A short-term model for the oil industry addressing commercial storage

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In memory of Maurizio Falcone

Outline

1. Introduction
2. The short-term model
3. Numerical simulations and discussion on the results

Models for the oil industry

The different agents.

- **A major agent : the OPEC cartel.**
  For now 30 years, the market share of OPEC has been stable around 42%
- **A competitive fringe of small producers.**
  For most small producers, investments depend on the price of the barrel. Producers invest at the same time, which generates investment inefficiency and inertia
- **A set of competitive risk-neutral physical arbitrageurs,** who store and sell the resource.
  Commercial storage is less than 10 % of the annual production. Nevertheless, commercial storage is key to understand short-term behaviours
- **The consumers.** After a suitable change of units, the demand of the resource is given by

$$D(p_t) = 1 - \epsilon p_t,$$

where p_t is the price of the resource

Two models

1. A long-term model for OPEC versus the competitive fringe of small classic-oil producers, on the time scale of several decades
2. A short-term model involving OPEC and the crowd of arbitrageurs who store and sell the resource:
   - the behaviour of the non-OPEC (small) producers is described in a simplified way
   - the time scale is of the order of some years
   - **Important feature:** the arbitrageurs will most often limit the price evolution.

     But, when storage capacity limits are reached, the strategic power of the cartel increases dramatically.

     When storage is at its minimal level, the cartel has the power to drive prices up by cutting production.
     Conversely, when storage is at its maximal level, the cartel can drive prices down by increasing production

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Two state variables

- k: **the level of commercial storage**
  (more exactly the ratio of the stored quantity of oil to the global level of demand, which can be considered constant up to a small error).

  Physical storage capacity limits: $k \in [k_{\min}, k_{\max}]$ **will play a key role**
- z: **the aggregate production of the non-OPEC producers**
  (more exactly, the ratio of the production of the non-OPEC producers to the global level of demand)

We look for **a stationary equilibrium between the cartel and the arbitrageurs :**

- the cartel solves an optimal control problem given the price of the resource.
  The optimal value is a function $U(k, z)$
- **The price of the resource**, described by a function $p(k, z)$, is fixed by the arbitrageurs when they are not bound by physical storage limits

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We look for **a stationary equilibrium between the cartel and the arbitrageurs :**

The functions U and p will satisfy a system of 2 coupled PDEs in $(k_{\min}, k_{\max}) \times (z_{\min}, z_{\max})$ with **non standard boundary conditions at** $k = k_{\min}$ **and** $k = k_{\max}$

The dynamics of z_t, the aggregate production of non-OPEC producers

For simplicity, we do not address the decision making process of non-OPEC producers; we rather assume that the dynamics of z_t is given:

$$dz_t = b(k_t, z_t, p_t)\, dt + \sqrt{2\nu_z}\, dB_t$$

with

$$b(k, z, p) = \lambda p - \mu \; + \; f(k) \; + \; \widetilde{b}(z)$$

- $\lambda p - \mu$ describes the direct impact of prices on the investments of the small producers, hence on their production capacity

- $f(k)$ is a proxy for the time delays between the investment decisions of non-OPEC producers and the actual creation of new capacities of production (inertia) :

$$f(k) = a_1 \left(\frac{k_{\max} - k}{k_{\max} - k_{\min}} \right)^2 - a_2 \left(\frac{k - k_{\min}}{k_{\max} - k_{\min}} \right)^2$$

with suitable $a_1 > 0$ and $a_2 > 0$.

The term $f(k)$ is significant only for $k \approx k_{\min}$ and $k \approx k_{\max}$, and accounts for a little increase (resp. decrease) in production capacities when the storage facilities are close to empty (resp. full).

The dynamics of k_t, the commercial storage

The control variable of the cartel is its production rate q_t
(more exactly, the ratio of the production to the global level of demand)

Matching demand and supply yields

$$dk_t + D(p_t)dt = (q_t + z_t)dt$$

The optimal control problem faced by the cartel

Costs

- a production cost: cq_t
- a penalty term $\alpha/2\,(q_t - q_\circ)^2$: $q_\circ$ is the target market share of the cartel. OPEC has had a market share of $\approx 42\%$ for more than three decades

Given $p_t = p(k_t, z_t)$, the cartel solves the optimal control problem

$$U(k,z) = \sup_{q_t} \mathbb{E}\left(\int_0^\infty e^{-rt} \left((p_t - c)q_t - \frac{\alpha}{2}(q_t - q_\circ)^2 \right) dt \quad \middle| z_0 = z, k_0 = k \right)$$

Defining the Hamiltonian

$$\begin{aligned} H(z,p,\xi) &= \sup_{q \geq 0} \left((p-c)q - \frac{\alpha}{2}(q - q_\circ)^2 + (q + z - D(p))\,\xi \right) \\ &= \frac{1}{2\alpha}(p - c + \xi)^2 + \xi(z - D(p)) + q_\circ(p - c - \xi), \end{aligned}$$

we get the HJB equation:

$$-rU + H(z, p, \partial_k U) + b(k,z,p)\partial_z U = 0$$

The arbitrageurs

Cost of storage: the cost of storing a unit of resource per unit of time is $g(k)$ when the level of storage is k

Since arbitrageurs are risk-neutral and face a cost of storage that depends on the amount stored, an equilibrium requires that

$$p_t = \mathbb{E}\left(e^{-r\delta t} p_{t+\delta t} - \int_t^{t+\delta t} g(k_s) ds \;\middle|\; (k_t, z_t) \right)$$

if $k_{\min} < k_t < k_{\max}$.

Ito calculus then leads to the equation:

$$-rp + D_\xi H(z, p, \partial_k U) \partial_k p + b(k, z, p) \partial_z p = g(k)$$

Summary

The system of PDEs satisfied by U, p is

$$\begin{cases} 0 & = & -rU + H(z,p,\partial_k U) + b(k,z,p)\partial_z U \\ 0 & = & -rp + D_\xi H(z,p,\partial_k U)\partial_k p + b(k,z,p)\partial_z p - g(k) \end{cases}$$

for $k_{\min} < k < k_{\max}$ and $z_{\min} < z < z_{\max}$

Connections with mean field games involving a major agent

- The system of PDEs couples a HJB equation for the cartel and an equation of the type *master equation* for the price of the resource
- The master equation does not model a crowd of players as in MFGs, but rather an equilibrium reached by the crowd of physical arbitrageurs
- It seems possible to refine the present model by considering that the arbitrageurs play a MFG. This would lead to a more involved model of a MFG with a major agent

Mathematically, little is known on such systems of PDEs; in particular, a notion of weak solution is lacking.

Boundary conditions at $k = k_{\min}$ and $k = k_{\max}$

Because of the state constraints $k_{\min} \le k_t \le k_{\max}$, it is useful to define:

- the nonincreasing and nondecreasing envelopes of $\xi \mapsto H(z,p,\xi)$:

$$\begin{aligned} H_\downarrow(z,p,\xi) &= \max_{q \le D(p)-z} \left(-\frac{\alpha}{2}(q-q_\circ)^2 + (p-c)q + \xi(q+z-D(p)\right) \\ H_\uparrow(z,p,\xi) &= \max_{q \ge D(p)-z} \left(-\frac{\alpha}{2}(q-q_\circ)^2 + (p-c)q + \xi(q+z-D(p)\right) \end{aligned}$$

$H_\downarrow(z,p,\xi)$ corresponds to the controls q such that the drift of k_t is nonpositive
$H_\uparrow(z,p,\xi)$ corresponds to the controls q such that the drift of k_t is nonnegative

- the quantity:

$$H_{\min}(z,p) = \min_\xi H(z,p,\xi) = -\frac{\alpha}{2}\left(D(p)-z-q_\circ\right)^2 + (p-c)(D(p)-z)$$

corresponds to the control $q = D(p) - z$ for which the drift of k_t vanishes

Boundary conditions at $k = k_{\min}$

$$k_t \geq k_{\min}, \quad \forall t \qquad \Rightarrow \qquad D_\xi H\Big(z, p(k_{\min}, z), \partial_k U(k_{\min}, z)\Big) \geq 0$$

Two cases may occur:

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$$D_\xi H(z, p(k_{\min}, z), \partial_k U(k_{\min}, z)) = 0 \qquad \Leftrightarrow \qquad q + z - D(p) = 0.$$

This implies that p can be considered as the control variable at $k = k_{\min}$. **In other words, the cartel directly controls the price in this situation.**

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In this case, we also expect that p **is nonincreasing with respect to** k **for** $k \approx k_{\min}$**,** which implies that

$$rp \geq b(k_{\min}, z)\partial_z p - g(k_{\min}).$$

Indeed, if p was increasing with respect to k at $k \approx k_{\min}$, then the agents owning the storage facilities would make an arbitrage and increase the stored quantity, and the drift of k_t would be positive.

Boundary conditions at $k = k_{\min}$, when $D_\xi H(z, p, \partial_k U) \leq 0$ for $k = k_{\min,+}$.

- Therefore, when $z_t = z$, among the strategies consisting of maintaining k_t at the minimal value $k_{\min}$, the optimal one is

$$\begin{cases} q^* &= 1 - z - \epsilon p^*, \\ p^* &= \underset{r\pi \geq b(k_{\min}, z)\partial_z \pi - g(k_{\min})}{\operatorname{argmax}} F(\pi, \partial_z U) \end{cases}$$

where

$$F(\pi, \partial_z U) = H_{\min}(z, \pi) + b(k_{\min}, z)\partial_z U$$

in view of the PDE for U.

Note that p^* depends on $\partial_z U, \partial_z p$.

- In this situation, the boundary condition for p at $k = k_{\min}$ is

$$p(k_{\min}, z) = p^*(\partial_z U, \partial_z p)$$

Summary

- The system of PDEs for $k_{\min} < k < k_{\max}$ is

$$\begin{cases} 0 = -rU + H(z, p, \partial_k U) + b(k, z)\partial_z U \\ \\ 0 = -rp + D_\xi H(z, p, \partial_k U)\partial_k p + b(k, z)\partial_z p - g(k) \end{cases}$$

- At $k = k_{\min}$,

$$rU = \max(A, B)$$

with

$$\begin{cases} A = H_\uparrow(z, p(k_{\min,+}, z), \partial_k U) + b(k_{\min}, z)\partial_z U & \text{(positive optimal drift)} \\ B = \max_{r\pi \geq b(k_{\min}, z)\partial_z \pi - g(k_{\min})} H_{\min}(z, \pi) + b(k_{\min}, z)\partial_z U & \text{(optimal drift=0)} \end{cases}$$

Boundary condition for p: in the weak sense,

$$p = p^*(\partial_z U, \partial_z p)$$

Comments on the boundary conditions

- The boundary conditions at $k = k_{\min}$ and $k = k_{\max}$ arise from the physical constraints on the storage capacity. These constraints play a key role. Indeed, in some situations and when the storage level is either minimal or maximal, the cartel directly controls the price of the resource
- These boundary conditions are very unusual from the mathematical point of view, in fact new to the best of our knowledge
- Obtaining a complete mathematical theory seems quite challenging
- On the theoretical viewpoint, our main results deal with asymptotic expansions of solutions

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The numerical scheme for the system of PDEs

Main ideas

- **A monotone finite difference method for the HJB equation** and the boundary conditions on U due to state constraints (upwinding in the discrete Hamiltonian)
- **A monotone finite difference method for the price p.** The advection speed in the discrete PDE is consistent with the discrete version of the Hamiltonian in the HJB equation
- The resulting discrete system is **solved by a long time approximation involving an explicit time scheme**
  - The reason for choosing an explicit scheme lies in the complexity of the boundary conditions
  - Finding an implicit or semi-implicit scheme consistent with the nonlinear boundary conditions seems challenging

The parameters of the model

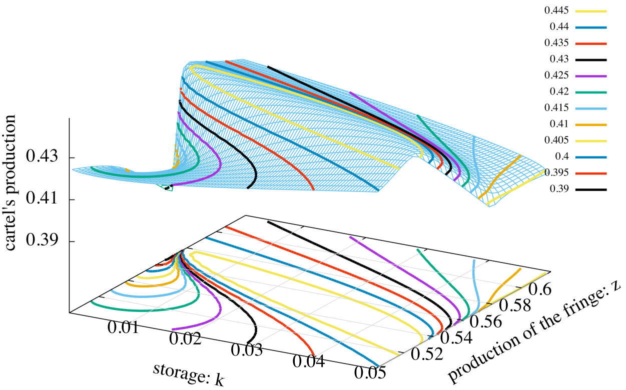
$$k_{\min} = 0, \quad k_{\max} = 0.05, \quad z_{\min} = 0.35, \quad z_{\max} = 0.75,$$

$$\begin{aligned} b(k,z,p) =& a\left(\frac{k_{\max}-k}{k_{\max}-k_{\min}}\right)^2 - a\left(\frac{k-k_{\min}}{k_{\max}-k_{\min}}\right)^2 + \kappa(\lambda p - \mu) \\ g(k) =& 0 \end{aligned}$$

with

$$\begin{aligned} r &= 0.1, \\ \epsilon &= 4.\ 10^{-4} \\ a &= 0.01 \\ \lambda = 8\ 10^{-4}, \quad \mu &= 0.05 \\ q_\circ = 0.42, \quad \alpha &= 10^4 \\ c &= 10 \\ \nu_z &\approx 10^{-4} \end{aligned}$$

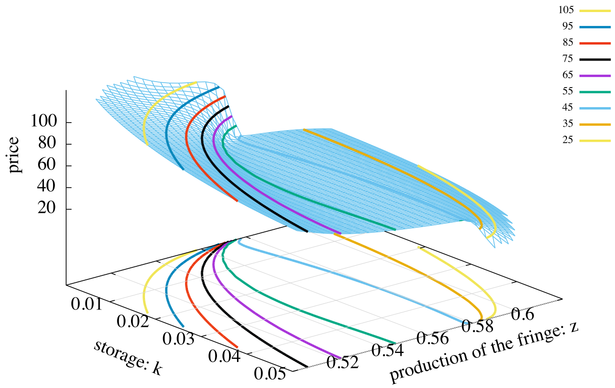
The optimal production of the cartel: $(k, z) \mapsto q^*(k, z)$



The optimal production level of the cartel as a function of k and z.

Note the shock, whose amplitude is maximal at $k = k_{\min}$ and vanishes at $k = k_{\max}$

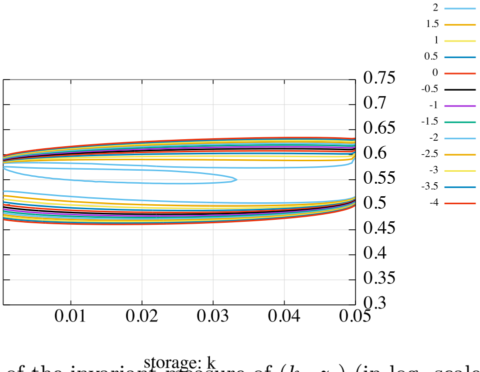
The price of the resource: $(k, z) \mapsto p(k, z)$



The price as a function of k and z

There is also a **shock in the price**

The invariant measure



The contours of the invariant measure of (k_t, z_t) (in log. scale)

The invariant measure is concentrated around a stable cycle

Within the cycle, the density is much higher in the region close to $k = k_{\min}$ and $k_{\max}$, because the evolution is slow there

Simulations of the trajectories from the optimal feedback laws

- With the functions U and p computed by the finite difference method, yielding the optimal feedback, we neglected the Gaussian noise (which is besides small in the present case) and simulated the evolution of k_t and z_t by means of a standard Euler scheme
- We see that after a small time, the trajectory becomes time-periodic, with a period of the order of 7.5 years
- In reality, there is noise. Thus the cycles are not so evident and so regular

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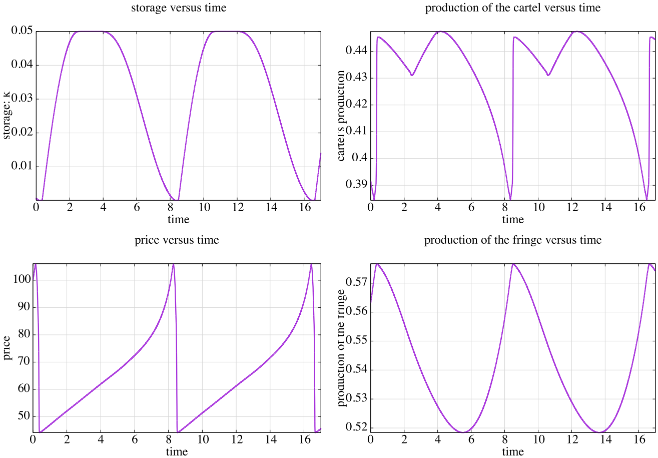
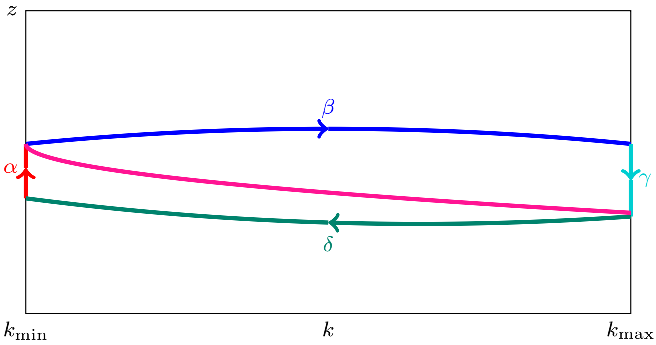


A stable cycle:
Top-Left: the level of storage vs. t. Top-Right: the production of the cartel vs. t.
Bottom-Left: the price vs. t. Bottom-Right: the production of the fringe vs. t.

The observed cycle and its interpretation

The cycle observed in the numerical simulations is drawn schematically. In the absence of significant randomness, we can make out four phases



The cycle $\alpha, \beta, \gamma, \delta$ in the (k, z) plane. In pink, the shock line

Interpretation of the first phase of the cycle: $k \approx k_{\min}$

Storage is close to minimal. **The monopolistic cartel has the power to drive the price up by maintaining a low level of production.**

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This explains the strategic shock that appears clearly in the simulation; indeed, it can be seen that when storage is minimal, the production of the monopolistic cartel increases brutally when z, non-OPEC production crosses a critical value. This brutal increase in production makes the price fall. This phenomenon has been observed in 2015 and 2020.

The last three phases of the cycle

Phase (β): When the price has fallen, the stored quantity of resource increases rapidly, and the state (k_t, z_t) is drifted to the right with a velocity nearly parallel to the k-axis. After having brutally increased its production, the cartel may let it decrease smoothly until storage gets full.

Phase (γ): Storage is full. The monopolistic cartel can now increase its production again and maintain the low level of price as long as necessary in order to deter non-OPEC producers from investing. Non-OPEC production z, decreases to the value that suits the cartel.

Phase (δ): Since the value of z is low enough, the monopolistic cartel may reduce its production. Then the stored quantity of resource decreases and the price is driven up more rapidly than in the phases β and γ.

Discussion on what happened in 2015

In 2015, OPEC decided to reconquer the market share that had been lost due to the fast development of the US shale industry from 2009 to 2015.

The price drop was then strong and sudden: prices dropped from \$100 to \$40 per barrel in a few months.

At that time, this price drop was analyzed as an attack against US shale. In the spirit of the present model, it was rather an attack against all competitors, aimed at recovering market share.

Indeed, the OPEC strategy had a strong impact not only on the US shale industry, which in fact, proved strong resiliency and coping abilities, but on many other fringe producers.

Discussion on what happened in 2020 (1/2)

In 2020, an exogenous shock to demand occurred in the first quarter of the year, with a magnitude of the order of ten times the standard deviation of demand.

Although it was not designed to handle such situations, our model seems to give an explanation of what happened.

Recall that in the present model, z (resp. q) is the ratio of the non OPEC capacity of production to the global level of demand, (resp. the ratio of the OPEC production to the global level of demand).

In the first semester of 2020, the sanitary crisis resulted in an unexpected and exceptional drop of the global demand, of the order of 10% to 15%. Therefore, the variable z got suddenly increased by 10% to 15%, and the monopolistic cartel got carried to the upper side of the shock.

In our model, the optimal response was to increase immediately production. This is precisely what happened to the surprise of many.

Discussion on what happened in 2020 (2/2)

Many observers considered that what seemed a conflict between OPEC and Russia, which led to an increase in production while the demand collapsed, was suicidal.

However, from the viewpoint of our model, this strategy was simply intended at reducing the capacity of production of the competitors as fast as possible.

Indeed, as soon as OPEC had increased its production after the collapse of the demand, storage became rapidly full. The maximal level $k_{\max}$ was reached in a few weeks, and the cartel could drive prices to a very low level (prices even went negative during a very short period).

Many planned investments stopped, and some production units were definitely closed.

After this period, OPEC started to strongly reduce its production.

Conclusion

- We propose a short term stylized model for the interaction of a cartel with a crowd of physical arbitrageurs
- Leads to a system of PDEs, comprising a HJB equation and some kind of master equation
- Completely new nonlinear boundary conditions
- Mathematical analysis: quite challenging
- Numerical simulations show
  - The production of the cartel is a discontinuous function of the state variables
  - The trajectories (at equilibrium) are cycles in the state variables
- These patterns help explain the swings in oil price that were observed in 2015 and 2020.

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