Photographic 3D-reconstruction: A Tour

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In collaboration with Maurizio and other people

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Outline

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3D-scanning $\equiv$ 3D-reconstruction + Colour

Some applications of 3D-reconstruction

- Architecture
- Augmented reality
- Metrology
- Heritage preservation

Different approaches of 3D-reconstruction

- Sensor $\equiv$ Mechanical process
- Kinect V1 $\equiv$ Projection of an infrared pattern
- Kinect V2 $\equiv$ Time-of-flight measurement of laser pulses
- Photographic techniques $\equiv$ Shape-from-X

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Shape-from-X Techniques

	Geometric techniques	Photometric techniques
Single pose Single image	Shape-from-contour Shape-from-shadows Shape-from-texture Structured light projection Shape-from-template Shape-from-defocus	Shape-from-shading
Single pose $N > 1$ images	Shape-from-focus	Photometric stereo Shape-from-polarization
$N > 1$ poses $N > 1$ images	Structure-from-motion Multi-view stereo Shape-from-silhouettes	

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The two most popular geometric techniques $\equiv$ Photogrammetry

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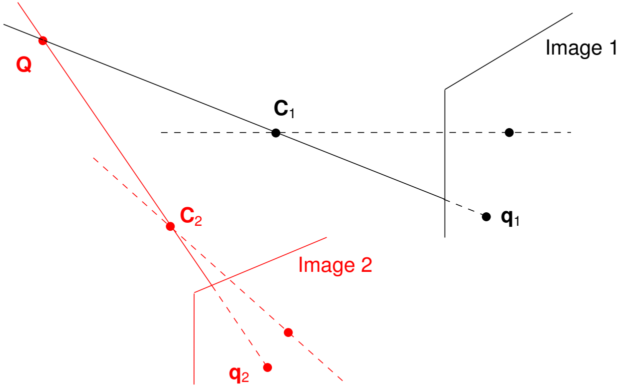
The two photometric techniques that are described in this talk

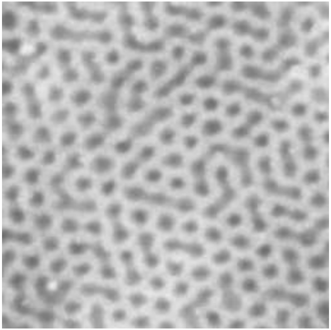
Photogrammetry: Matching + Triangulation



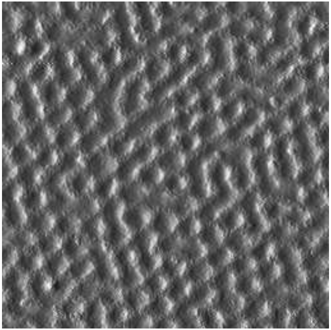
1- Matching: 2D-points $\mathbf{q}_1$ and $\mathbf{q}_2$ correspond to the same 3D-point
2- Triangulation: 3D-point $\mathbf{Q}$ at the intersection of $(\mathbf{C}_1\mathbf{q}_1)$ and $(\mathbf{C}_2\mathbf{q}_2)$

Photometric Techniques: Modelling the Greylevel

Pair of images (depth + greylevel) from an atomic force microscope



Depth



Greylevel

Paradox: the 3D-shape is better perceived from the greylevel image

Photogrammetry vs. Photometric Techniques

Photogrammetry

- Principle: matching + triangulation
- Main tools: linear algebra
- Pros: operational softwares; can be used outdoors
- Cons: matching is difficult; colour is not estimated

Photometric techniques

- Principle: modelling the greylevel
- Main tools: nonlinear PDE solvers
- Pros: no matching; colour can be estimated
- Cons: lighting estimation; difficult to use outdoors

Historical Landmarks of Photometric Techniques

- [Van Diggelen, 1951] First paper on photoclinometry (ancestor of shape-from-shading)
- [Horn, 1970] First explicit mention of shape-from-shading
- [Woodham, 1980] First paper on photometric stereo
- [Rouy, Tourin, Lions, 1988] Solving SfS with viscosity solutions
- [Hayakawa, 1994] Renewed interest for photometric stereo (related to the democratization of digital photography?)
- Since 2000: More papers on photometric stereo than on shape-from-shading
- Since 2010: First applications of photometric stereo
- Since 2020: Deep learning approach

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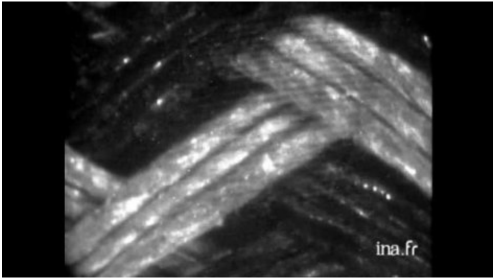
Less and less mathematics, more and more applications

Outline

Shape-from-shading (SfS): Back to the Origin

Horn's PhD thesis (MIT, 1970)

- Goal: facial recognition system
- Means: 3D-reconstruction from a single image
- Result: failure, but laid the foundations of SfS



French TV game show from the 70's: « Which object is it? »
Obviously, going through 3D-reconstruction is not necessary!

Shape-from-shading: A very Intuitive Technique

It is easy to deduce that the wall is not perfectly flat...

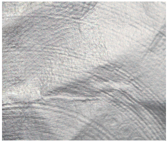
Modelling the Greylevel: Lambert's Law

Greylevel

$=$

Albedo

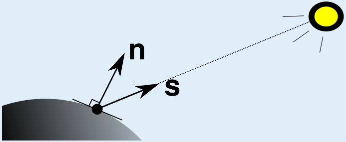
$\times$



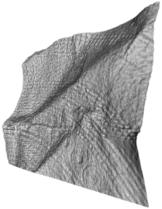
Shading

$$\underbrace{I}_{\text{Greylevel}} = \underbrace{\rho}_{\text{Albedo}} \times \underbrace{\mathbf{s} \cdot \mathbf{n}}_{\text{Shading}}$$

- **s**: lighting vector
- **n**: surface normal ($\approx$ 3D-shape)



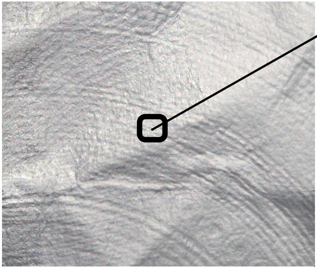
$\Downarrow$ SfS



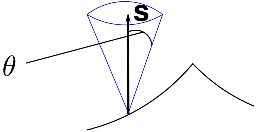
3D-shape

From a Simple Equation in the Normal **n**...

- Data: greylevel I
- Assumption on the albedo: $\rho \equiv 1$
- Unknown: normal **n** such that $\|\mathbf{n}\| = 1$



$$I = \mathbf{s} \cdot \mathbf{n} = \|\mathbf{s}\| \cos\theta$$



The model specifies only one out of the two degrees of freedom of **n**

The local estimation of **n** is ill-posed!

... To a Nonlinear PDE in the Depth z

Linear equation in **n**

$$I = \mathbf{s} \cdot \mathbf{n} \qquad (1)$$

Additional assumption on the lighting

Frontal lighting: $\mathbf{s} = [0, 0, 1]^\top \qquad (2)$

Additional assumption on the camera

Orthogonal projection: $\mathbf{n} = \frac{1}{\sqrt{\|\nabla z\|^2 + 1}} \begin{bmatrix} \nabla z \\ 1 \end{bmatrix} \qquad (3)$

(1) + (2) + (3) $\Rightarrow$ Nonlinear PDE in z

$$\|\nabla z\| = \sqrt{\frac{1}{I^2} - 1}$$

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(1) + (2) + (3) $\Rightarrow$ Nonlinear PDE in z

$$\|\nabla z\| = \sqrt{\frac{1}{I^2} - 1}$$

Eikonal equation

Shape-from-shading: Three Major Defects

Assumption on the albedo ρ

- $\rho \equiv 1 \Rightarrow$ Arbitrary
- Estimate $\rho \Rightarrow$ Difficult

Assumption on the lighting $\mathbf{s}$

- $\mathbf{s} = [0, 0, 1]^\top \Rightarrow$ Arbitrary
- Estimate $\mathbf{s} \Rightarrow$ Calibration

Elk image in Maurizio's office

But the problem is still ill-posed!

How to make the solution unique?

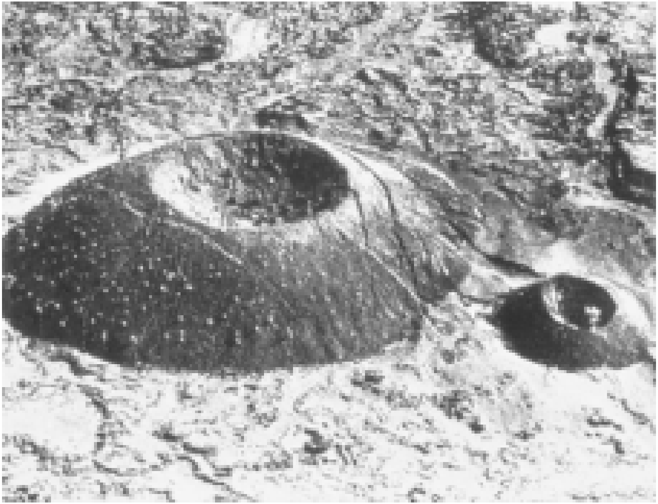
- Equation $I = \mathbf{s} \cdot \mathbf{n} \quad \Rightarrow$ Regularization (inaccurate)
- Eikonal equation $\Rightarrow$ Boundary condition (arbitrary)

Illustration of the SfS Ill-posedness

This 3D-shape has nothing to do with Lena, and yet...

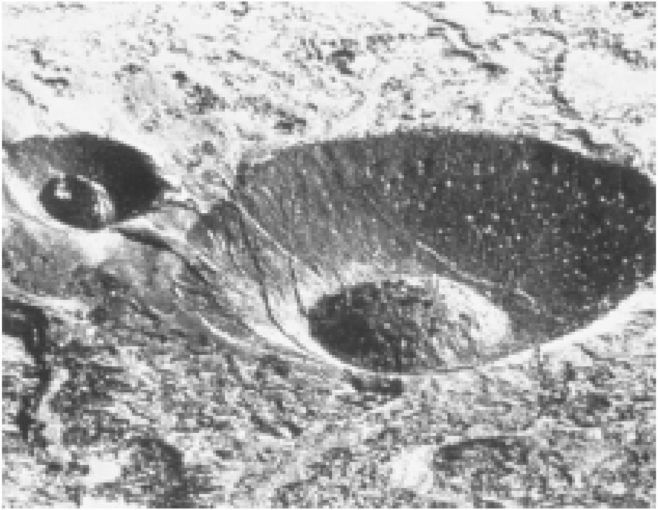
$\Rightarrow$ Video Lena.m4v

Concave / Convex Ambiguity



Photograph of a volcano in Hawaii (copyright Whitman Richards)

Concave / Convex Ambiguity



After rotating the image, we see another 3D-shape!

A Surprising Consequence of this Ambiguity

Concave Beethoven's bust under frontal lighting

$\Rightarrow$ Video Beethoven.mp4

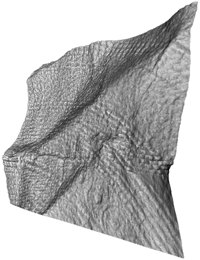
Outline

Photometric Stereo: Real 3D-scanning Technique

$N > 1$ lightings $\mathbf{s}^1, \ldots \mathbf{s}^N$

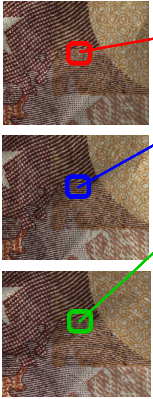
$\Longrightarrow$

Albedo ρ



Depth z

A Linear Problem Easy to Solve Locally



$$I^1 = \rho\,\mathbf{s}^1 \cdot \mathbf{n} = \mathbf{s}^1 \cdot \mathbf{m}$$

$$I^2 = \rho\,\mathbf{s}^2 \cdot \mathbf{n} = \mathbf{s}^2 \cdot \mathbf{m}$$

$$I^3 = \rho\,\mathbf{s}^3 \cdot \mathbf{n} = \mathbf{s}^3 \cdot \mathbf{m}$$

In each pixel, let us define $\mathbf{m} = \rho\,\mathbf{n}$:

$$I^i = \mathbf{s}^i \cdot \mathbf{m}, \quad i \in \{1, \dots, N\}$$

$N \geq 3$ non-coplanar lighting vectors $\mathbf{s}^i$:

- Exact solution if $N = 3$
- Approximate solution if $N > 3$

Finally: $\rho = \|\mathbf{m}\|$ and $\mathbf{n} = \frac{\mathbf{m}}{\|\mathbf{m}\|}$

Photometric Stereo: Three Advantages over SfS

No assumption on the albedo $\rho \Rightarrow$ Realistic

- Now, ρ is an additional unknown
- Technique which can estimate the albedo (3D-scanning)

Well-posed problem $\Rightarrow$ Accurate

- No regularization, no boundary condition
- Only need to know the N lightings $\mathbf{s}^i$ (by calibration)

Local linear problem $\Rightarrow$ Fast solving

- Easy problem to solve at each point: $\min_{\mathbf{m}} \sum_{i=1}^{N} \left(I^i - \mathbf{s}^i \cdot \mathbf{m} \right)^2$
- But integrating the normals $\mathbf{n}$ has to be done afterwards

Example: Plaster, Convex Beethoven's Bust

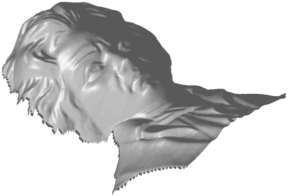
$I^1 = \mathbf{s}^1 \cdot \mathbf{m}$

$I^2 = \mathbf{s}^2 \cdot \mathbf{m}$

$I^3 = \mathbf{s}^3 \cdot \mathbf{m}$

Albedo ρ

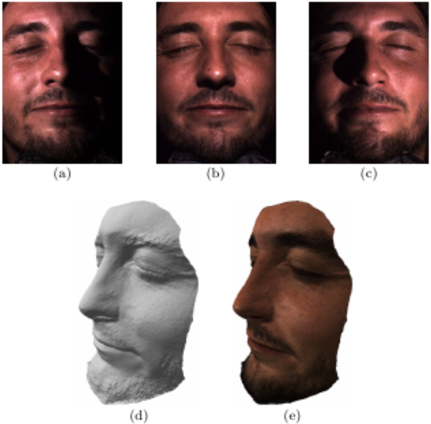
Normals $\mathbf{n}$
(RGB representation)



Depth z

Application to Augmented Reality

Showroom in Toulouse

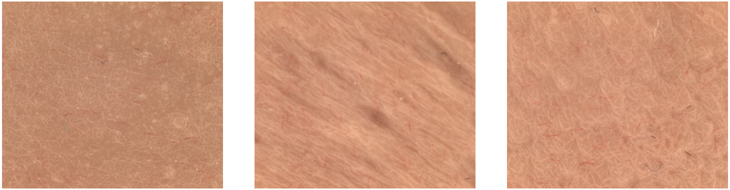


(a-b-c) Three photographs of a face
(d-e) 3D-shape and 3D-model
(accuracy $\approx$ 0.8 mm)

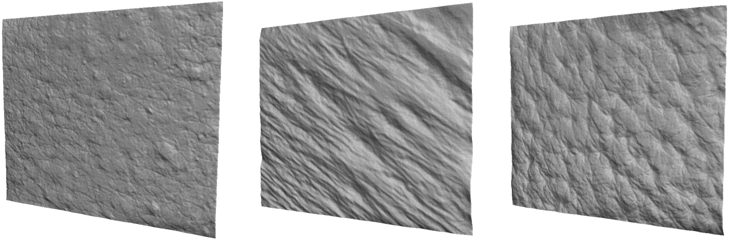
Application to Metrology

Dermatological camera transformed into a 3D-scanner

Application to Metrology



Three samples of fake skin



3D-reconstructions (accuracy $\approx$ 10 micrometers)

Application to Metrology

1 euro (Italy)

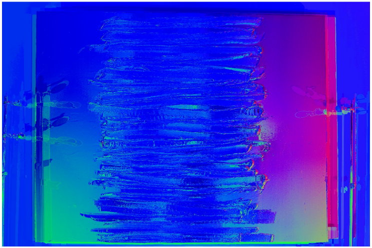
50 cents (Spain)

1 yuan (China)

3D-reconstructions (accuracy $\approx$ 10 micrometers)

Application to Heritage Preservation

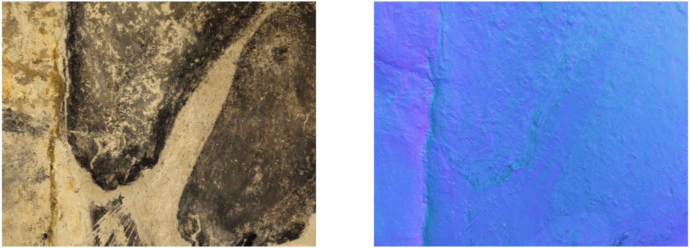

A painting by Pierre Soulages, known as the « master of black »

The lack of salient points makes photogrammetry unusable

Application to Heritage Preservation

Painted / engraved horse in the Chauvet cave (−38000 years)



PS is well-suited to separate painting from engraving

Outline

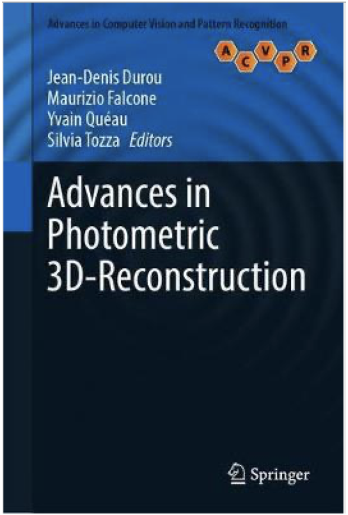
A Funny Photometric Stereo Illusion

Chagall

Cézanne

$\Rightarrow$ Video Chazanne.mp4

Collaborative Book Published in 2020



Merci Maurizio !