First order problem on networks and applications in memory of Maurizio Falcone

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Abstracts

Fabio Bagagiolo
Università di Trento

Two origin-destination problems on congested networks.

In this talk I will present two origin-destination problems on congested networks. In the first case, a time-continuous routing problem is treated where the agents have at their disposal some path-preference information, depending on the actual state of the congestion in the network. In the second case, a switching problem, coming from an optimal visiting problem, is recast in a controlled jumping dynamics on the congested nodes of a network, where the agents have at their disposal a time for deciding the switch and a time for performing the switch. For both problems, the existence of a mean-field equilibrium is proved under some hypotheses and possible approximations.
The results are in collaboration with Rosario Maggistro, Luciano Marzufero and Raffaele Pesenti.

Raul Borsche
TU Kaiserslautern

Kinetic layers and coupling conditions

In this talk we consider kinetic models to derive coupling conditions for the corresponding macroscopic equations on networks. In the limit process layers can form at the junctions. By a detailed analysis and suitable approximations of the associated half-space problems the kinetic coupling conditions can be connected to the macroscopic states. This approach is applied to linear hyperbolic systems and scalar conservation laws. Numerical comparisons between the solutions of the macroscopic equation and the kinetic models confirm the accuracy of the proposed approximations.

Simone Cacace
Sapienza Università di Roma
A numerical solver for Hamilton-Jacobi equations on stratified domains

In optimal control problems defined on stratified domains, the dynamics and the running cost may have discontinuities on a finite union of sub-manifolds of the state space. The corresponding value function can be characterized as the unique viscosity solution of a discontinuous Hamilton-Jacobi equation satisfying additional viscosity conditions on the sub-manifolds. In this talk, I will introduce HJSD, an easy-to-use software for the numerical solution of such optimal control problems in two and three dimensions. The core algorithm is based on a semi-Lagrangian approximation scheme for the stratified Hamilton-Jacobi equation, whose convergence is proved relying on a classical stability argument in viscosity solution theory. I will also present some implementation details and some simulations, showing the particular phenomena that can arise with respect to the classical continuous framework. This is a joint work with Fabio Camill

Fabio Camilli
Sapienza Università di Roma
A continuous dependence estimate for viscous Hamilton-Jacobi equations on networks with applications

We study continuous dependence estimates for viscous Hamilton- Jacobi equations defined on a network G. Given two Hamilton-Jacobi equations, we prove an estimate on the C^2-norm of the difference between the corresponding solutions in terms of the L^∞ distance among the coefficients. We also provide two applications of the previous estimate: the first one is an existence and uniqueness result for a quasi-stationary Mean Field Games defined on the network G; the second one is an estimate of the rate of convergence for homogenization of viscous Hamilton-Jacobi equations defined on a periodic network, when the size of the cells vanishes and the limit problem is defined in the whole Euclidean space.

Elisabetta Carlini
Sapienza Università di Roma
A numerical scheme for evolutive Hamilton Jacobi equations on Networks

We present a numerical scheme for evolutive Hamilton-Jacobi equations on networks. The method is based on a recent definition of viscosity solution and consists of two steps. First, the individual equations, defined on each branch of the network, are approximated by a Semi-Lagrangian scheme. Then, the multiple values obtained at the vertices, one for each branch incident on a given vertex, are processed to select a single value that verifies the definition of viscosity solution. The main advantage over the pure semi-Lagrangian scheme is in terms of computational cost. We present a convergence analysis and some numerical tests.
Joint work with A. Siconolfi

Rinaldo Colombo
Università di Brescia

Conservation Laws & Hamilton-Jacobi Equations: Connections and Inverse Design

We identify a framework where, in the space dependent case, Cauchy problems for both conservation laws and for Hamilton-Jacobi equations are well posed and are equivalent to each other, without any convexity requirement. Instrumental in all this, in the case of the conservation laws, is the construction of a "foliation" consisting of stationary solutions, possibly displaying (entropic) shocks and with unbounded total variation. On these basis, in the convex case, we solve the "inverse design" problem, which consists in the characterization first of the attainable sets and then of the initial data developing into a given profile. Finally, an explicit example shows the deep, somewhat counterintuitive, effects of the x dependence. Joint work with Vincent Perrollaz, Abraham Sylla

Adriano Festa
Politecnico di Torino

Navigation system based routing strategies in traffic flows on networks

Navigation choices play an important role in modeling and forecasting traffic flows on road networks. We introduce a macroscopic differential model coupling a conservation law with a Hamilton-Jacobi equation to respectively model the nonlinear transportation process and the strategic choices of users. Furthermore, the model is adapted to the multi-population case, where every population differs in the level of traffic information about the system.
Joint work with Paola Goatin (INRIA Nice), Fabio Vicini (Politecnico di Torino)

Nicolas Forcadel
University of Rouen

Microscopic derivation of traffic flow models on networks

The goal of this talk is to present a rigorous derivation of a macroscopic traffic flow model with a bifurcation or a local perturbation from a microscopic one. The microscopic model is a simple follow-the-leader with random parameters. The random parameters are used as a statistical description of the road taken by a vehicle and its law of motion. The limit model is a deterministic and scalar Hamilton-Jacobi equation on a network with a flux limiter, the flux-limiter describing how much the bifurcation or the local perturbation slows down the vehicles. The proof of the existence of this flux limiter relies on a concentration inequality and on a delicate derivation of a super-additive inequality.
This is a joint work with P. Cardaliaguet.

Theo Girard (University of Tours)

An abstract existence for generalised Hughes' model

The Hughes' model is a model for the dynamics of pedestrian flows. In the one dimensional case, it represents the evacuation of agents in a corridor through either one of the exits. This model couples two PDEs : a discontinuous-flux conservation law and an eikonal equation. After a brief review about what's known on the subject, we propose an abstract existence result for solution to generalized Hughes' model. We also present three applications of this existence result and an extension with constrained evacuation at exits.

Martin Gugat (FAU Erlangen)

On the synchronization of observer systems for isothermal flow in gas networks

Pipeline networks correspond to graphs where the edges are given by the pipes that form the network. The flow of gas through the pipes can be modeled by the isothermal Euler equations. A model for the flow through the network is obtained by coupling the PDEs that describe the flow through the pipes by algebraic node conditions that model the flow through the vertices of the graph. Measurements of the state in the network are available only at certain points in space. Based on these nodal observations, the full system state can be approximated using an observer system. We present an observer system for general graphs and prove that the state of the observer system approaches the original state exponentially fast.

Shi Jin (Shanghai Jiao Tong University)

Quantum Computation of partial differential equations

Quantum computers have the potential to gain algebraic and even up to exponential speed up compared with its classical counterparts, and can lead to technology revolution in the 21st century. Since quantum computers are designed based on quantum mechanics principle, they are most suitable to solve the Schrodinger equation, and linear PDEs (and ODEs) evolved by unitary operators. The most efficient quantum PDE solver is quantum simulation based on solving the Schrodinger equation. It became challenging for general PDEs, more so for nonlinear ones. Our talk will cover three topics:

1. We introduce the "warped phase transform" to map general linear PDEs and ODEs to Schrodinger equation or with unitary evolution operators in higher dimension so they are suitable for quantum simulation;

2. For (nonlinear) Hamilton-Jacobi equation and scalar nonlinear hyperbolic equations we use the level set method to map them exactly to phase space linear PDEs so they can be implemented with quantum algorithms and we gain quantum advantages for various physical and numerical parameters.

3. For PDEs with uncertain coefficients, we introduce a transformation so the uncertainty only appears in the initial data, allowing us to compute ensemble averages with multiple initial data with just one run, instead of multiple runs as in Monte-Carlo or stochastic collocation type sampling algorithms.

Paola Mannucci
Università di Padova

Optimal control problems and deterministic Mean Field Games with control on the acceleration and state constraints

I will talk about a joint research project with Y. Achdou, C. and N. Tchou about the study of optimal controlÂ problems and deterministic MFGs with control on the acceleration.

This research is included in a larger project devoted to optimal control problemsÂ and some models of deterministic MFGs where the Hamiltonian is not coercive in the gradient term because the dynamics of the generic player must fulfill some constraints, holonomic or non-holonomic, or fail to be controllable. In these models the generic player has some forbidden directions.

In the case where the generic player can control its state only through the acceleration, I consider both the case where the agents can move in the whole space and the case where the agents are constrained to remain in a given bounded region. In particular I will explain in detail the case where the agents are constrained to remain in an interval. These results are the first step to study optimal control problem and deterministic MFGs in a network with control on the acceleration. For these constrained optimal control problems we are interested to obtain a closed graph result on the multivalued map which associates to an initial condition the set of the optimal trajectories starting from that point. We will apply this result to the MFG model to get the existence of relaxed equilibria in the Lagrangian interpretation which describes the game in terms of a probability measure on optimal trajectories.

Claudio Marchi
Università di Padova

First order Mean Field Games on networks

The theory of Mean Field Games studies the asymptotic behaviour of differential games (mainly in terms of their Nash equilibria) as the number of players tends to infinity. In these games, each player aims at choosing its trajectory so to minimize a cost which depends on the trajectory itself and on the distribution of the whole population of agents.

We focus our attention on deterministic Mean Field Games with finite horizon in which the states of the players are constrained in a network (in our setting, a network is given by a finite collection of vertices connected by continuous edges) and the cost may change from edge to edge. As in the Lagrangian approach, we introduce a relaxed notion of Mean Field Games equilibria which describe the game in terms of probability measures on trajectories instead of time-dependent probability measures on the network.

Our first main result is to establish the existence of such a MFG equilibrium. Afterward, to each MFG equilibrium, can be naturally associated a cost, the corresponding value function and optimal trajectories (chosen by the agents). We prove that the optimal trajectories starting at time t=0 are Lipschitz continuous, locally uniformly with respect to the initial position. As a byproduct, we obtain a ”Lipschitz” continuity of the MFG equilibrium: its push-forward through the evaluation-function at each time gives rise to a Lipschitz continuous function from the time interval to the space of probability measures on the network.

The second main result is to prove that this value function is Lipschitz continuous and solves a Hamilton-Jacobi partial differential equation in the network.

This is a joint work with: Y. Achdou (Univ. of Paris), P. Mannucci (Univ. of Padova) and N. Tchou (Univ. of Rennes).

Marco Pozza
Tor Vergata
Large Time Behavior of Solutions to Hamilton-Jacobi Equations on Networks

Starting from Namah, Roquejoffre (1999) and Fathi (1998), the large time asymptotic behavior of solutions to Hamilton-Jacobi equations has been extensively investigated by many authors, mostly on smooth compact manifolds or the N-dimensional torus. Following recent development due to Pozza, Siconolfi (to appear), we extended this asymptotic analysis to time dependent problems on networks. The main difference between this and more traditional settings is that, for the well posedness of the evolutive problem on networks, the equation must be coupled with a ”flux limiter”, that is the choice of appropriate constants on each vertex of the network. These constants, among other things, bond from above the time derivatives of any subsolution on the vertices. In this talk we will show how this new condition impact the asymptotic behavior of the solutions to the Hamilton-Jacobi problem on networks.

Gabriella Puppo
Sapienza
Coupling conditions for shallow water equations at channel junctions

Coupling conditions in networks depend on the particular system of equations that must be coupled. Here I will describe an approach specific for shallow water. The model used is one-dimensional along each channel, except at junctions. Here the coupling conditions are given by the solution of the half Riemann problems occurring at the interfaces between the junction and each of the adjoining channels, completed by the application of the conservation principles at the heart of the model. Thus, at the junction we preserve mass and both components of momentum. As a consequence, we find that the solution depends on the geometry of the network. In particular, the solution at the junction depends on the angles formed by the adjoining channels.

Alfonso Sorrentino
Tor Vergata
The Hamilton-Jacobi equation on networks: from Aubry-Mather theory to Homogenization

Over the last few years, there has been an increasing interest in studying the Hamilton-Jacobi Equation on networks and related questions. These problems, in fact, involve several subtle theoretical issues and have a significant impact on the applications in various fields, for example, data transmission, traffic management problems, etc. While locally - i.e., on each branch of the network (arcs) - the study reduces to the analysis of 1-dimensional problems, the main difficulties arise in matching together the information converging at the juncture of two or more arcs, and relating the local analysis at a juncture with the global structure/topology of the network. In this talk, I shall discuss several results related to the global analysis of this problem, obtained in collaboration with Antonio Siconolfi; more specifically, we developed analogs of the so-called Weak KAM theory and Aubry-Mather theory in this setting. The salient point of our approach is to associate the network with an abstract graph, encode all of the information on the complexity of the network, and relate the differential equation to a discrete functional equation on the graph. In particular, I shall describe an ongoing project in collaboration with Marco Pozza and Antonio Siconolfi to prove a Homogenization result in this context.

Andrea Tosin
Poltecnico di Torino
Boltzmann-type kinetic equations on networks

In this talk, we present a Boltzmann-type kinetic approach to networked interactions in multi-agent systems. We discuss the use of Boltzmann-type collisional equations to: (i) describe binary interactions mediated by a graph structure of the connections among the agents; (ii) investigate the impact of such a graph structure on the emergence of aggregate trends at the statistical and macroscopic levels. We treat both kinetic equations on finite graphs, suitable to model few compartments in which agents may be grouped, and kinetic equations incorporating a statistical description of the agent connectivity, suitable to address large networks such as e.g., social networks. The main motivating applications are infectious disease transmission and opinion formation.
This talk is based on joint works with Rossella Della Marca (SISSA), Tommaso Lorenzi (Politecnico di Torino), Nadia Loy (Politecnico di Torino), Marco Nurisso (Politecnico di Torino), Elisa Paparelli (Politecnico di Torino), Matteo Raviola (EPFL).