Brandenburgische Technische Universität Cottbus - Senftenberg

12. Februar 2021

Mathematics for Printing Metallic Structures

IndAM Workshop, 2021

Michael Breuß

Acknowledgements

Brandenburgische Technische Universität Cottbus - Senftenberg

The talk encompasses joint work with:

Martin Bähr

Markus Bambach

Johannes Buhl

Armin Fügenschuh

Pascal Peter

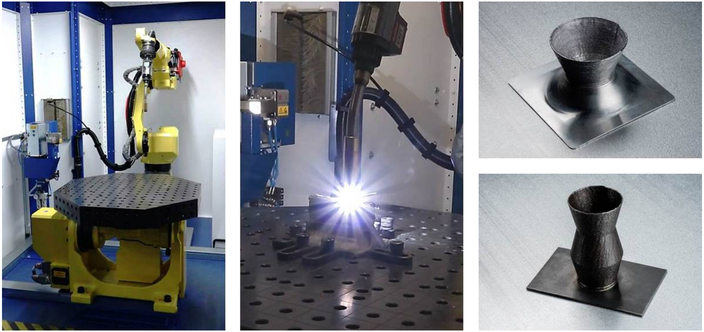
Johannes Schmidt

Georg Radow

Ashkan Mansouri Yarahmadi

Introduction

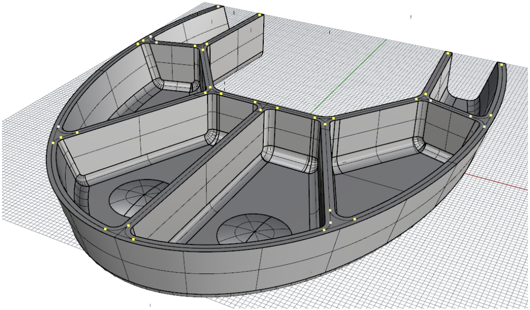
Application context: Wire-arc additive manufacturing



Printing robot and work pieces, by courtesy of Johannes Buhl, BTU Cottbus-Senftenberg

Introduction

Brandenburgische Technische Universität Cottbus - Senftenberg



Example workpiece: Horseshoe, CAD plot

Challenges:

- Determine structures of high strength-to-weight ratio
- Enhance printability by specific routing scheme

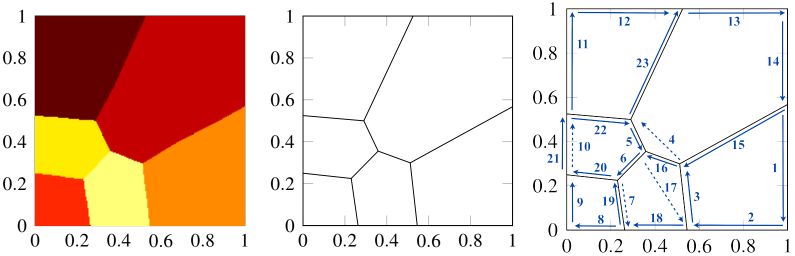
Introduction

Contents of the talk: Some ways to tackle the challenges, work in progress

- **First stage (discussed in some detail):** Stable structure of printed objects
  - Two approaches borrowed from image processing techniques are discussed and investigated
- **Second stage (just touched):** Optimal printing path and process
  - Incorporates many mathematical techniques and models

Brandenburgische
Technische Universität
Cottbus - Senftenberg

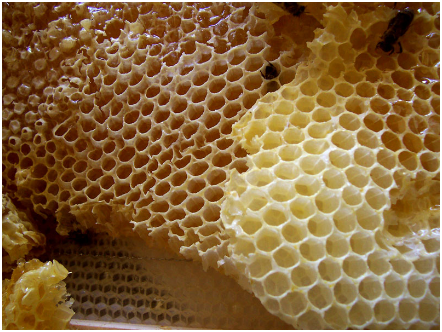
The desired complete pipeline, sketched using a few images



Constructed structure, extracted graph, and computed trajectory

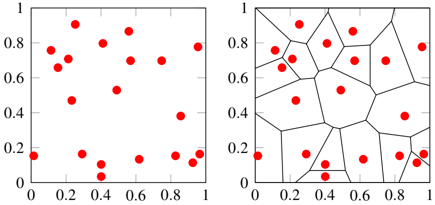
Honeycomb Structure

Interesting structure in nature: Honeycombs



Voronoi tesselation (VT):

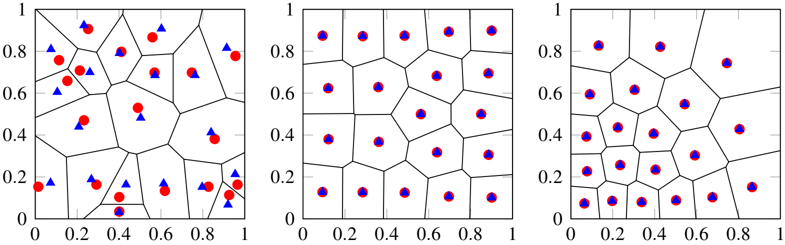
- Partition of a region $\Omega \subset \mathbb{R}^2$ into Voronoi cells $V_i \subset \Omega$ depending on generators $x_i \in \Omega$
- $V_i = \{x \in \Omega : |x - x_i| < |x - x_j|, j \neq i\}$



$m = 20$ generators in $\Omega = [0,1] \times [0,1]$, corresponding Voronoi tesselation

Centroidal Voronoi tesselation (CVT):

Special type of VT, generators coincide with centroids, possibly addressing a stress map or density ρ



Left: VT, constant density. **Middle:** CVT, constant density. **Right:** CVT, Gaussian density

Finding a CVT corresponds to energy minimisation:

$$\min_{x_1,\dots,x_m\in\mathbb{R}^2} \sum_{i=1}^{m} \int_{V(x_i)} \rho(x)\,|x-x_i|^2\,\mathrm{d}x$$

Lloyd's algorithm (1982): Iterate the following steps

1. Given x_i, generate Voronoi cells V_i.
2. Given V_i, replace x_i with the center of mass in V_i: $$x_i \leftarrow \frac{\int_{V_i}\rho(x)x\mathrm{d}x}{\int_{V_i}\rho(x)\mathrm{d}x}$$

Generating Voronoi Tesselations

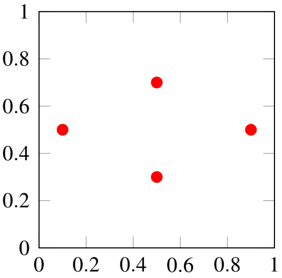
Approaches to Voronoi tessellation

Geometric Optimisation	PDE-based Approach
Employ Delaunay Triangulation (Barber et al., 1996)	Discretise Eikonal Equation (e.g., Cristiani and Falcone, 2007)

Question: Which approach is better for finding CVTs?

Delaunay triangulation (DT):

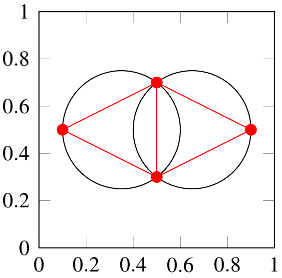
- Triangulation of the generators, such that no generator is inside the circumcircle of any triangle
- Circumcenters are vertices of VT



Generators

Delaunay triangulation (DT):

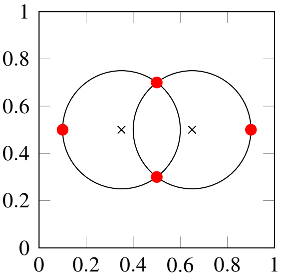
- Triangulation of the generators, such that no generator is inside the circumcircle of any triangle
- Circumcenters are vertices of VT



Generators, DT

Delaunay triangulation (DT):

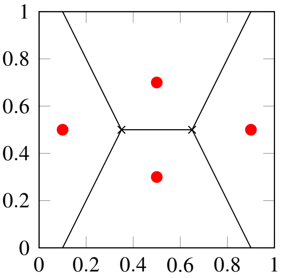
- Triangulation of the generators, such that no generator is inside the circumcircle of any triangle
- Circumcenters are vertices of VT



Generators, circumcenters

Delaunay triangulation (DT):

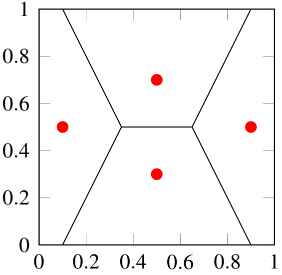
- Triangulation of the generators, such that no generator is inside the circumcircle of any triangle
- Circumcenters are vertices of VT



Generators, circumcenters and VT

Delaunay triangulation (DT):

- Triangulation of the generators, such that no generator is inside the circumcircle of any triangle
- Circumcenters are vertices of VT



Generators, VT

PDE-based Approach

PDE-based approach: Solve eikonal equation (e.g. by fast marching)

$$|\nabla d(x)| = 1, \qquad x \in \Omega \setminus X$$

with the boundary condition

$$d(x_i) = 0, \qquad x_i \in X$$

to generate Voronoi cells $V_i = \{x \in \Omega : d(x,x_i) < d(x,x_j),\ j \neq i\}$

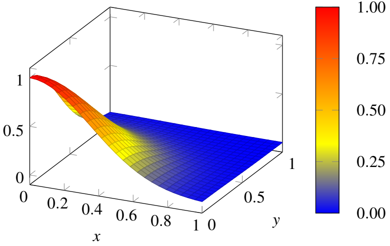
Question: Is high accuracy required here (i.e. use of high-order discretisation)?

Question: What is the impact of grid resolution?

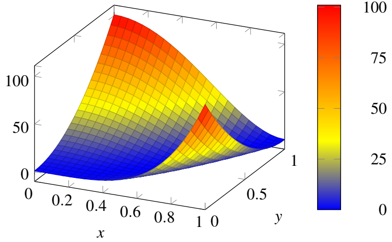
Model for Evaluation

Brandenburgische Technische Universität Cottbus - Senftenberg

Density functions: constant density $\rho \equiv 1$ and



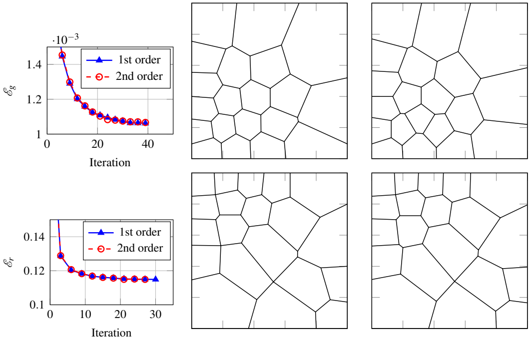
a Gaussian,
$\rho = \exp(-4(x^2 + y^2))$



b Rosenbrock,
$\rho = (1-x)^2 + 100(y-x^2)^2$

PDE: Numerical Accuracy

Brandenburgische Technische Universität Cottbus - Senftenberg

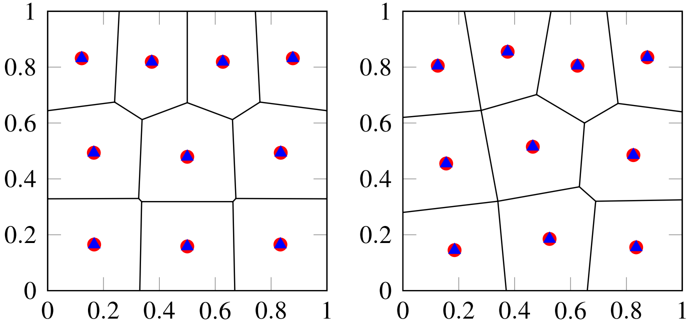


Energy progression and PDE-based CVT ($\Theta = [0,1]^2$, 200×200 grid), FM of 1st vs. 2nd order, for *(top)* Gaussian ρ^g and *(bottom)* Rosenbrock function ρ^r

Implication: First-order FM will do for our purpose (so we stick to it)

Geometric vs. PDE

Impact of mesh size: Constant density



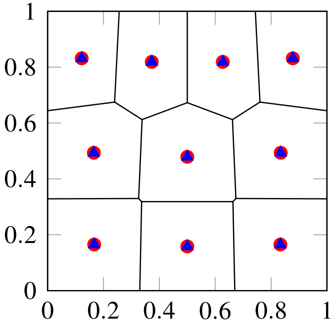
a $m = 10$, geometric
$E = 1.7062 \cdot 10^{-2}$

c $m = 10$, 100×100 grid
$E = 1.7180 \cdot 10^{-2}$

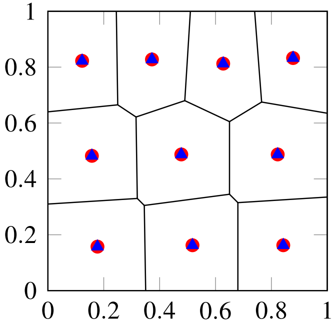
Geometric vs. PDE

Brandenburgische Technische Universität Cottbus - Senftenberg

Impact of mesh size: Constant density



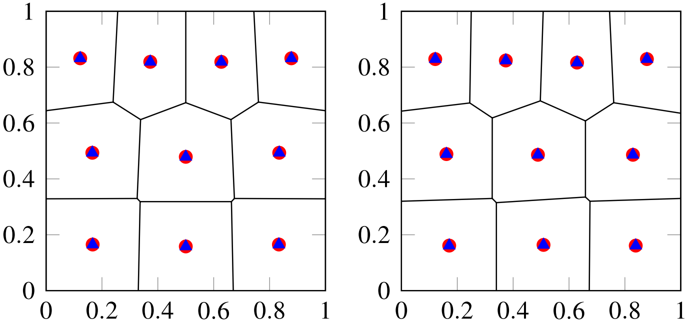
a $m = 10$, geometric
$E = 1.7062 \cdot 10^{-2}$



d $m = 10$, 200×200 grid
$E = 1.7083 \cdot 10^{-2}$

Geometric vs. PDE

Impact of mesh size: Constant density

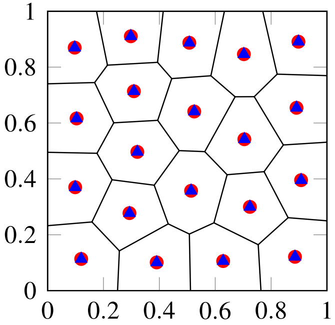


a $m = 10$, geometric
$E = 1.7062 \cdot 10^{-2}$

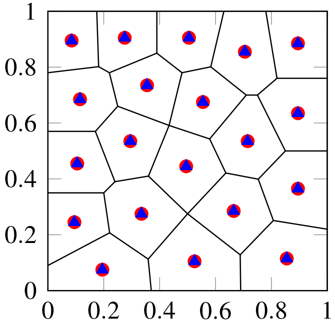
e $m = 10$, 400×400 grid
$E = 1.7068 \cdot 10^{-2}$

Geometric vs. PDE

Impact of mesh size: Constant density



b $m = 20$, geometric
$E = 8.4304 \cdot 10^{-3}$

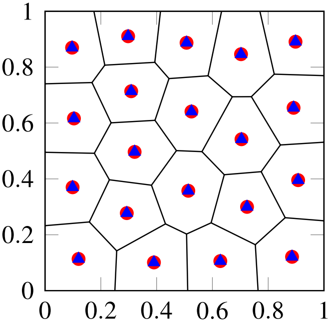


f $m = 20$, 100×100 grid
$E = 8.7656 \cdot 10^{-3}$

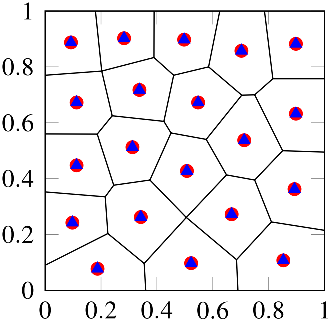
Geometric vs. PDE

Brandenburgische Technische Universität Cottbus - Senftenberg

Impact of mesh size: Constant density



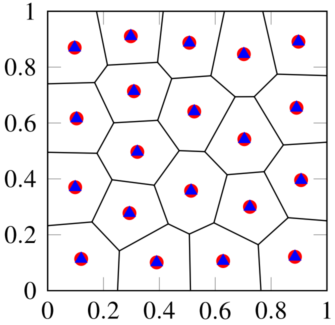
b $m = 20$, geometric
$E = 8.4304 \cdot 10^{-3}$



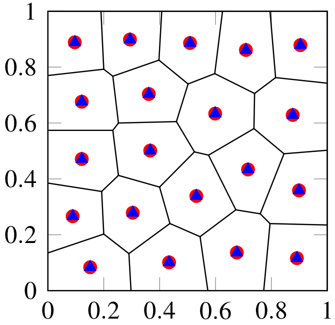
g $m = 20$, 200×200 grid
$E = 8.7127 \cdot 10^{-3}$

Geometric vs. PDE

Impact of mesh size: Constant density



b $m = 20$, geometric
$E = 8.4304 \cdot 10^{-3}$



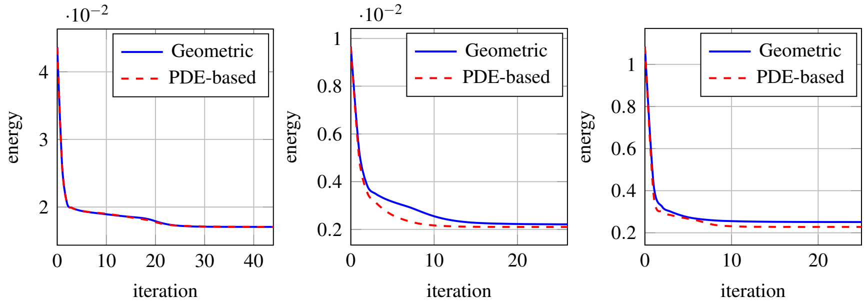
h $m = 20$, 400×400 grid
$E = 8.5067 \cdot 10^{-3}$

Implication: $\uparrow$ Number of seeds $\Longrightarrow$ mesh size $\downarrow$

Geometric vs. PDE

Brandenburgische Technische Universität Cottbus - Senftenberg

Evaluation of energy: $m = 10$, 400×400 grid



a Constant density

b Gaussian density

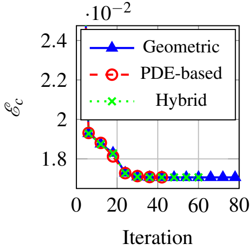
c Rosenbrock density

Implication: PDE-based approach performs better for complicated densities

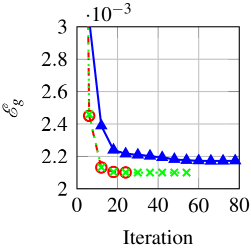
Geometric vs. PDE

Brandenburgische Technische Universität Cottbus - Senftenberg

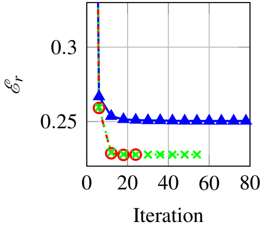
Evaluation of energy: $m = 20$, 400×400 grid



a Constant density



b Gaussian density

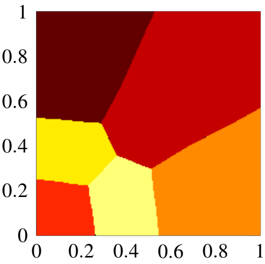


c Rosenbrock density

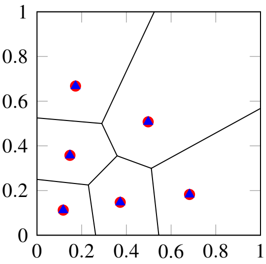
Implication: It is possible to combine advantages of geometric/PDE-based approach (hybrid scheme)

Optimal Printing Process

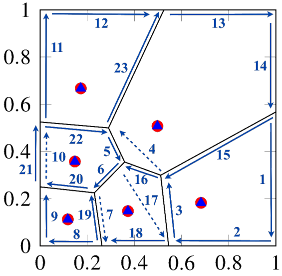
Application of trajectory optimisation



a Computed CVT



b Extracted graph



c Computed trajectory

Problem: Nodes of odd degree

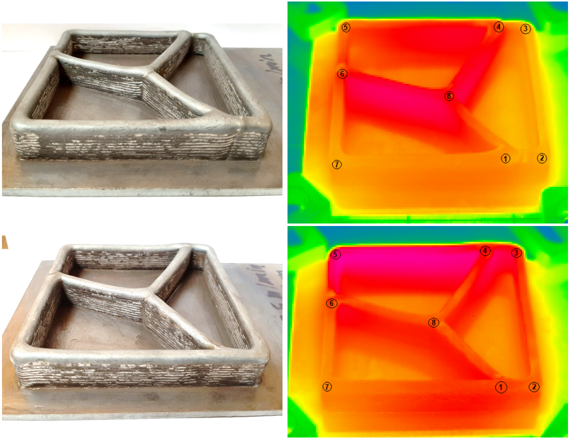
Preferable solution: Remove nodes/edges but keep stability

Optimal Printing Process

What means trajectory optimisation?

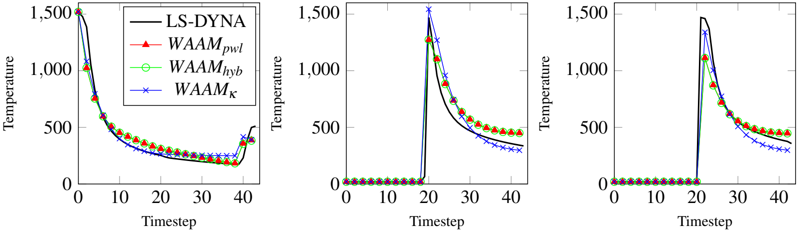
- **Goal:** Find best welding path in terms of temperature distribution
- Consider connected sequence of welding and deadheading moves
- **Minimise** temperature gradient i.e. minimising material stress (cf. Young's modulus)
- **Features of the approach:**
  - includes discrete approximations of heat conduction within printed material w.r.t. substrate plate
  - includes approximate radiation (heat transfer between separate, printed material surfaces), especially view factor evaluation (Quasi-Monte-Carlo-Method)
  - includes model for heat distribution from welding torch
  - material data estimation using LS-DYNA (cf. Buhl et al., in Journal of Machine Engineering, 2019)
- Mathematical model combines (after many approximations) to mixed-integer linear optimisation problem (solved with BARON, 2018)

Evaluation of Combined Approach



Real workpiece, with CVT internal structure: (top) optimal path, (bottom) worst path. Heat distributions (snapshots) are acquired with a thermal camera.

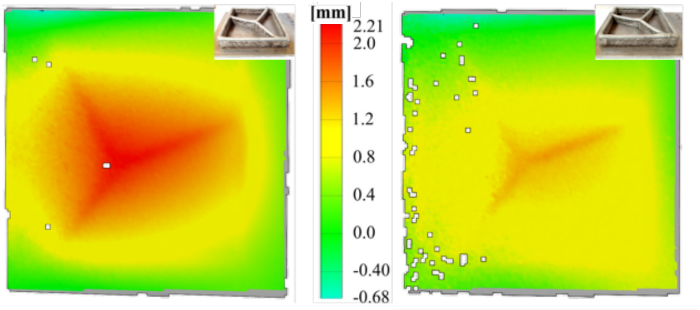
Evaluation of Combined Approach



Comparison of computed temperatures with LS-DYNA for the first three nodes and developed models

Evaluation of Combined Approach

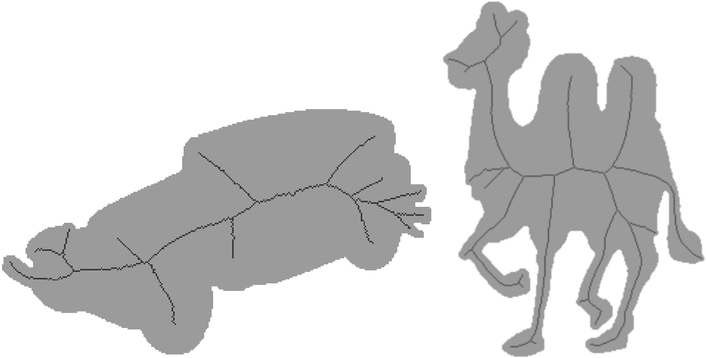
Independent indicator for quality of the printing process: Distortion of substrate plate



Distortion distribution of the substrate plate for the worst *(left)* and the best welding path *(right)* according to the thermal calculation. Distortion can be reduced by $\sim 30\%$ by path optimisation.

Alternative Approach: Skeletonisation

Reminding of a biological skeleton: Medial axis transform



Typical skeletonisation result (medial axis transform) from image processing:

The skeleton has width of one pixel, is connected and does not touch the object boundary

Skeleton Computation

Brandenburgische Technische Universität Cottbus - Senftenberg

First solve eikonal equation

$$|\nabla d(x)| = 1, \qquad x \in \Omega \setminus X$$

with the boundary condition

$$d(x_i) = 0, \qquad \forall x_i \in \partial X$$

to compute distance map d.

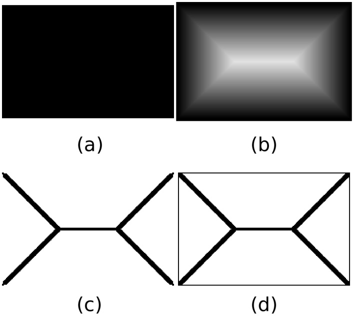
Then compute average outward flux F in each point p to determine sinks:

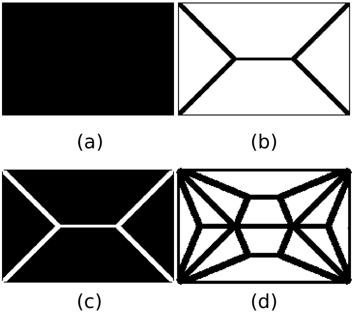
$$F(p) = \frac{1}{8} \sum_{i=1}^{8} \nabla d(p) \cdot N(n_i(p))$$

where $n_i(p)$ are neighbours of p, and $N(n_i(p))$ is the normalised vector pointing from p to $n_i(p)$

Then define a *suitable* (thinning) threshold number τ and apply a flux-ordered homotopic thinning

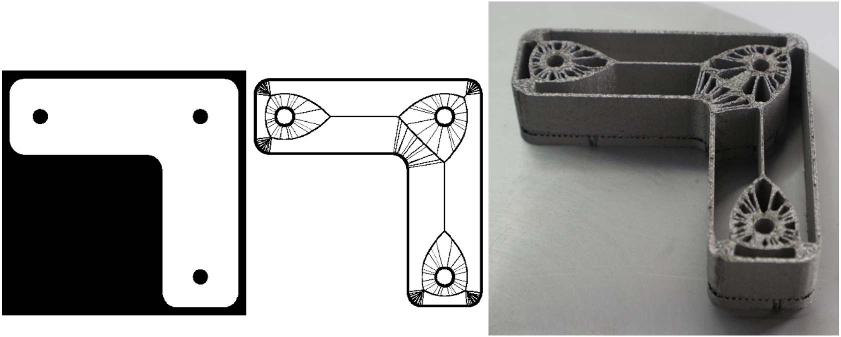
Illustration of Skeletonisation





Left: Skeleton, enhanced by morphological thickening. **Right:** Recursive skeletonisation.

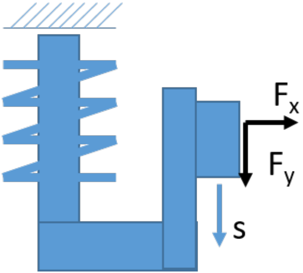
Evaluation of the Approach

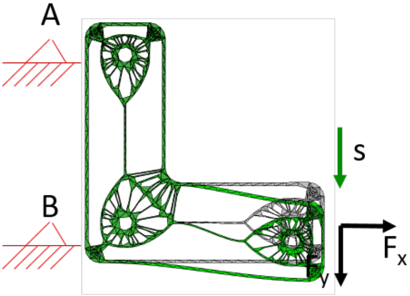


Shape of a workpiece and its skeleton, plus printed version

Evaluation of the Approach

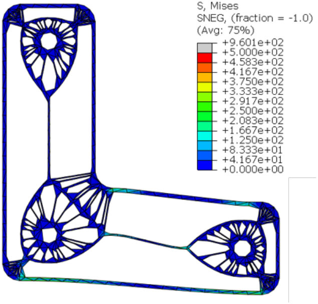
Brandenburgische Technische Universität Cottbus - Senftenberg

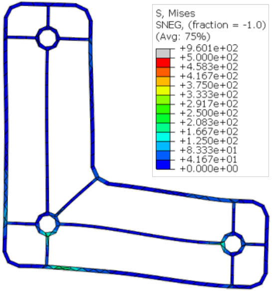




Printed nodes can be connected with guiding and spring function, subject to elastic deformation test

Evaluation of the Approach





Dimensions of test geometries: $100 \times 100 \times 10$mm, simulation performed using Abaqus.
We measure the maximum displacement s_y for Von Mises Stress at a force of 200N in y-direction:
$s_y = 0.0009$mm for skeleton-based geometry, $s_y = 0.023$mm for standard geometry.

Publications

Brandenburgische Technische Universität Cottbus - Senftenberg

M. Bähr, J. Buhl, G. Radow, J. Schmidt, M. Bambach, M.B. and A. Fügenschuh:
Stable honeycomb structures and temperature based trajectory optimization for wire-arc additive manufacturing. Optimization and Engineering, Springer Science and Business Media LLC, 2020

M.B., J. Buhl, A.M. Yarahmadi, M. Bambach and P. Peter:
A simple approach to stiffness enhancement of a printable shape by Hamilton-Jacobi skeletonization. Procedia Manufacturing 47 (2020), 1190–1196

Many thanks for your attention!