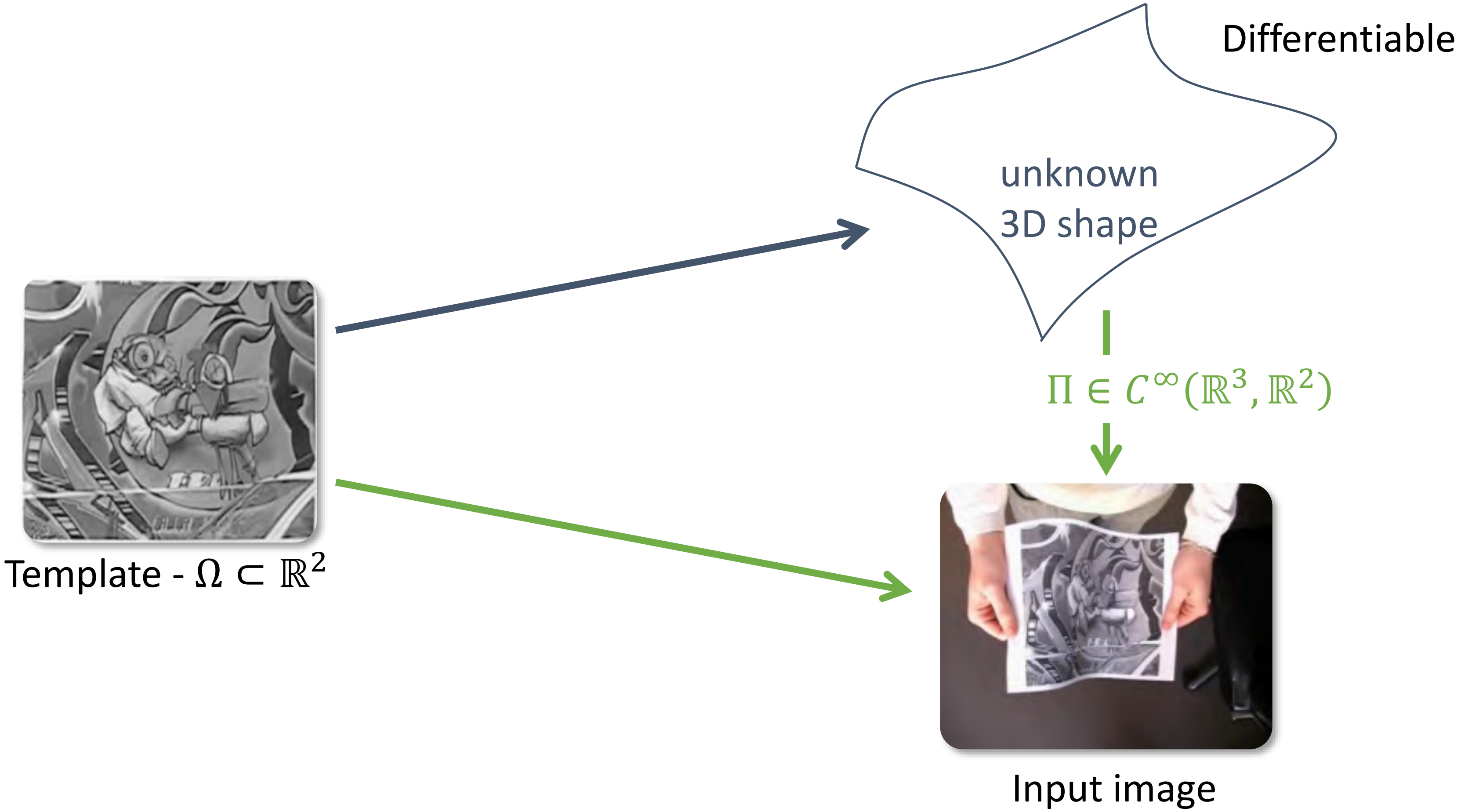
- Shape-from-Template
  - **The short story: an algebraic derivation**
  - The whole story: Riemannian geometry

Bartoli et al, PAMI 2015

- Non-Rigid Structure-from-Motion
  - The very short story: a partial algebraic derivation

Parashar et al, PAMI 2018

1. Modeling

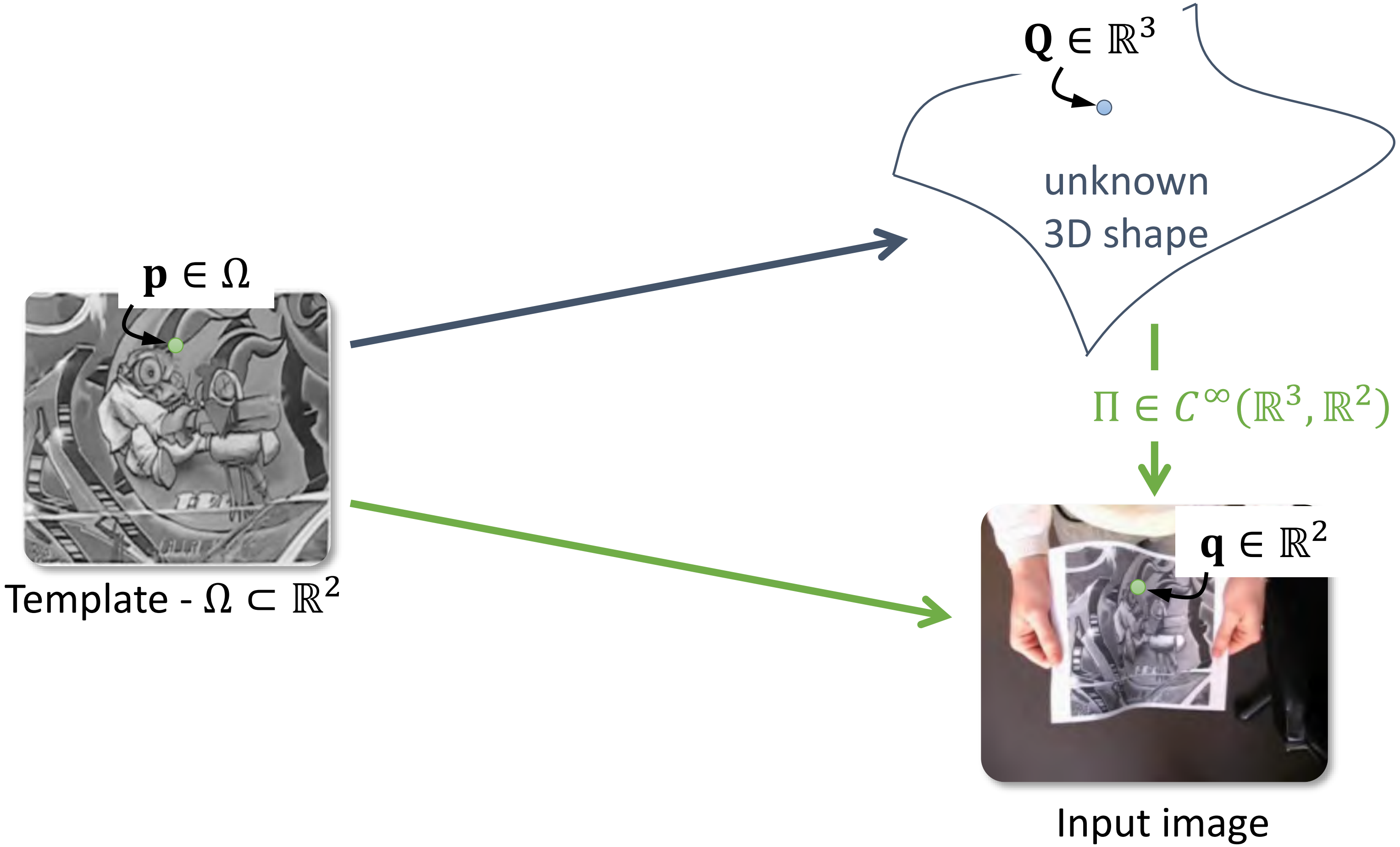


= known

= unknown

2. Data and Unknowns

Find $\mathbf{Q} \in \mathbb{R}^3$ (given $\mathbf{p} \leftrightarrow \mathbf{q}$ and Π)



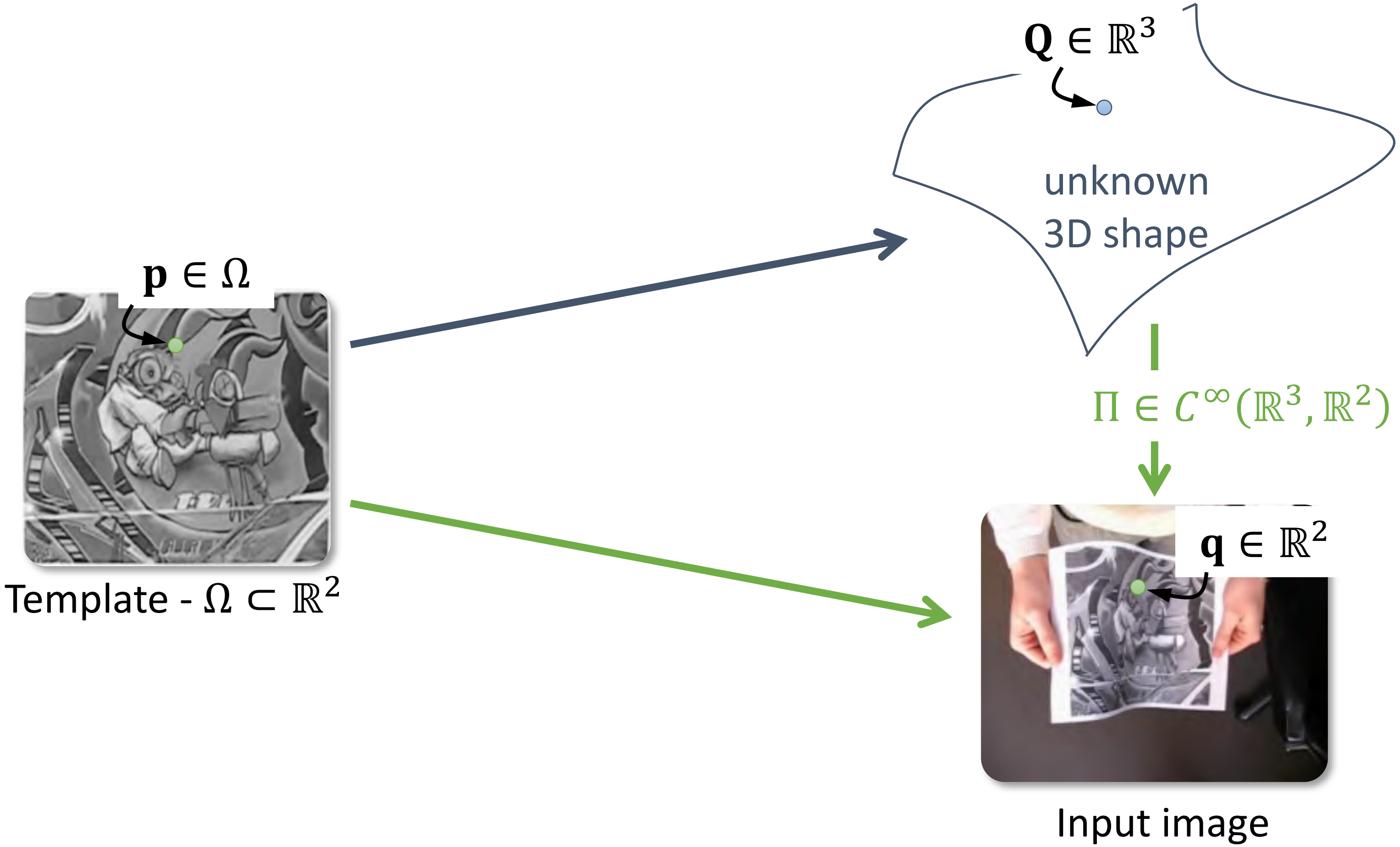
= known

= unknown

3. The Reprojection Constraint, Order 0

Find $\mathbf{Q} \in \mathbb{R}^3$

$$\Big| \; \Pi(\mathbf{Q}) = \mathbf{q}$$



= known

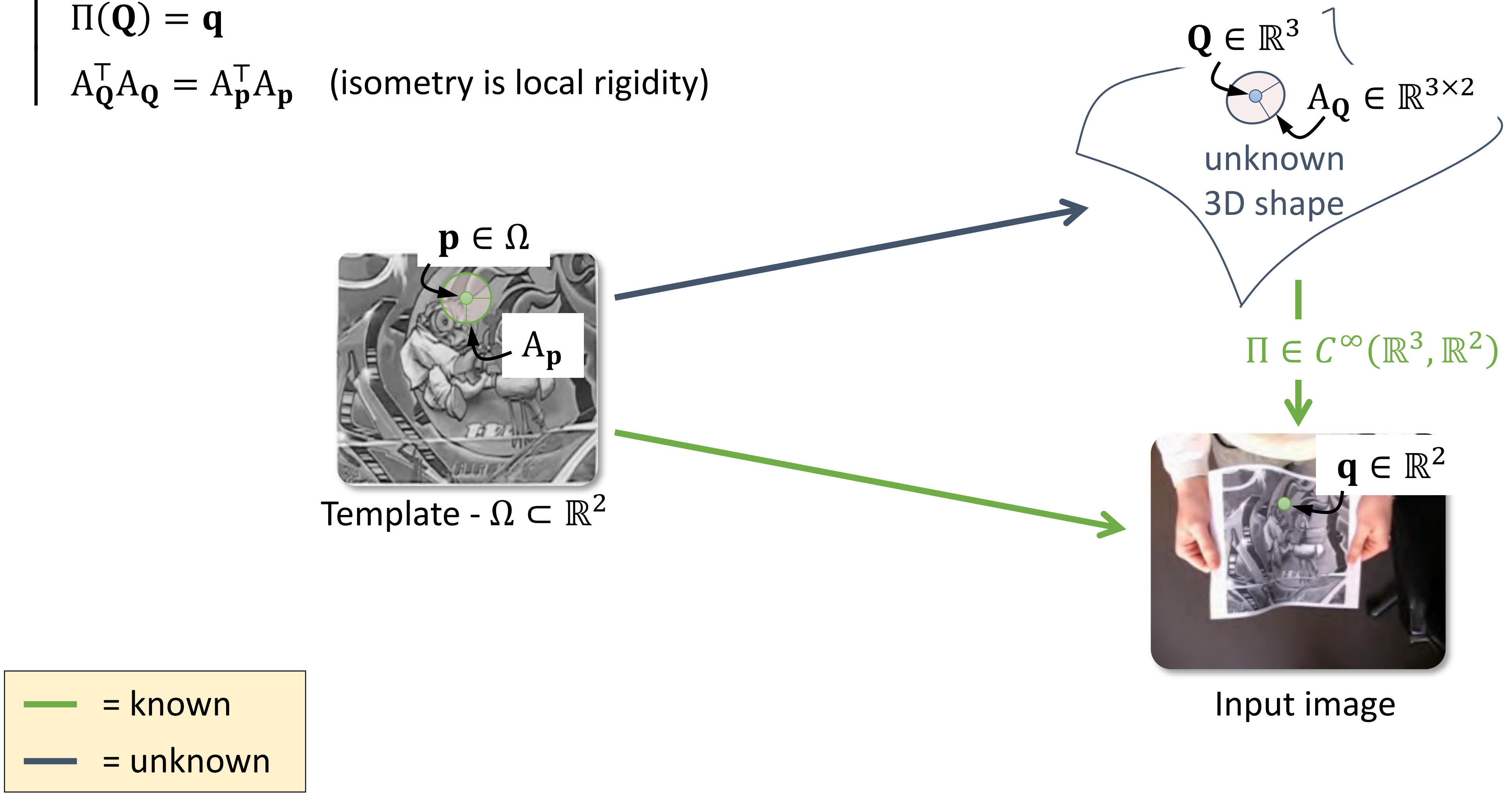
= unknown

4. The Deformation Constraint

Find $\mathbf{Q} \in \mathbb{R}^3$, $\mathrm{A}_\mathbf{Q} \in \mathbb{R}^{3\times2}$

$$\begin{cases} \Pi(\mathbf{Q}) = \mathbf{q} \\ \mathrm{A}_\mathbf{Q}^\top \mathrm{A}_\mathbf{Q} = \mathrm{A}_\mathbf{p}^\top \mathrm{A}_\mathbf{p} \end{cases}$$

(isometry is local rigidity)

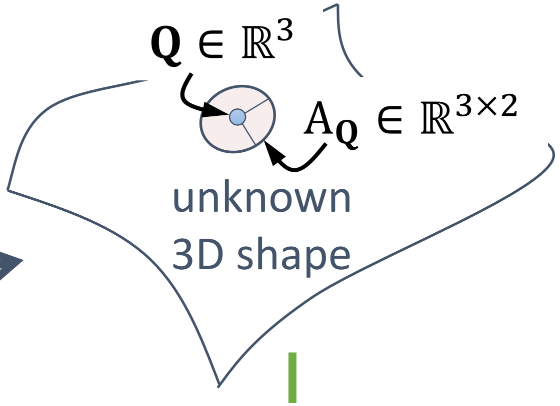


5. Solving?

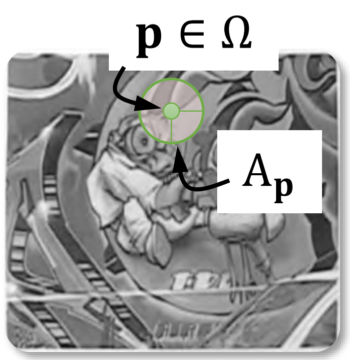
Find $\mathbf{Q} \in \mathbb{R}^3, \mathrm{A}_\mathbf{Q} \in \mathbb{R}^{3\times 2}$ → 9 unknowns

$$\begin{cases} \Pi(\mathbf{Q}) = \mathbf{q} \\ \mathrm{A}_\mathbf{Q}^\top \mathrm{A}_\mathbf{Q} = \mathrm{A}_\mathbf{p}^\top \mathrm{A}_\mathbf{p} \end{cases}$$ → 5 constraints

Underconstrained



$\Pi \in C^\infty(\mathbb{R}^3, \mathbb{R}^2)$



Template - $\Omega \subset \mathbb{R}^2$

Input image

= known

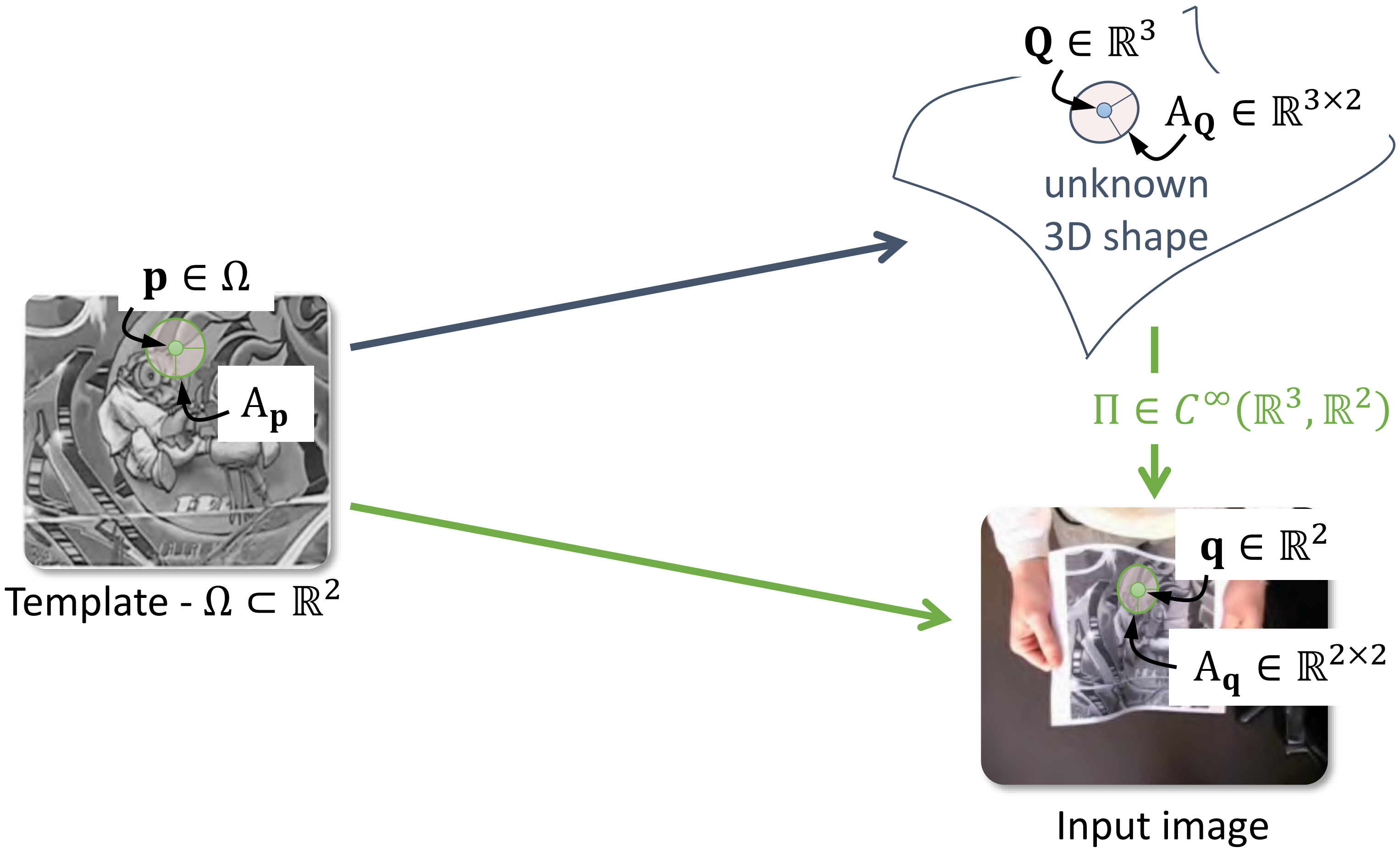
= unknown

6. The Reprojection Constraint, Order 1

Find $\mathbf{Q} \in \mathbb{R}^3$, $A_\mathbf{Q} \in \mathbb{R}^{3\times 2}$

$\mathcal{A}$

$$\Pi(\mathbf{Q}) = \mathbf{q}$$
$$A_\mathbf{Q}^\top A_\mathbf{Q} = A_\mathbf{p}^\top A_\mathbf{p}$$
$$\nabla\Pi(\mathbf{Q}) A_\mathbf{Q} = A_\mathbf{q}$$



= known

= unknown

7. Solving

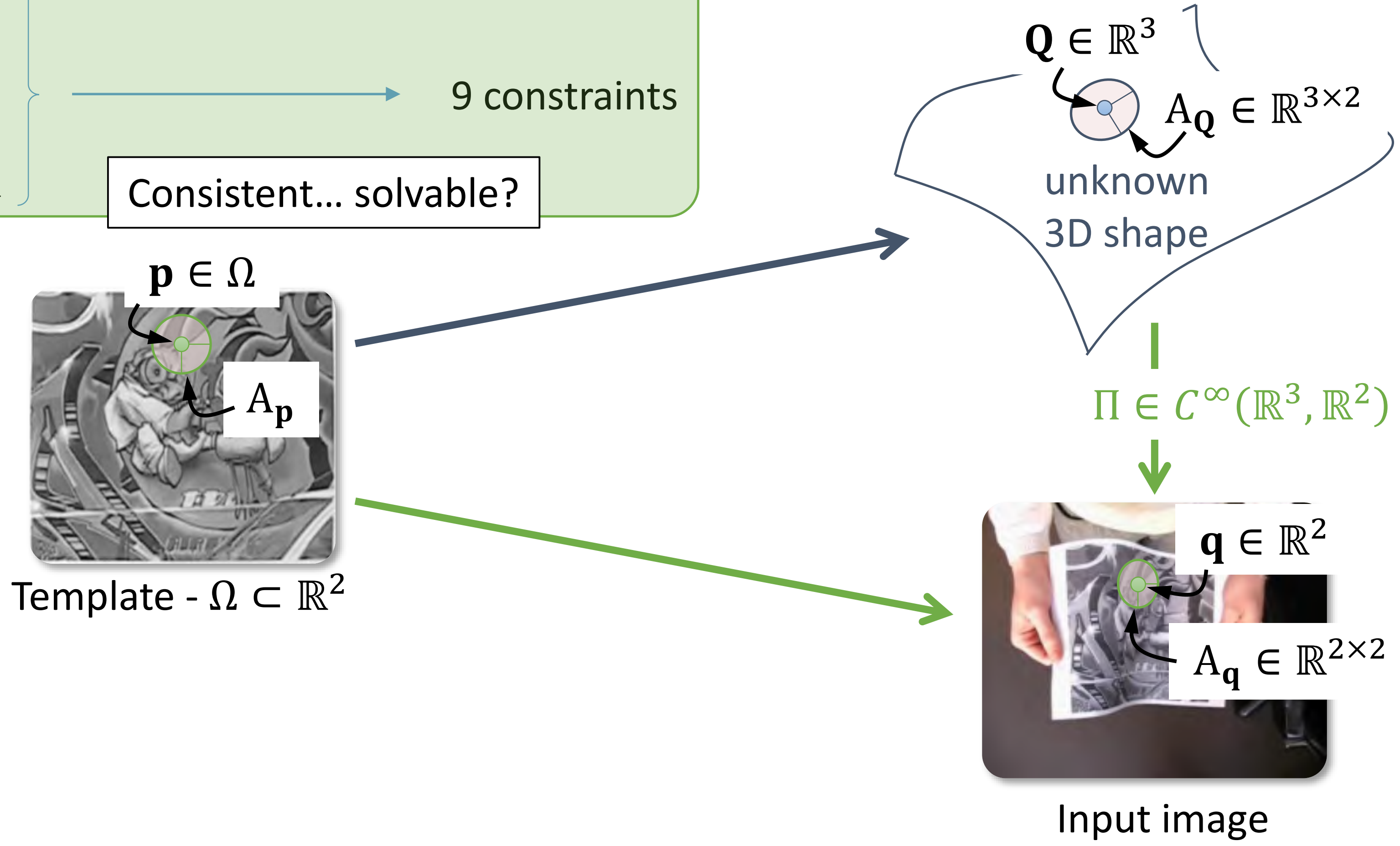
Find $\mathbf{Q} \in \mathbb{R}^3$, $A_{\mathbf{Q}} \in \mathbb{R}^{3\times 2}$ → 9 unknowns

$\mathcal{A}$

$$\Pi(\mathbf{Q}) = \mathbf{q}$$
$$A_{\mathbf{Q}}^\top A_{\mathbf{Q}} = A_{\mathbf{p}}^\top A_{\mathbf{p}}$$
$$\nabla\Pi(\mathbf{Q}) A_{\mathbf{Q}} = A_{\mathbf{q}}$$

→ 9 constraints

Consistent... solvable?



= known

= unknown

7. Solving

Find $\mathbf{Q} \in \mathbb{R}^3$, $\mathrm{A}_{\mathbf{Q}} \in \mathbb{R}^{3\times 2}$

$\mathcal{A}$

$\Pi(\mathbf{Q}) = \mathbf{q}$ → Linear

$\mathrm{A}_{\mathbf{Q}}^\top \mathrm{A}_{\mathbf{Q}} = \mathrm{A}_{\mathbf{p}}^\top \mathrm{A}_{\mathbf{p}}$ → Quadratic

$\nabla\Pi(\mathbf{Q})\mathrm{A}_{\mathbf{Q}} = \mathrm{A}_{\mathbf{q}}$ → Quadratic (for the perspective camera)

$\mathcal{A}$ is a **quadratic** algebraic system

Bézout's bound on the number of solutions is $2^7 = 128$.

Theorem. The algebraic system $\mathcal{A}$ has a unique solution for $\mathbf{Q}$ and two solutions for $\mathrm{A}_{\mathbf{Q}}$.

Theorem. The algebraic system $\mathcal{A}$ represents P3P for infinitesimally close points.

Result. The algebraic system $\mathcal{A}$ has a simple analytic solution.

8. Practical Considerations

For a point $\mathbf{p} \in \Omega$ we have

$$\mathbf{q} \in \mathbb{R}^2, \mathrm{A}_{\mathbf{q}} \in \mathbb{R}^{2\times 2} \xrightarrow{\text{Analytic solution}} \mathbf{Q} \in \mathbb{R}^3, \mathrm{A}_{\mathbf{Q}+}, \mathrm{A}_{\mathbf{Q}-} \in \mathbb{R}^{3\times 2}$$

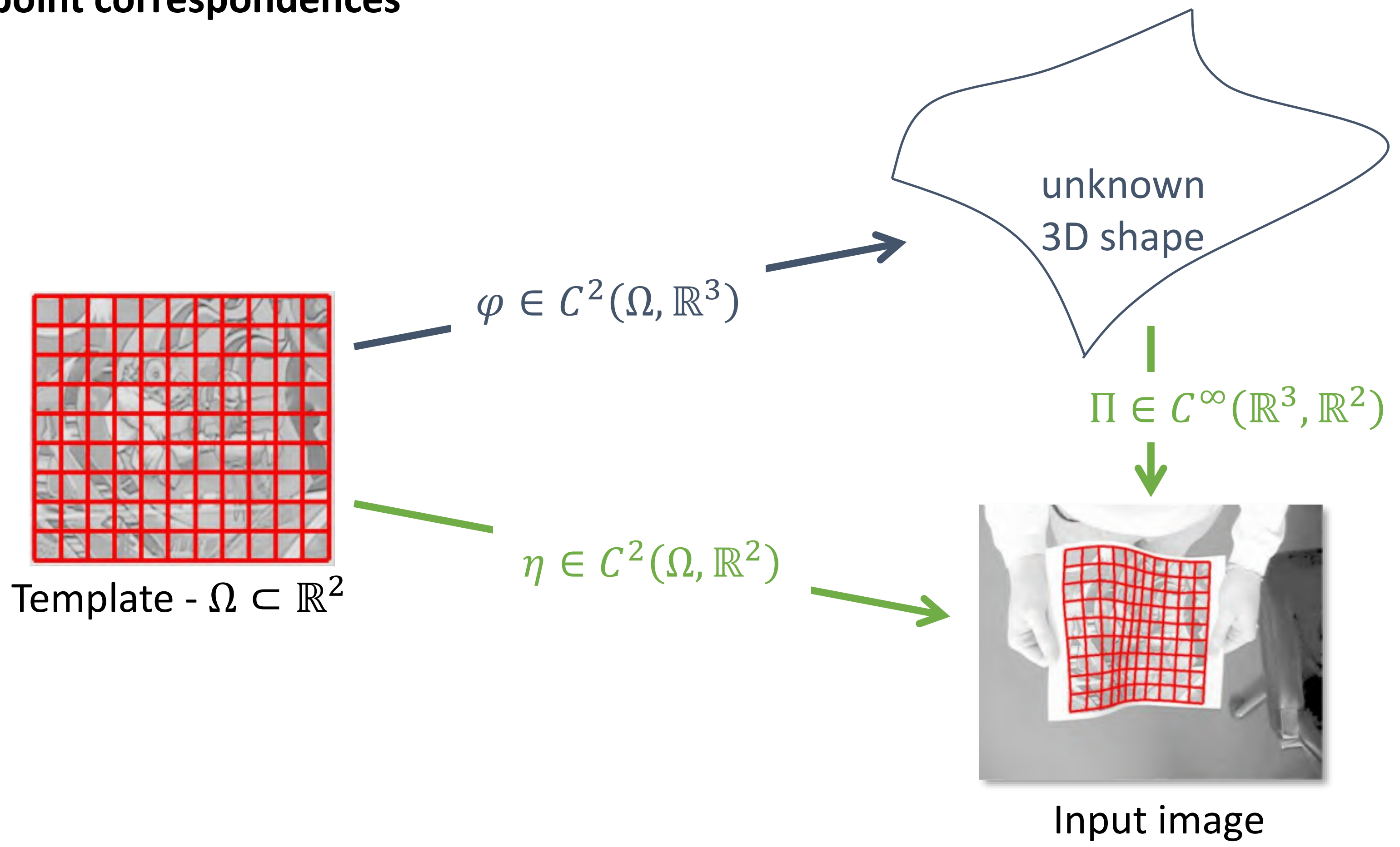
A first-order differential correspondence, represented by a 2D affine transform

Computable in at least three ways:

1. Detect and match affine covariant keypoints
2. Use dense optic flow correspondences
3. Use a warp function fitted to keypoint correspondences

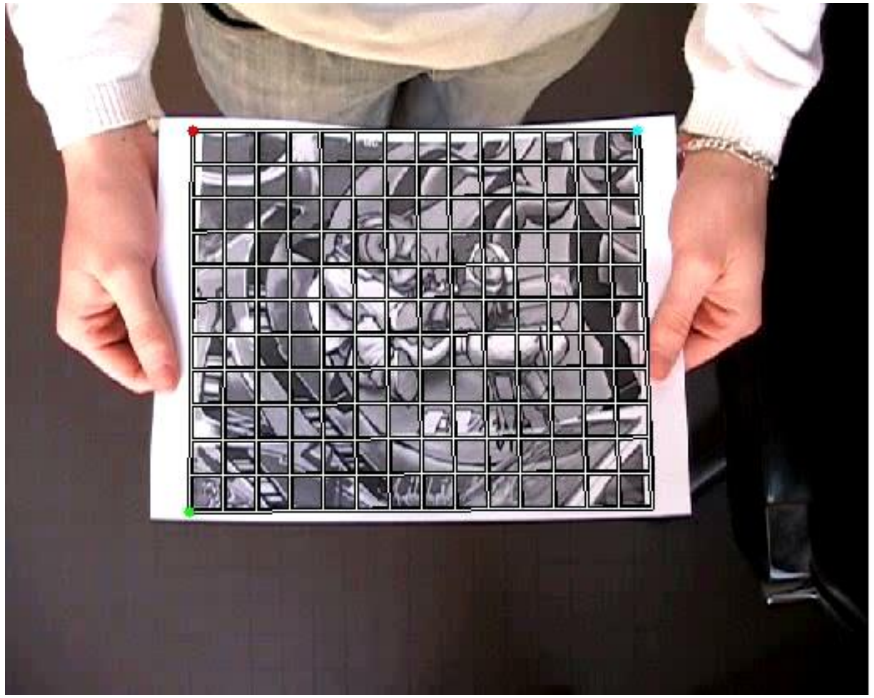
9. The Warp Function

Example of a **Thin-Plate Spline**
Computed from **keypoint correspondences**



= known
= unknown

Reconstruction Results



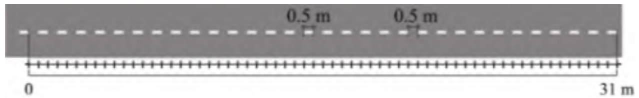
Gay-Bellile et al, PAMI 2010 ; Perriollat et al, IJCV 2011

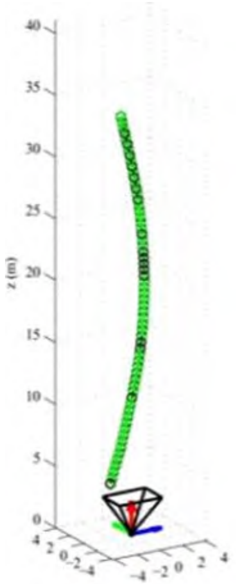
Reconstruction Results

Bartoli et al, PAMI 2015

Extensions

- Full 3D templates – same framework
- Isometric deformation and focal length – second-order local solution
- Conformal deformation (angle-preserving) – first-order local solution for normal
- Equiareal deformation (area-preserving) – no local solution found
- Metric affine cameras – local solutions
- 1D structures – no local solution

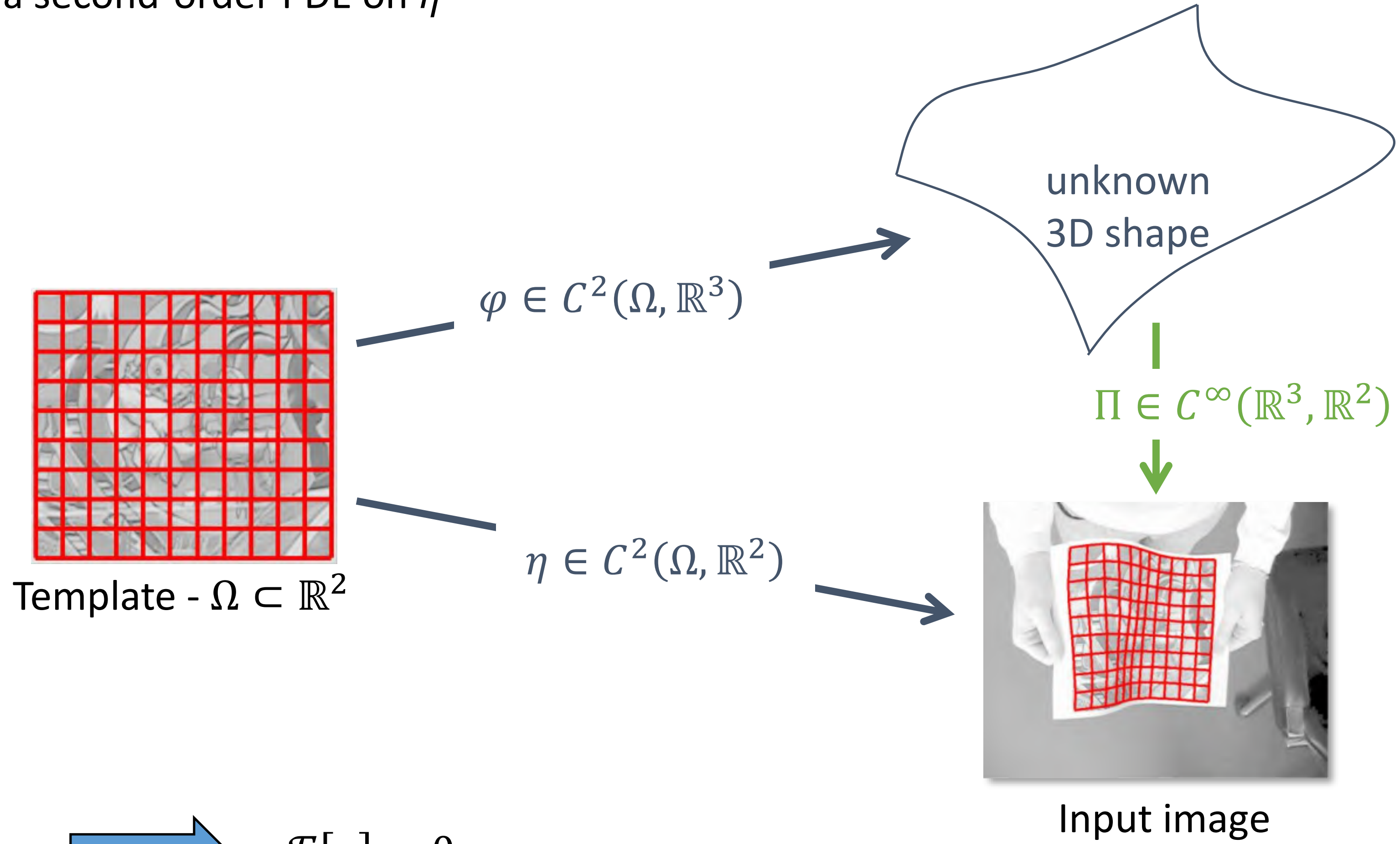




Bartoli et al, ICCV 2013, PAMI 2015 ; Casillas-Perez et al, JMIV 2019 ; Gallardo et al, IJCV 2020

Is the Warp η an Arbitrary Smooth 2D Function?

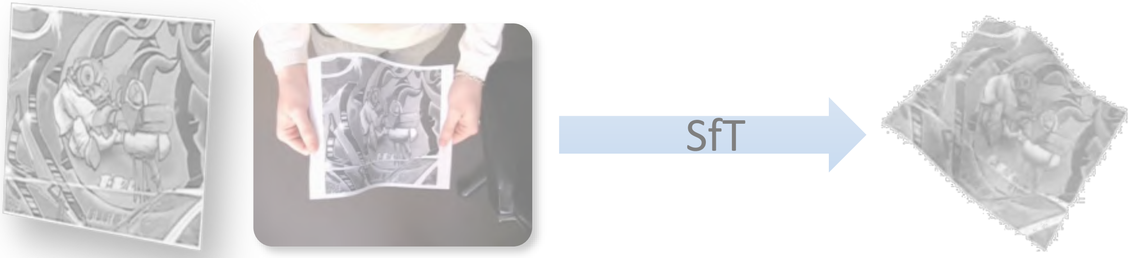
- Isowarp: perspective projection of isometric deformation
- Characterisation: a second-order PDE on η



$\eta = \Pi \circ \varphi$ ⇨ $\mathcal{F}[\eta] = 0$

Pizarro et al, BMVC 2013

Can we Reconstruct from **One** Point Correspondence?



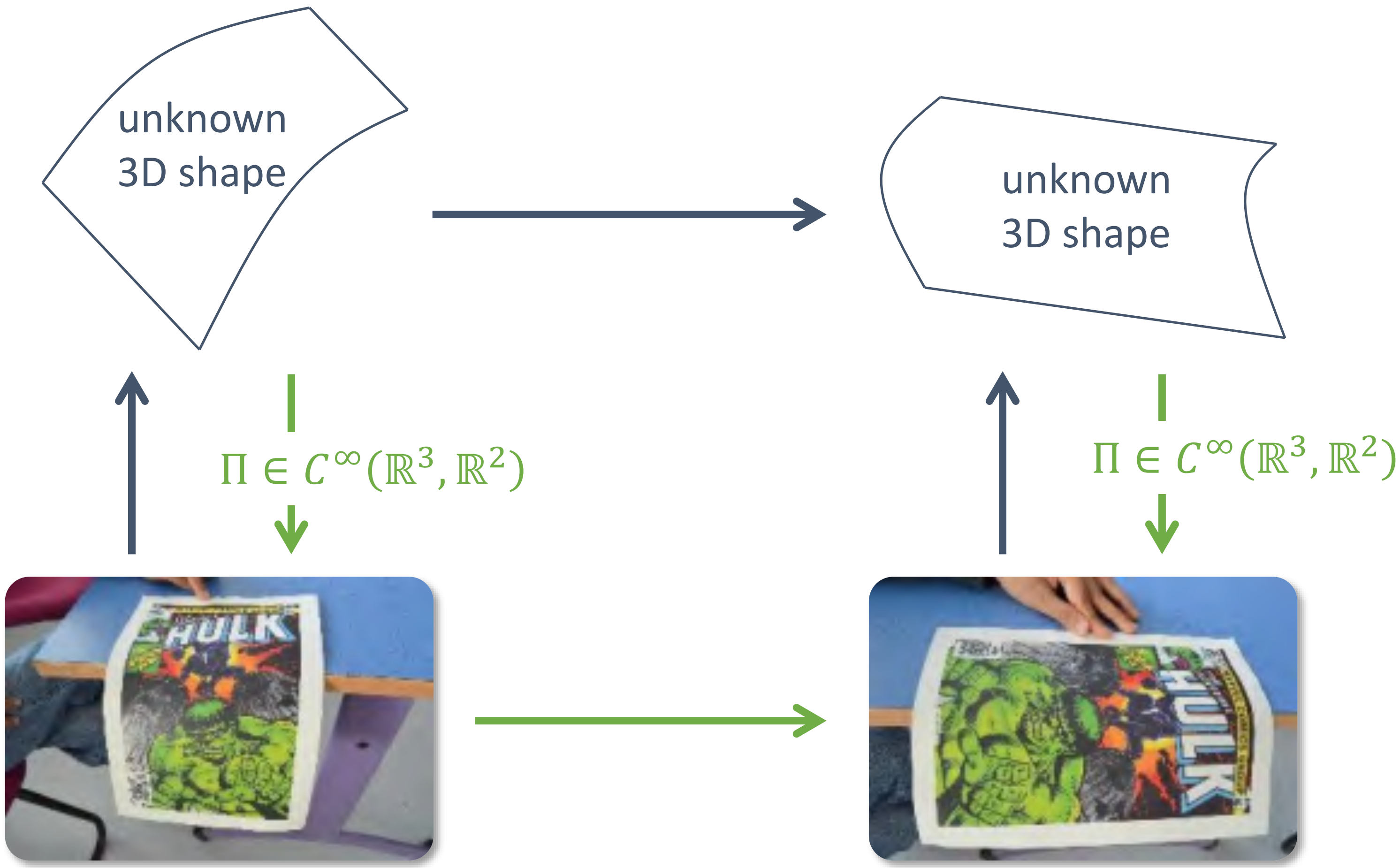
- Shape-from-Template
  - **The short story: an algebraic derivation**
  - The whole story: Riemannian geometry

Bartoli et al, PAMI 2015

- Non-Rigid Structure-from-Motion
  - **The very short story: a partial algebraic derivation**

Parashar et al, PAMI 2018

Modeling



Input image 1

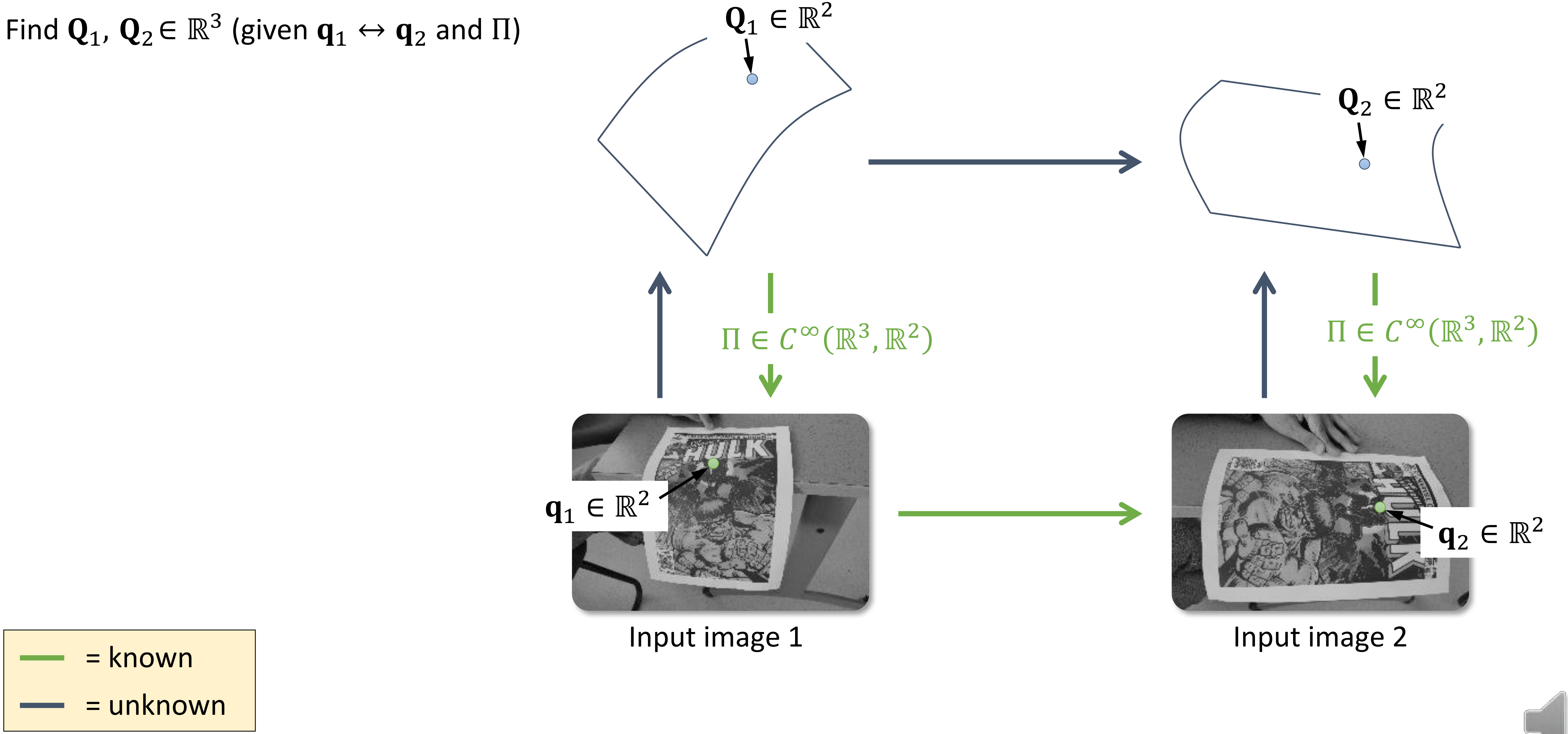
Input image 2

= known

= unknown

Data and Unknowns

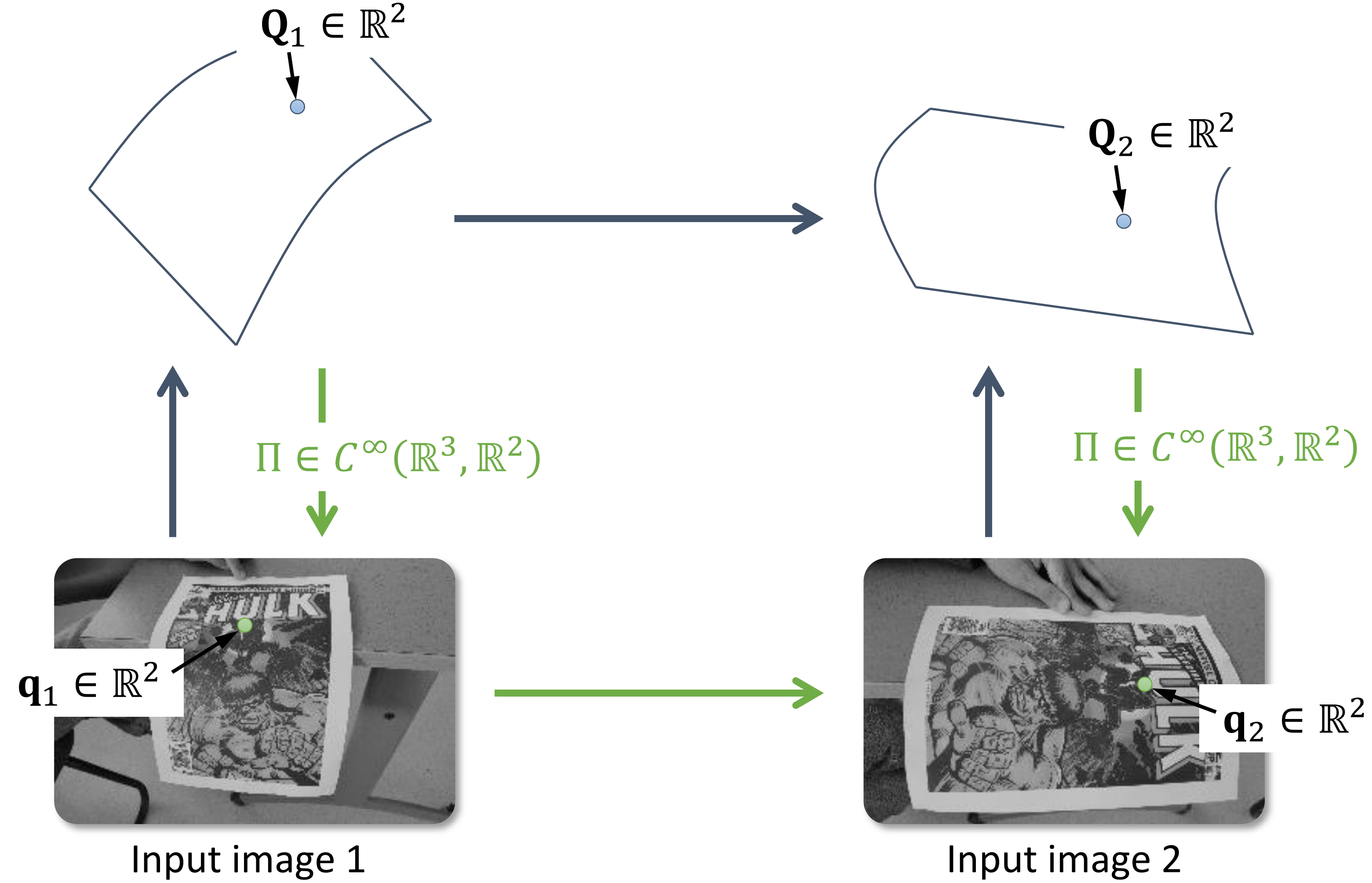
Find $\mathbf{Q}_1$, $\mathbf{Q}_2 \in \mathbb{R}^3$ (given $\mathbf{q}_1 \leftrightarrow \mathbf{q}_2$ and Π)



The Reprojection Constraints, Order 0

Find $\mathbf{Q}_1, \mathbf{Q}_2 \in \mathbb{R}^3$

$$\left| \begin{array}{l} \Pi(\mathbf{Q}_1) = \mathbf{q}_1 \\ \Pi(\mathbf{Q}_2) = \mathbf{q}_2 \end{array} \right.$$



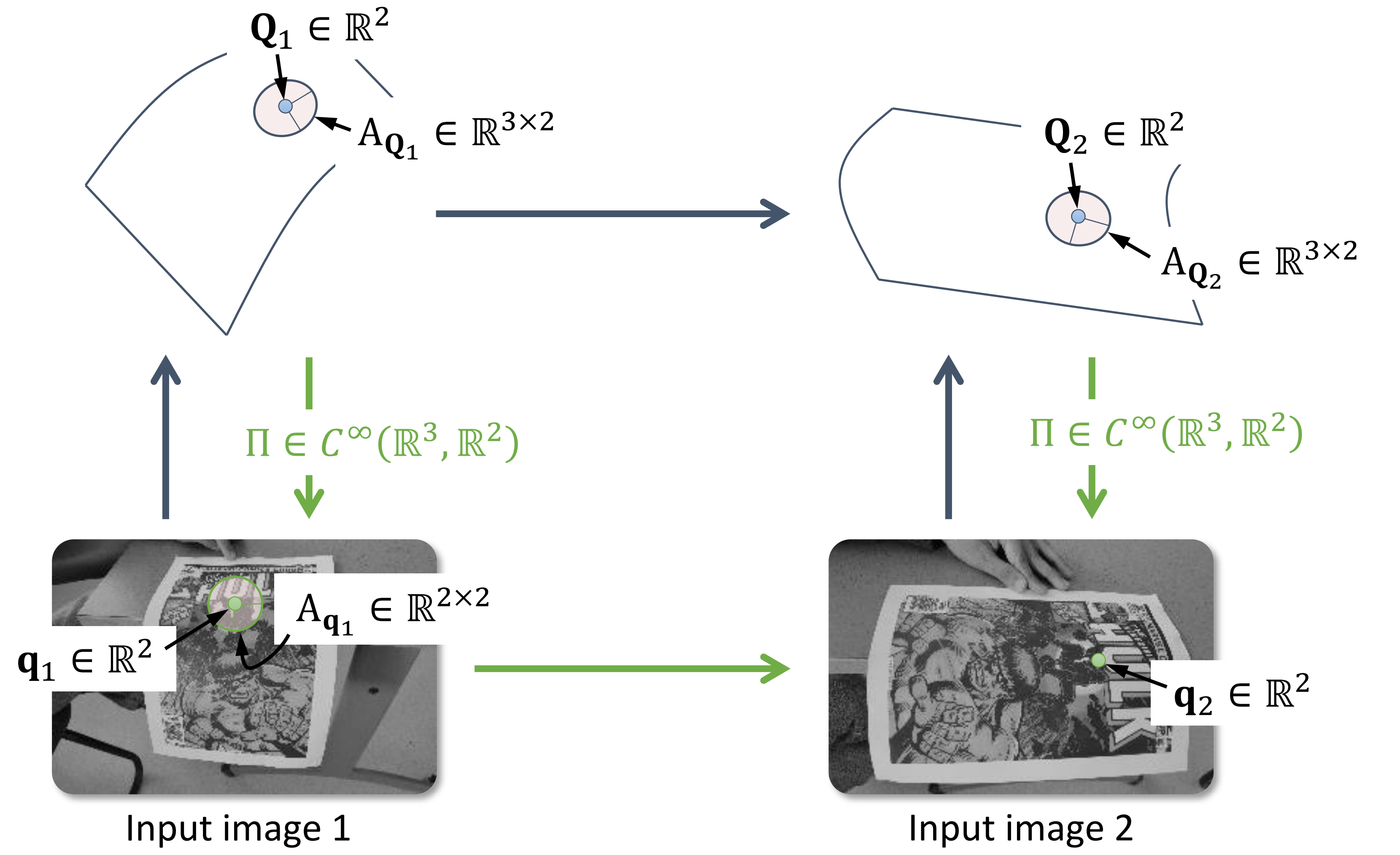
(green line)	= known
(dark line)	= unknown

The Deformation Constraint

Find $\mathbf{Q}_1$, $\mathbf{Q}_2 \in \mathbb{R}^3$, $\mathrm{A}_{\mathbf{Q}_1}, \mathrm{A}_{\mathbf{Q}_2} \in \mathbb{R}^{3\times 2}$

$\mathcal{A}_0$

$$\Pi(\mathbf{Q}_1) = \mathbf{q}_1$$
$$\Pi(\mathbf{Q}_2) = \mathbf{q}_2$$
$$\mathrm{A}_{\mathbf{Q}_1}^\top \mathrm{A}_{\mathbf{Q}_1} = \mathrm{A}_{\mathbf{Q}_2}^\top \mathrm{A}_{\mathbf{Q}_2}$$
$$\nabla\Pi(\mathbf{Q}_1)\mathrm{A}_{\mathbf{Q}_1} = \mathrm{A}_{\mathbf{q}_1}$$



= known

= unknown

Solving?

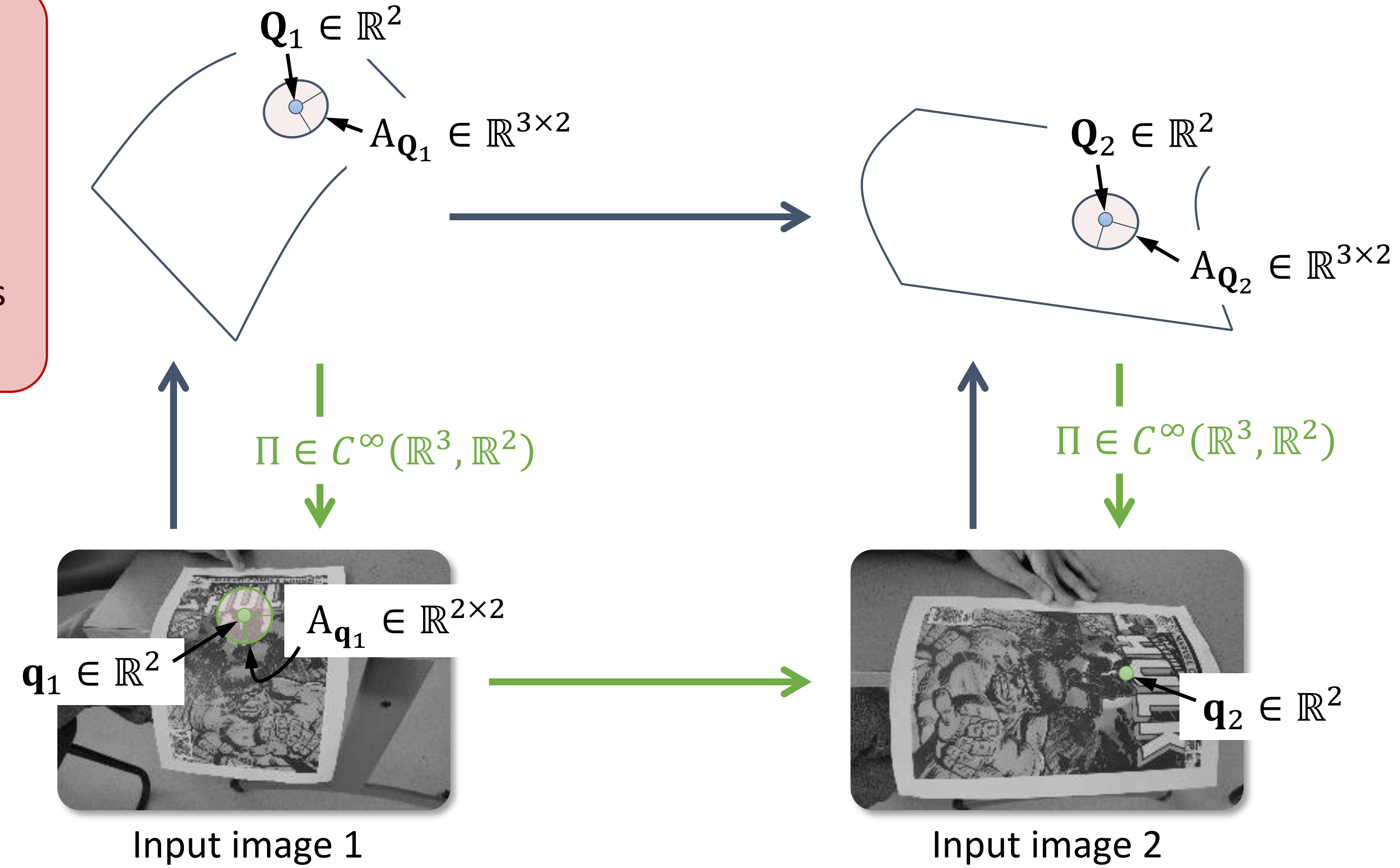
Find $\mathbf{Q}_1, \mathbf{Q}_2 \in \mathbb{R}^3, \mathrm{A}_{\mathbf{Q}_1}, \mathrm{A}_{\mathbf{Q}_2} \in \mathbb{R}^{3\times 2}$

$$\mathcal{A}_0 \quad \begin{cases} \Pi(\mathbf{Q}_1) = \mathbf{q}_1 \\ \Pi(\mathbf{Q}_2) = \mathbf{q}_2 \\ \mathrm{A}_{\mathbf{Q}_1}^\top \mathrm{A}_{\mathbf{Q}_1} = \mathrm{A}_{\mathbf{Q}_2}^\top \mathrm{A}_{\mathbf{Q}_2} \\ \nabla\Pi(\mathbf{Q}_1)\mathrm{A}_{\mathbf{Q}_1} = \mathrm{A}_{\mathbf{q}_1} \end{cases}$$

18 unknowns

11 constraints

Underconstrained



= known

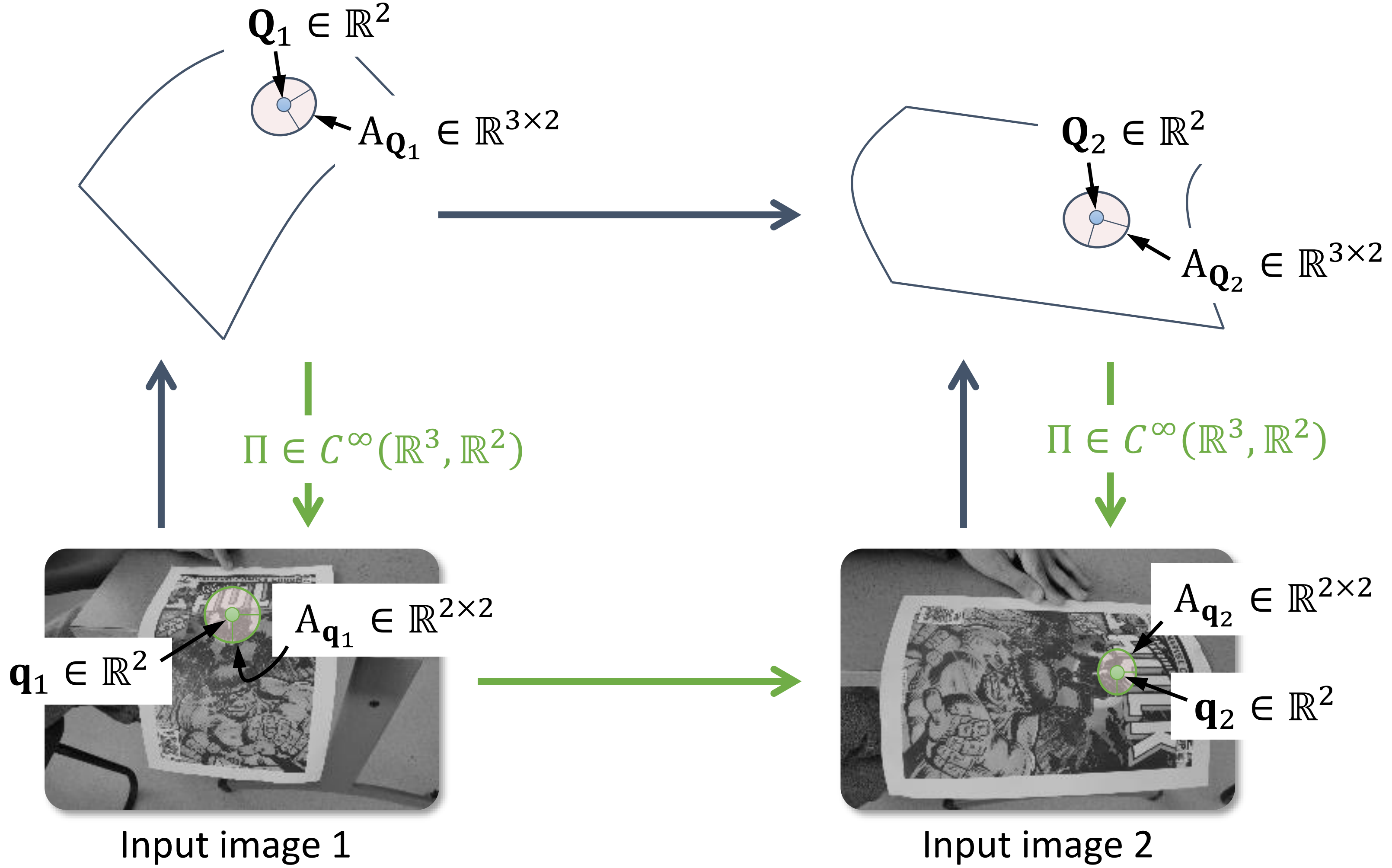
= unknown

The Reprojection Constraint, Order 1

Find $\mathbf{Q}_1, \mathbf{Q}_2 \in \mathbb{R}^3, \mathrm{A}_{\mathbf{Q}_1}, \mathrm{A}_{\mathbf{Q}_2} \in \mathbb{R}^{3\times 2}$

$\mathcal{A}_1$

$$\Pi(\mathbf{Q}_1) = \mathbf{q}_1$$
$$\Pi(\mathbf{Q}_2) = \mathbf{q}_2$$
$$\mathrm{A}_{\mathbf{Q}_1}^\top \mathrm{A}_{\mathbf{Q}_1} = \mathrm{A}_{\mathbf{Q}_2}^\top \mathrm{A}_{\mathbf{Q}_2}$$
$$\nabla\Pi(\mathbf{Q}_1)\mathrm{A}_{\mathbf{Q}_1} = \mathrm{A}_{\mathbf{q}_1}$$
$$\nabla\Pi(\mathbf{Q}_2)\mathrm{A}_{\mathbf{Q}_2} = \mathrm{A}_{\mathbf{q}_2}$$



= known

= unknown

Solving?

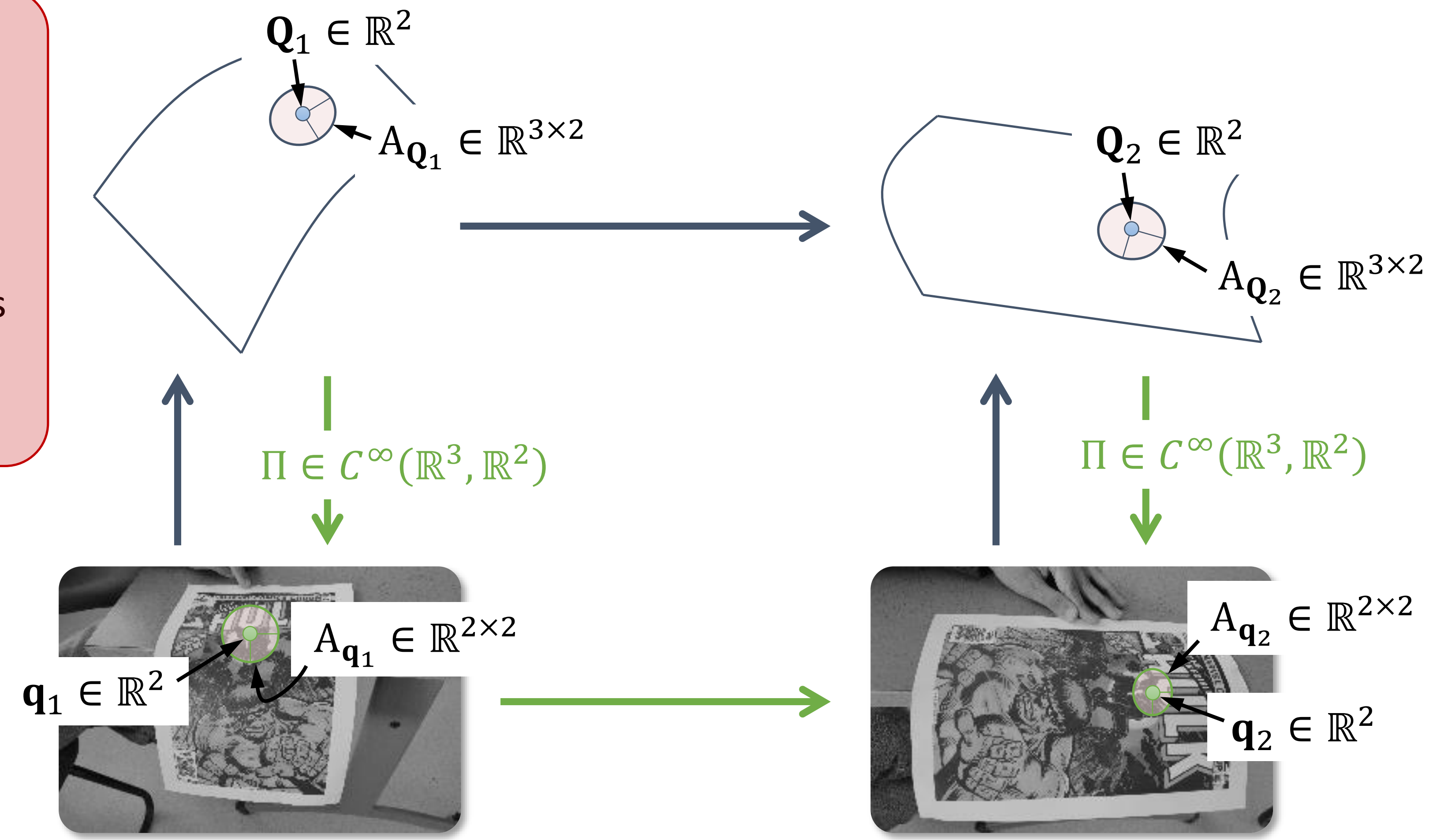
Find $\mathbf{Q}_1, \mathbf{Q}_2 \in \mathbb{R}^3, \mathrm{A}_{\mathbf{Q}_1}, \mathrm{A}_{\mathbf{Q}_2} \in \mathbb{R}^{3\times2}$ → 18 unknowns

$\mathcal{A}_1$

$$\Pi(\mathbf{Q}_1) = \mathbf{q}_1$$
$$\Pi(\mathbf{Q}_2) = \mathbf{q}_2$$
$$\mathrm{A}_{\mathbf{Q}_1}^\top \mathrm{A}_{\mathbf{Q}_1} = \mathrm{A}_{\mathbf{Q}_2}^\top \mathrm{A}_{\mathbf{Q}_2}$$
$$\nabla\Pi(\mathbf{Q}_1)\mathrm{A}_{\mathbf{Q}_1} = \mathrm{A}_{\mathbf{q}_1}$$
$$\nabla\Pi(\mathbf{Q}_2)\mathrm{A}_{\mathbf{Q}_2} = \mathrm{A}_{\mathbf{q}_2}$$

→ 15 constraints

Underconstrained



Input image 1

Input image 2

= known

= unknown

Synthesis and Happy End

Result. The algebraic systems $\mathcal{A}_0$ and $\mathcal{A}_1$ are underconstrained.

Result. The algebraic system $\mathcal{A}_2$ is consistent.

Result. The algebraic system $\mathcal{A}_2$ can be reduced to two cubics in two variables under infinitesimal planarity.

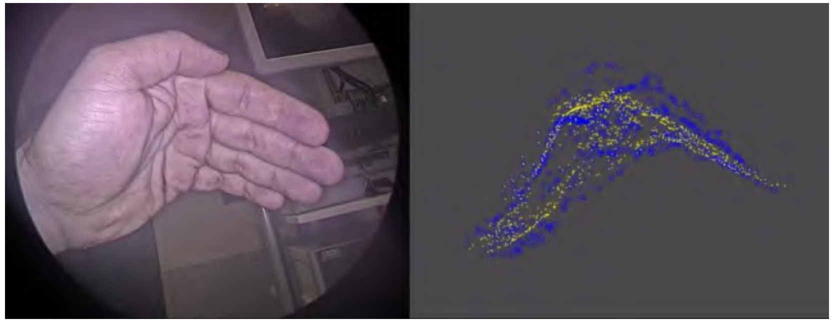
Theorem. The algebraic system $\mathcal{A}_2$ has between 2 and 18 solutions for $\mathrm{A}_{\mathbf{Q}_1}$ and $\mathrm{A}_{\mathbf{Q}_2}$ while $\mathbf{Q}_1$ and $\mathbf{Q}_2$ cannot be resolved.

Result. Using a third image generally disambiguates $\mathrm{A}_{\mathbf{Q}_1}$, $\mathrm{A}_{\mathbf{Q}_2}$ and $\mathrm{A}_{\mathbf{Q}_3}$ but does not resolve $\mathbf{Q}_1$, $\mathbf{Q}_2$ and $\mathbf{Q}_3$.

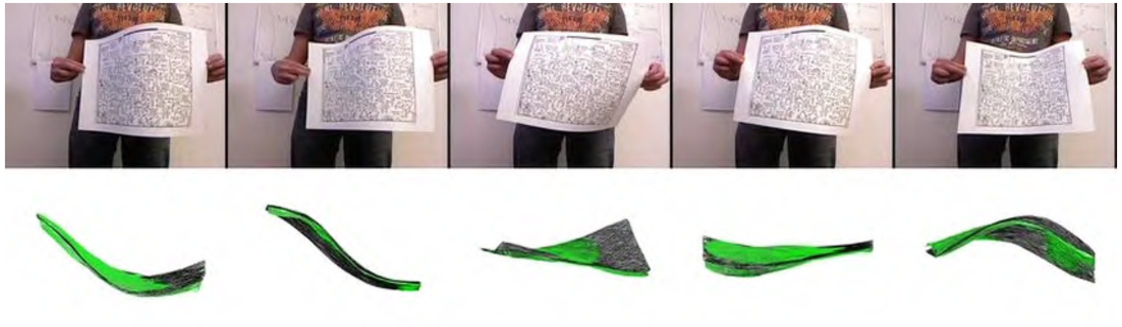
Conjecture. The algebraic system $\mathcal{A}_2$ represents planar SfM for infinitesimally close points.

Results. The algebraic system $\mathcal{A}_2$ can be solved numerically using the theory of resultants by finding the roots of a nonic.

Results from Isometric Non-Rigid Structure-from-Motion (IsoSfM)



Reconstruction Groundtruth Error: 3.23 to 5.72 mm



Reconstruction
Groundtruth
Error: 4.55 to 6.50 mm

Chhatkuli et al, PAMI 2013

Extensions

- Isometric deformation and focal length – second-order local solution
- Conformal deformation (angle-preserving) – second-order local solution for normal
- Equiareal deformation (area-preserving) – no local solution found
- Metric affine cameras – local solutions

Parashar et al, ECCV 2018 ; Parashar et al, PAMI 2018

Robustification of SfT and NRSfM

- We have 'minimal' very local solutions

$$\mathbf{q} \in \mathbb{R}^2, \mathrm{A}_{\mathbf{q}} \in \mathbb{R}^{2\times 2} \xrightarrow{\text{Analytic solution}} \mathbf{Q} \in \mathbb{R}^3, \mathrm{A}_{\mathbf{Q}+}, \mathrm{A}_{\mathbf{Q}-} \in \mathbb{S}_{3\times 2}$$

$$\mathbf{q}_1, \mathbf{q}_2 \in \mathbb{R}^2, \mathrm{A}_{\mathbf{q}_2} \in \mathbb{R}^{2\times 2}, \mathrm{B}_{\mathbf{q}_2} \in \mathbb{R}^6 \xrightarrow{\text{Analytic solution}} \left\{\mathrm{A}^i_{\mathbf{Q}_1} \in \mathbb{R}^{3\times 2}, \mathrm{A}^i_{\mathbf{Q}_2} \in \mathbb{R}^{3\times 2}\right\}_{i=1}^{k}, 2 < k \leq 18$$

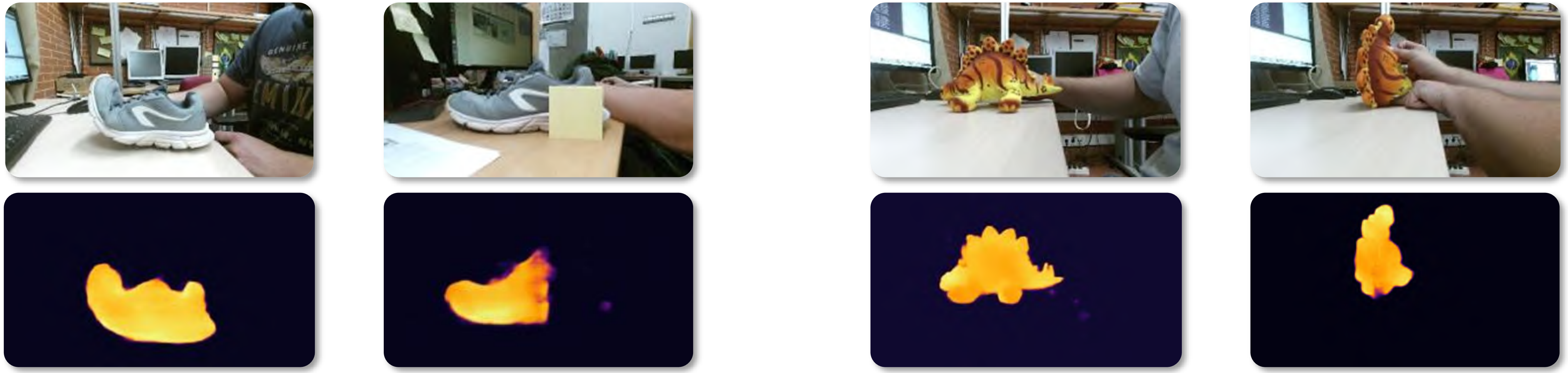
$$\mathbf{q}_1, \dots, \mathbf{q}_n \in \mathbb{R}^2, \mathrm{A}_{\mathbf{q}_2}, \dots, \mathrm{A}_{\mathbf{q}_n} \in \mathbb{R}^{2\times 2}, \mathrm{B}_{\mathbf{q}_2}, \dots, \mathrm{B}_{\mathbf{q}_n} \in \mathbb{R}^6 \xrightarrow{\text{Analytic solution}} \mathrm{A}_{\mathbf{Q}_1}, \dots, \mathrm{A}_{\mathbf{Q}_n} \in \mathbb{R}^{3\times 2}$$

- Robust statistics
- Isometric consistency checks

Collins et al, ECCV 2014

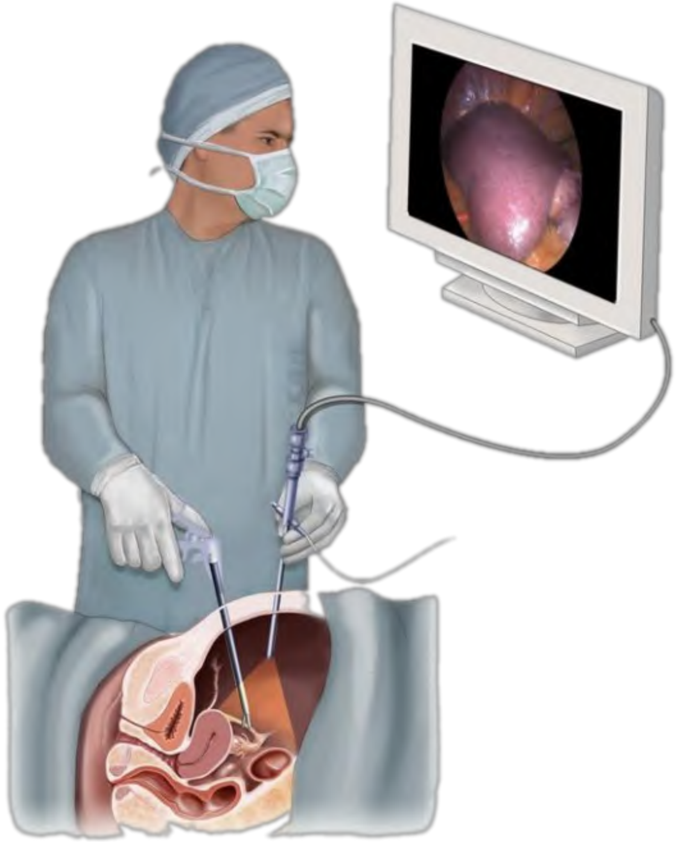
DeepSfT: Deep Neural Network based solution to SfT

- Use the template to simulate training data under many conditions
- One network = one template = one object
- Use a few Kinect depthmaps for domain adaptation
- Compute depth and registration

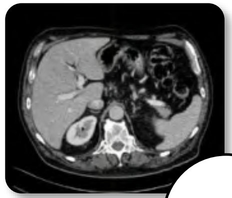


Fuentes-Jimenez et al, arxiv, 2020

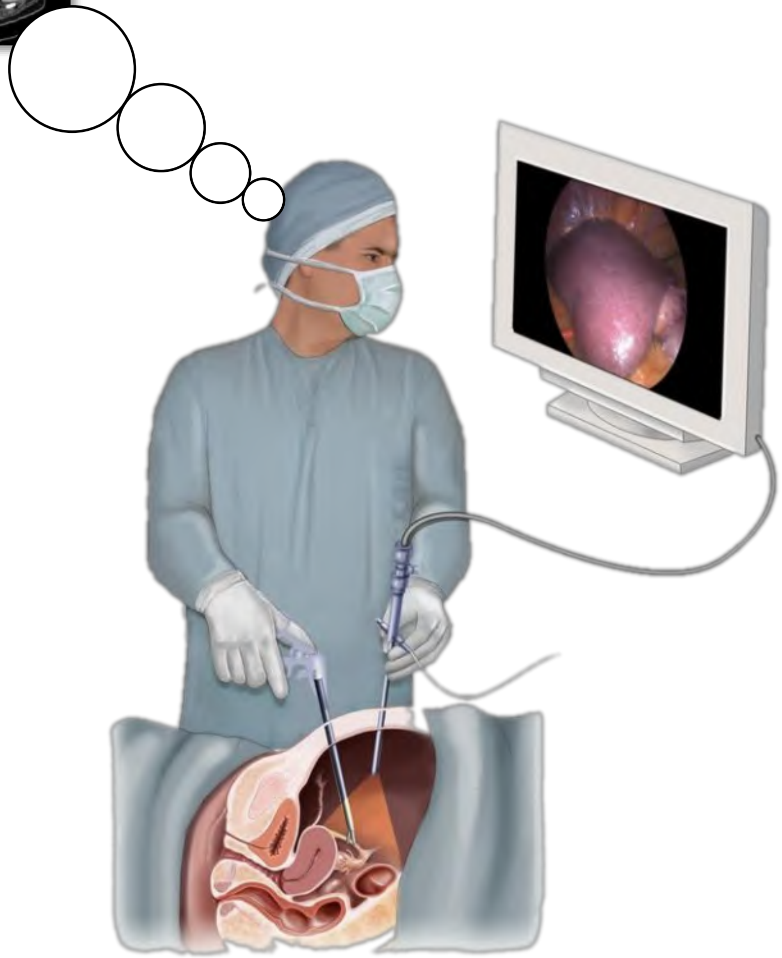
Laparoscopy



Laparoscopy



Preoperative CT scan
Diagnosis



Classical laparoscopy

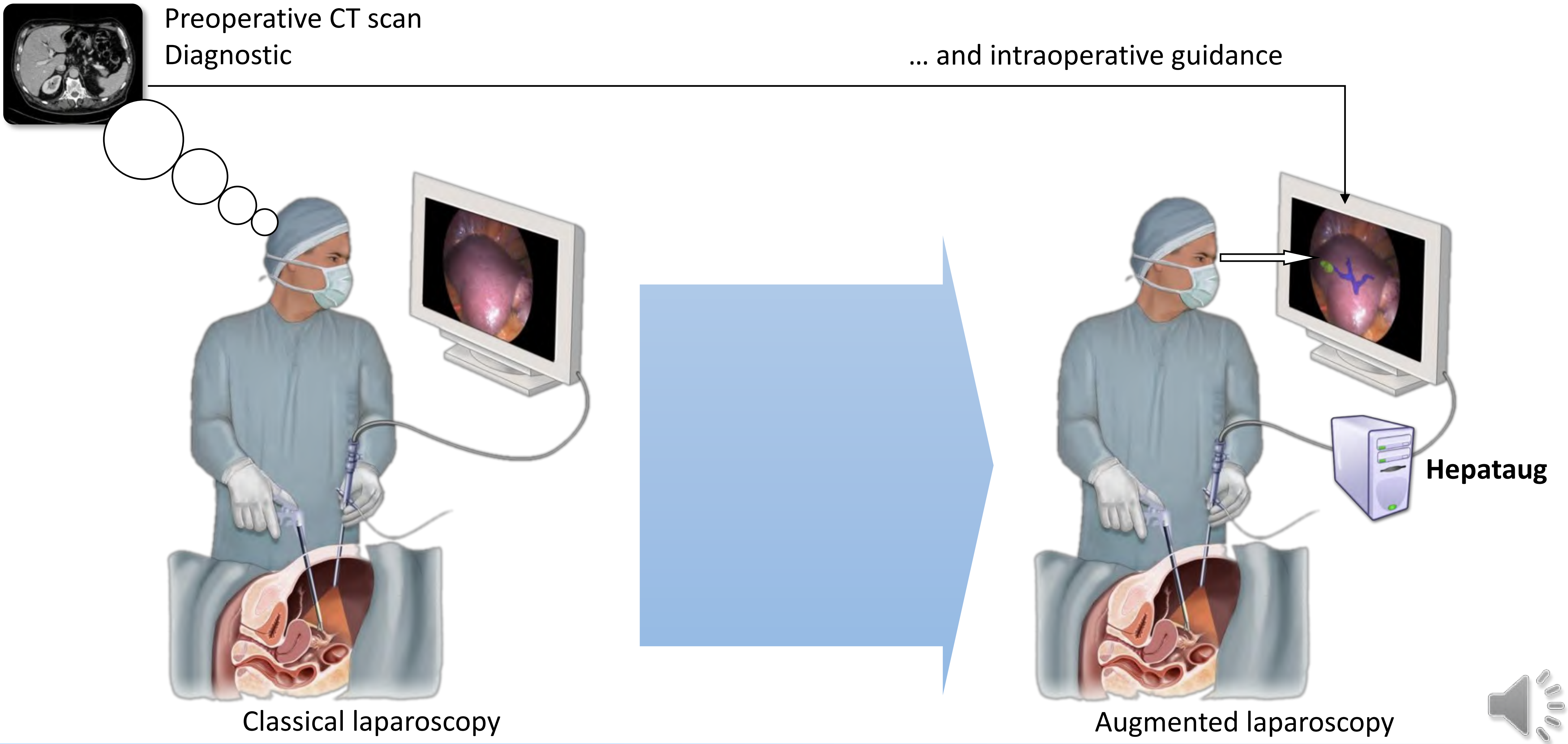
Finding the Tumours

The tumours are **endophytic**, so invisible

Endoscopic US does not help; palpation is obviously not an option

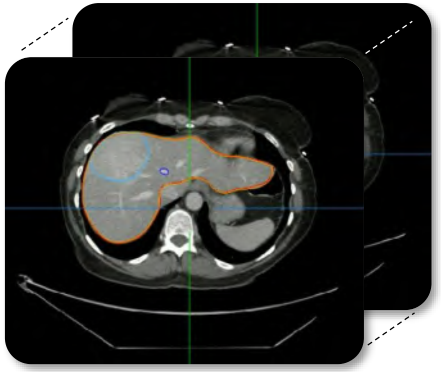
Mentally aligning the CT to laparoscopy is impossible

Proposed Approach: **Augmented Laparoscopy**



Problem Statement

Main difficulty: no fixed structures such as bones, non-rigidities (pneumoperitoneum, mobilisation)



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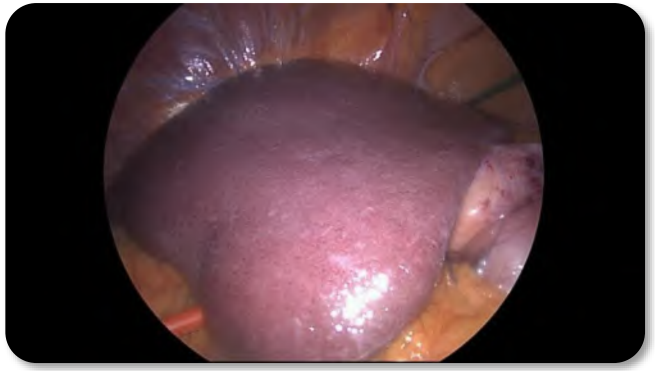

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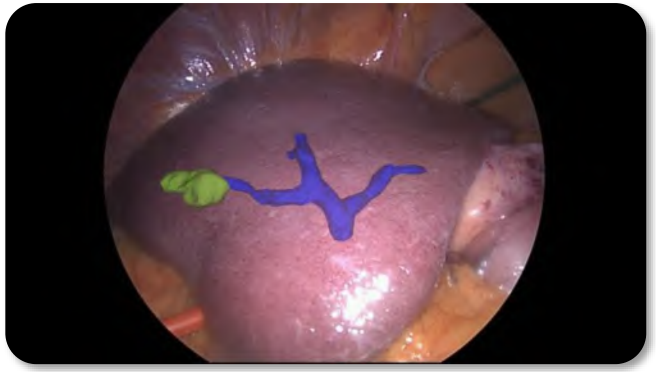
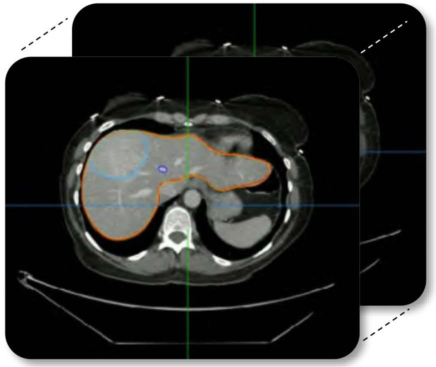


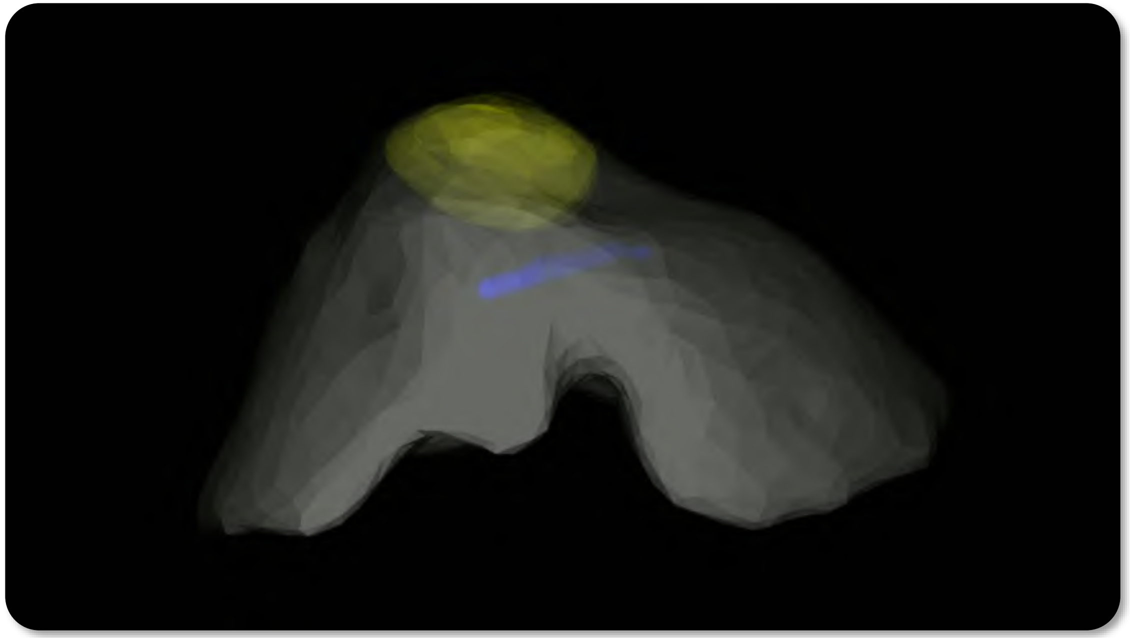
Preoperative 3D reconstruction:
Shape of the organ and internal structures
→ Biomechanical preoperative 3D model

Intraoperative registration
Correspondence between preoperative 3D model and intraoperative laparoscopy image
→ 3D non-rigid flow

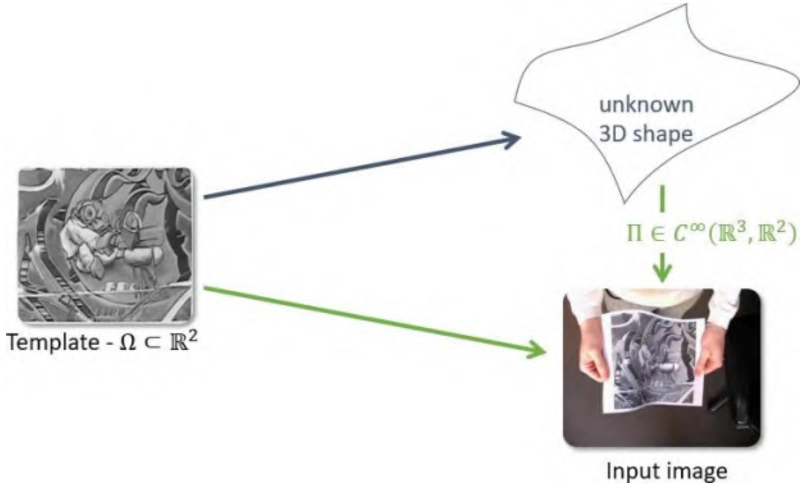
Preoperative 3D Reconstruction

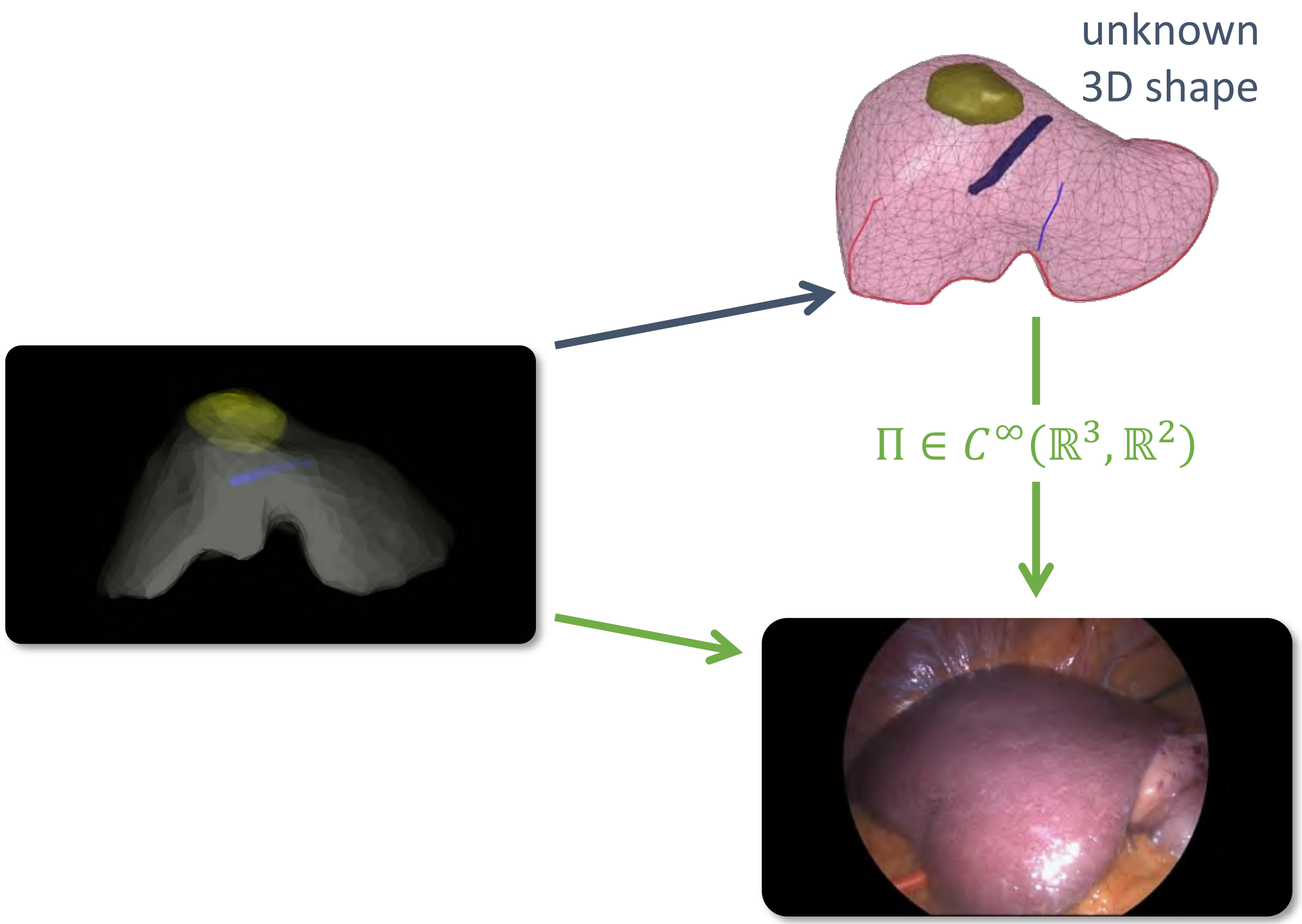


MITK

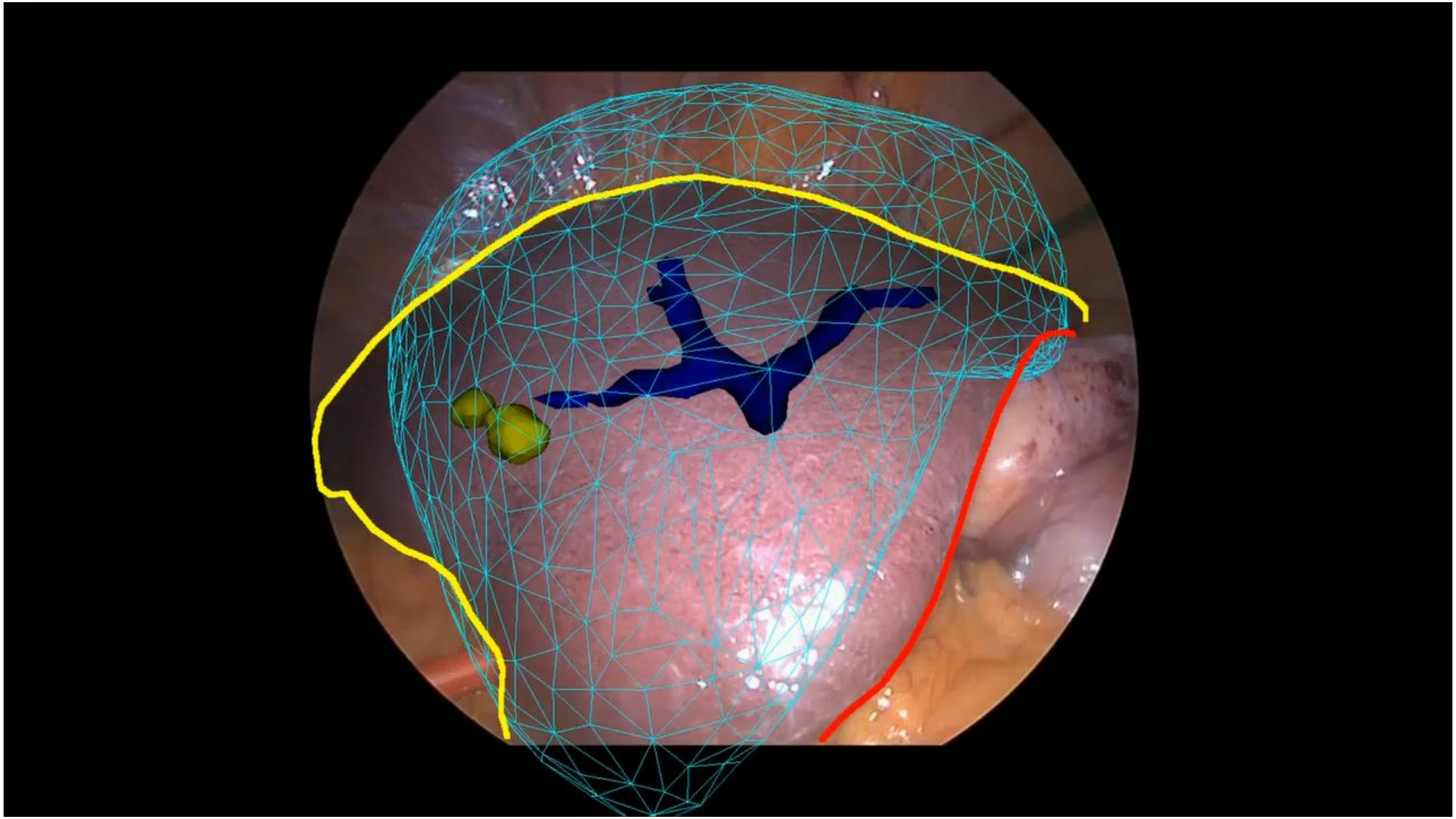


Equivalence to Textureless SfT

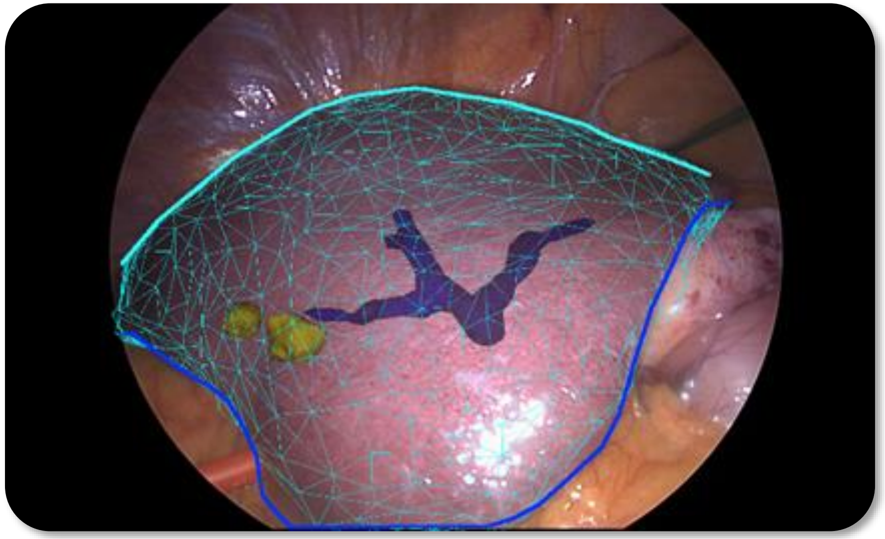


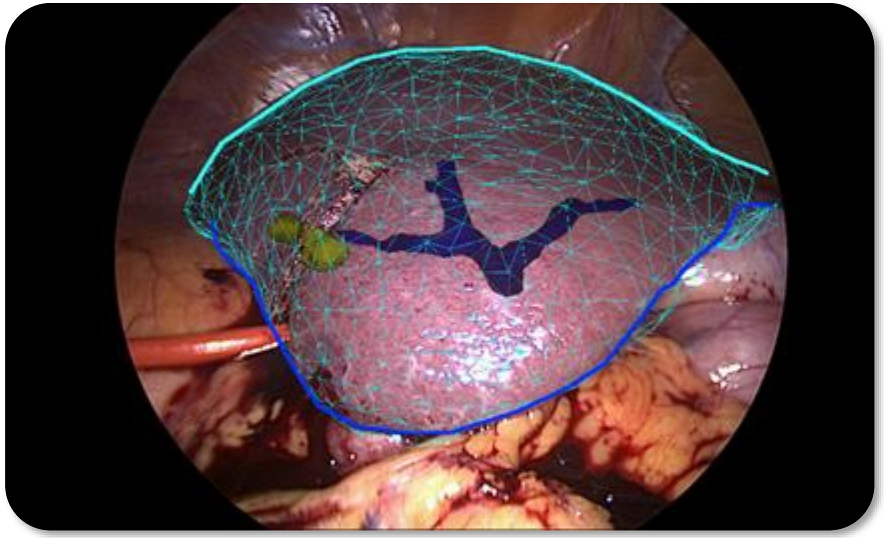


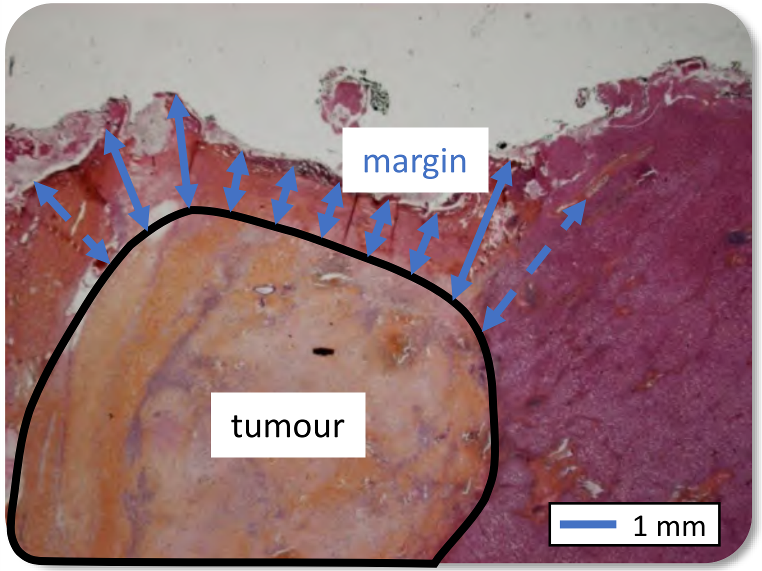
Segmentectomy 6 Case



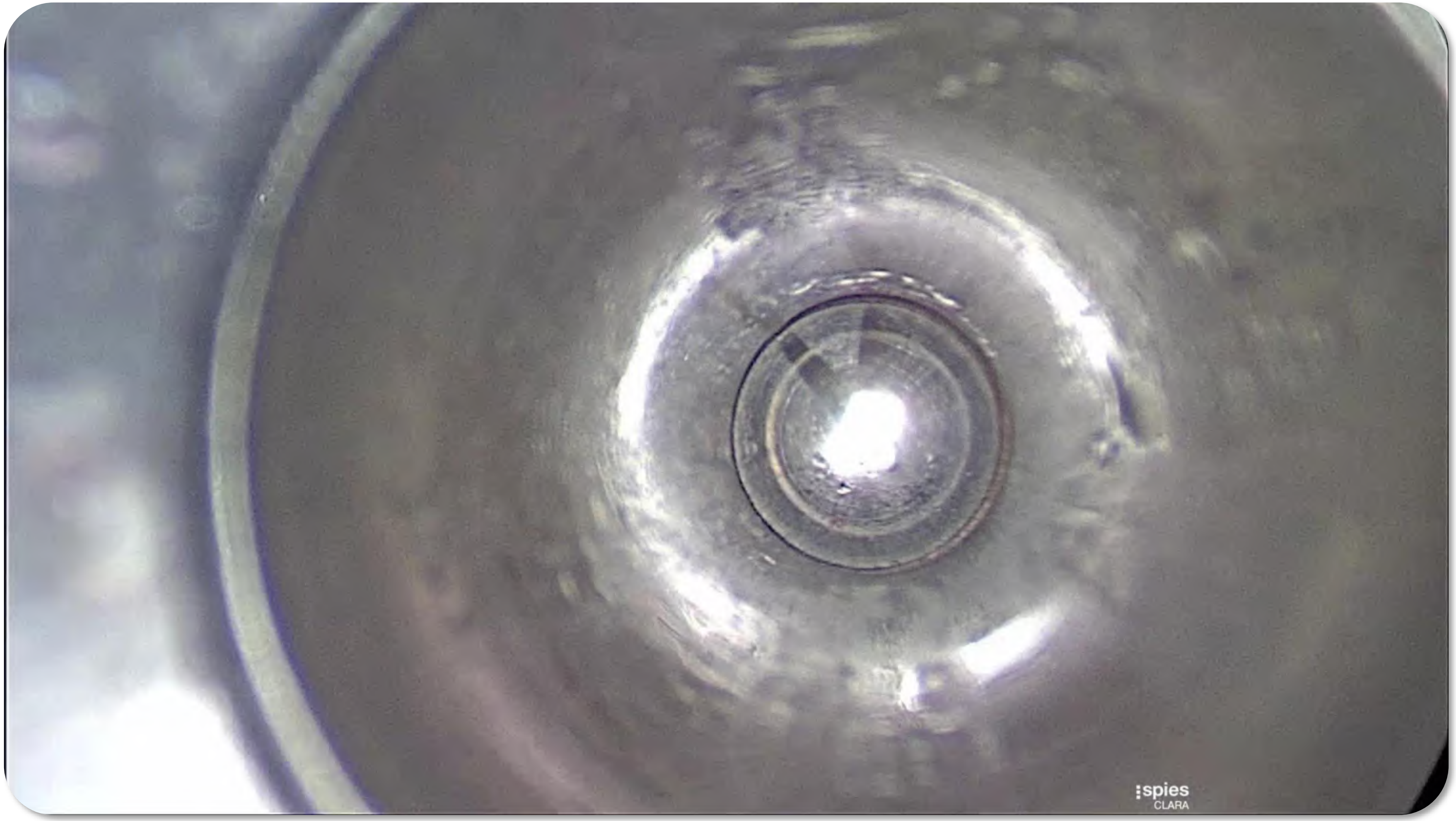
Segmentectomy 6 Case







Myomectomy Application



Seeing in 3D from a Single Image with Geometric Priors

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