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Quasi Sasakian manifolds endowed with a 1-conformal cosymplectic structure

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*Alla memoria della cara impareggiabile amica M.A. Sneider
sempre viva nei miei ricordi e nel mio cuore*

RIASSUNTO – *In questo lavoro, che va inquadrato nel vasto campo delle varietà Sasakiane, vengono studiate varietà quasi Sasakiane dotate di una struttura 1-conforme cosimplettica. I risultati ottenuti: diverse proprietà del campo vettoriale di struttura ξ e del tensore di curvatura R di una tale varietà, riguardano – in generale – le strutture cosimplettiche.*

ABSTRACT – *Let $M(\Omega,\eta,\xi,g)$ be an almost cosymplectic manifold defined by the pairing: $g\Omega \in \Lambda^2 M$, $\eta \in \Lambda^1 M$. If u is a certain closed 1-form and d^u denotes the cohomology operator associated with u, i·e $d^u\alpha = d\alpha + u\wedge d$; $\alpha \in \Lambda M$; then $M(\Omega,\eta,\xi,g)$ is said to be endowed with a 1-conformal cosymplectic structure if Ω and η satisfy*

$$d^u\eta = 0 \quad , \quad d\Omega = 0 .$$

In this case differente properties of the structure vector field ξ (ξ: dual of η with respect to g) and of the curvature tensor R of M are discussed.

KEY WORDS – *Cohomology operator - Quasi Sasakian manifold - Quasi concircular pairing - Exterior recurrent - Exterior concurrent - Symplectic adjoint - Symplectic harmonic - Anti-invariant - Soldering form.*

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– Introduction

In the last two decade a series of papers have been devoted to *almost cosymplectic* manifolds.

Let $M(\Omega,\eta,\xi,g)$ be a $(2m+1)$-dimensional Riemannian C^∞-manifold endowed with an almost cosymplectic structure $1\times S_p(m;\mathbb{R})$ (the structure 2-form $\Omega\in\Lambda^2 M$ and the structure 1-form $\eta\in\Lambda^1$ satisfy $\Omega^m\wedge\eta\neq 0$).

If d^u is the *cohomology operator* (F. Guedira and A. Lichnerowicz [1]) then if

$$(a)\quad d^u\eta=0,\quad d\Omega=0,\quad u\in\Lambda^1 M$$

we say that the pairing (η,Ω) defines a *1-conformal cosymplectic structure* (abr. (1-c.c.)-structure). In this case u and it's dual vector field U are called the *associated tensor* with this structure.

In the present paper we study the case of a *quasi Sasakian* manifold $M(\Phi,\eta,\xi,g)$ endowed with a (1-c.c.)-structure. We are quoting here the following properties:

(i) The structure vector fields ξ, and U define a *quasi concircular pairing* [9] and U defines an *infinitesimal conformal transformation* of the *Reeb vector field* ξ (or the structure vector field.

(ii) U is a *geodesic* and ΦU is a *parallel* vector field.

(iii) The *Ricci curvatures* of ξ and U are $\mathrm{Ric}(\xi)=-1$, $\mathrm{Ric}(U)=+1$.

(iv) The curvature tensor field R of M satisfies $R(Z,Z')W+\Phi R(Z,Z')\,\Phi W+R(Z,Z')U=0$ where $Z,Z',W\in xM$ are any vector fields, and the *vertical curvature* 2-forms of M are *exterior recurrent* [2].

(v) The Lie derivatives $L_X\eta$, where X is any *infinitesimal automorphism* of u, are as η, d^u-closed, i.e. $d^u(L_X\eta)=0$,

(vi) Any manifold $M(\Phi,\eta,\xi,g)$ is foliated by *geodesic hypersurfaces* M_h normal to ξ, and if M_I is any *invariant submanifold* [3] of M then the immersion $x\colon M_I\to M$ is *minimal*. It should be noticed that this property is similar to that of invariant submanifolds immersed in a quasi-Sasakian manifold carrying a cosymplectic structure [4].

1 – Preliminaries

Let $M(\Phi,\eta,\xi,g)$ be an oriented paracontact C^∞-manifold with tangent bundle TM and denote by $\Gamma TM=xM$, the set of the sections of TM.

Following W.A. Poor [5] we denote by $\flat : TM \to T^*M$ the *musical isomorphism* defined by the metric tensor g and write $A^q(M,TM) = \Gamma Hom(\Lambda^q TM, TM)$. We notice that elements of $A^q(M,TM)$ are vector valued q-forms on M. Next if ∇ is the *covariant derivative* operator defined by g, then

$$d^\nabla : A^q(M,TM) \to A^{q+1}(M,TM)$$

defines the *exterior covariant derivative operator* with respect to ∇. Generally $d^{\nabla^2} = d^\nabla \circ d^\nabla \neq 0$, and we assume in this paper that ∇ is *symmetric*. Let $dp \in A^1(M,TM)$ be the *soldering form* of M [6] (as is known [6] dp is a canonical vector valued 1-form). Since the manifold M we are going to discuss is connected, we shall denote following F. Guedira and A. Lichnerowicz [1], by $d^u = d + e(u)$; $e(u)$: exterior product by the closed 1-form u the *cohomology operator*.

Any form $\Phi \in \Lambda M$ such that $d^u\Phi = 0$ is said to be *d^u-closed*.

Let $F = \sum a_A Z_A \otimes \omega^A \in A^1(m,TM)$ ($a_A \in C^\infty M$, $Z_A \in xM$, $\omega^A \in \Lambda^1 M$) be any vector valued 1-form F. If F satisfies

$$d^{\nabla^q} F = \Phi_q \wedge dp \in A^{q+1}(M,TM) \tag{1.1}$$

for some q–form $\Phi_q \in \Lambda^q M$, then F is defined as *q–exterior concurrent* (M. Petrovic, R. Rosca, L. Verstraelen [7]).

Let $0_\Phi = \text{vect}\{e_a, \Phi e_a = e^{a*}, e_0 = \xi / a = 1, \dots m; a^* = a + m\}$ be an adapted local field of 0_Φ-orthonormal frames on M [3], and let $0^*_\Phi = \text{covect}\{\omega^A | A = 0, 1 \dots, 2m\}$ be it's associated coframe. Then Cartan's structure Eqs. written in index less form are

$$\nabla e = \theta \otimes e \in A^1(M,TM), \tag{1.2}$$

$$d\omega = -\theta \wedge \omega, \tag{1.3}$$

$$d\theta = -\theta \wedge \theta + \Theta. \tag{1.4}$$

In this above Eqs. θ (resp.Θ) are the local connection forms in the bundle $0(M)$ (resp. the curvature forme on M).

2 – Quasi Sasakian manifolds endowed with a 1-conformal cosymplectic structure

Let $M(\Phi,\eta,\xi,g)$ be a $(2m+1)$-dimensional quasi Sasakian manifold. Then the structure tensor fields (Φ,η,ξ,g) satisfy (see also [4])

$$\left\{\begin{array}{l}\Phi^2 = -Id + \eta\otimes\xi, g(Z,Z') - \eta(Z)\eta(Z') = g(\Phi Z,\Phi Z')\\ \eta(Z) = g(\xi,Z), \Phi\xi = 0, \eta(\xi) = 1, \forall Z,Z' \in xM\,.\end{array}\right. \tag{2.1}$$

Following [1] the structure 1-form η is d^u-closed, for some *semi-basic unit closed* 1-form u iff:

$$d^u\eta = d\eta + u\wedge\eta = 0\,. \tag{2.2}$$

If we set

$$u = \sum u_\alpha\omega^\alpha\,;\ u_\alpha \in C^\infty M\,,\ \alpha\in\{a,a^*\} \tag{2.3}$$

then by (2.2) and the structure Eqs. (1.3) one gets

$$\theta_0^\alpha = u_\alpha\eta\,. \tag{2.4}$$

Next we denote by $U = \flat^{-1}u \in xM$ the dual vector field of u. Then by the structure Eqs. (1.2) and by (2.4) the covariant derivative of the structure vector field ξ is expressed by

$$\nabla\xi = U\otimes\eta\,. \tag{2.5}$$

Next by (2.1) and (2.5) one derives

$$(\nabla\Phi)Z = -g(U,\Phi Z)\xi\otimes\eta - \eta(Z)\Phi U\otimes\eta\,. \tag{2.6}$$

Denote now by

$$\Omega = \sum\omega^a\wedge\omega^{a^*} \tag{2.7}$$

the globally defined 2-form, which defines wirh η an *almost cosymplectic structure* $1\times S_p(m;\mathbb{R})$, i.e.

$$\Omega^m\wedge\eta\neq 0\quad,\quad i_\xi\Omega = 0\,.$$

By (2.4) and (2.5) and making use of (1.3) one gets by exterior differentiation of (2.7)

$$d\Omega = 0 \tag{2.8}$$

i.e. Ω is a *presymplectic* form [8].

In the following any quasi Sasakian manifold $M(\Phi, \eta, \xi, g)$ such that Eqs. (2.1), (2.2) and (2.6) hold good, will be called a *1-conformal cosymplectic quasi Sasakian manifold* (abr. 1-c.c.q.S).

In addition u (resp. $U = \flat^{-1}u$) will be called the *associated 1-form (resp. associated vector field)*of the (1-c.c.)-structure.

Denote by $D_h = \{Z \in xM; \eta(Z) = 0\}$ the $2m$-distribution annihilated by η (the *horizontal* distribution).

Since the covariant differential ∇U of the unit vector field $U \in D_h$ is *self adjoint* [3] one finds by (1.2) and (2.4)

$$\nabla U = -\xi \otimes \eta . \tag{2.9}$$

Since ξ and U are two mutually orthogonal sections, then by references to [9], Eqs. (2.5) and (2.9) show that (ξ, U) defines a *quasi-concircular pairing*. Further one easly derives from (2.5), (2.9)

$$[U, \xi] = L_U \xi = \xi \tag{2.10}$$

which proves that, in the case under consideration, the vector field U defines an *infinitesimal conformal transformation* i.c.t.) of ξ.

In addition one finds

$$\nabla_U U = 0 \tag{2.11}$$

and making use of (2.6) one gets

$$\nabla \Phi U = 0 . \tag{2.12}$$

Hence it follows from the above, that U and ΦU is a *geodesic* and a *parallel* vector field respectively.

We shall now give the following

DEFINITION. *Any vector field $V \in \mathcal{X}M$ such that it's q^{th} covariant derivative is expressed by*

$$\nabla^q V = v \wedge \nabla^{q-1} V \tag{2.13}$$

for some 1-form v is said to be $(q-1)$-covariant recurrent.

Since the operator ∇ acts inductively one derives from (2.5) and (2.9)

$$\nabla^2 \xi = (\eta \wedge u) \otimes U = -u \wedge \nabla \xi \,. \tag{2.14}$$

and

$$\nabla^2 U = (u \wedge \eta) \otimes \xi = -u \wedge \nabla U \,. \tag{2.15}$$

Hence one may say that ξ and U are both 1-covariant recurrent. As a consequence of this property, consider the vector valued 1-form

$$F = \xi \wedge U = \flat(U) \otimes \xi - \flat(\xi) \otimes U = u \otimes \xi - \eta \otimes U \in A^1(M, TM).$$

Operating F by d^{∇^2}, one finds by (2.2), (2.14), (2.15) (and taking account of $du = 0$)

$$d^{\nabla^2} F = \nabla^2 \xi \wedge u - \nabla^2 U \wedge \eta = 0$$

which shows that F is d^{∇^2}-closed.

Now since in general for any $Z \in \mathcal{X}M$ one has

$$\operatorname{div} Z = \operatorname{tr}[\nabla Z], \quad \text{at any} \quad p \in M$$

one derives from (2.5) and (2.9)

$$\operatorname{div} \xi = 0 \tag{2.16}$$

and

$$\operatorname{div} U = -1 \tag{2.17}$$

Hence one may say that ξ and U defines an *infinitesimal automorphism* and an *infinitesimal homothety* respectively, of the volume element of M.

Making use of the general formula of K. Yano (see also [3])

$$\operatorname{div}(\nabla_Z Z) - \operatorname{div}((\operatorname{div} Z)Z) + (\operatorname{div} Z)^2 =$$
$$\operatorname{Ric}(Z) + \sum g(\nabla_{e_A} Z, e_B) g(e_A, \nabla_{e_B} Z); \quad \operatorname{Ric}(Z): \text{ Ricci curvature}$$

one finds by (2.5), (2.9), (2.16) and (2.17)

$$\operatorname{Ric}(\xi) = -1 \quad , \quad \operatorname{Ric}(U) = +1 .$$

We recall also that if $R \in \Gamma\mathrm{end}\Lambda TM$ is the *curvature operator* on M, one has the general formula

$$\nabla^2 W(Z, Z') = R(Z, Z')W \quad ; \quad \forall\, Z, Z', W \in xM .$$

Setting in the above $W = U$ one finds on behalf of (2.15)

$$R(Z, Z')U = (g(U, Z)\eta(Z') - g(U, Z')\eta(Z))\xi \tag{2.18}$$

i.e. $R(Z, Z')U$ is colinear to ξ.

Further since the *sectional curvature* $K_{Z\wedge U}$ with respect to the 2-plane spaned by any vector field Z and by U is expressed by

$$K_{Z\wedge U} = \frac{< R(Z, U)U, Z >}{\|U\|^2\|Z\|^2 - < U, Z >^2}; <> \ : \ \text{instead of } g$$

one finds with the help of (2.19)

$$K_{Z\wedge U} = \frac{(\eta(Z))^2}{g(U, Z)^2 - g(Z, Z)^2} .$$

Thus one may say that $K_{Z\wedge U}$ vanishes for all horizontal vector fields $Z \in D_h$.

If δ denotes the *adjoint* operator of d with respect to g, then by (2.16) and (2.17) one has

$$\delta u = 1 \quad , \quad \delta\eta = 0 \tag{2.19}$$

and so by (2.2) and taking account of $du = 0$, a short calculation gives

$$\Delta\eta = 0 \quad , \quad \Delta u = 0 .$$

Hence both forms η and U are *harmonic*.

Further if ω is a closed 1-form and $\alpha \in \Lambda M$, any form, the differential operator of degree -1, δ^ω has been defined in [1]

$$\delta^\omega \alpha = \delta\alpha + i_{\flat^{-1}(\omega)}\alpha$$

and the *generalized Laplacian* Δ^ω by

$$\Delta^\omega = d^\omega \delta^\omega + \delta^\omega d^\omega .$$

Setting in the above Eqs. $\omega = u$ one finds by (2.19)

$$\Delta^u \eta = 0 .$$

Let us now go back to Eq. (2.14). Then by the general formula

$$\Delta^2 Z = Z^A \Theta_A^B \otimes e_B \in A^2(M, TM)$$

one finds

$$\Theta_0^\alpha = u_\alpha \eta \wedge u \quad ; \quad \alpha \in \{a, a^*\} \tag{2.20}$$

where Θ_0^α may be called the *vertical curvature 2-forms*.

Since by (2.2) $\eta \wedge u$ is a closed 2-form, one readly derives from (2.20)

$$d\Theta_0^\alpha = d \lg U_\alpha \wedge \Theta_0^\alpha \iff d^{-d \lg U_\alpha} \Theta_0^\alpha = 0$$

which shows that all the forms Θ_0^α are *exterior recurrent*. further since by (2.1) and (2.6) and making use of (1.2) one has

$$\theta_b^a = \theta_{b^*}^{a^*} \quad , \quad \theta_b^{a^*} = \theta_a^{b^*} \tag{2.21}$$

then by (2.4) and the structure Eqs. (1.4) one finds

$$\Theta_b^a = \Theta_{b^*}^{a^*} \quad , \quad \Theta_b^{a^*} = \Theta_a^{b^*} . \tag{2.22}$$

Now by (2.18) and (2.22) one finds the following general formula for the curvature tensor of $M(\Phi, \eta, \xi, g)$

$$R(Z, Z')W + \Phi R(Z, Z')\Phi W + R(Z, Z')U = 0 \tag{2.23}$$

where $W, Z, Z' \in \mathcal{X}M$ are any vector fields.

We shall discuss now some properties of the Lie Algebra involving the tensor fields $U \in \mathcal{X}M, u, \eta \in \Lambda^1 M$ and $\Omega \in \Lambda^2 M$.

By (2.1), (2.9), (2.7) and (2.8) one has

$$(2.24) \qquad i_U \Omega = \flat(\Phi U) \quad , \quad d\flat(\Phi U) = 0 \rightarrow L_U \Omega = 0$$

and since by definition $du = 0$, one also has

$$(2.25) \qquad i_{\Phi U} \Omega = -u \rightarrow L_{\Phi U} \Omega = 0 .$$

Consequently both vector fields U and ΦU define infinitesimal automorphism of the structure 2-form Ω.

In addition, since $U(\Phi U) = g(U, \Phi U) = 0$ one also quickly finds by (2.6)

$$L_{\Phi U} \eta = 0 .$$

Therefore one may say that ΦU defines an infinitesimal automorphism of the considered (1-c.c)-structure defined by the pairing (η, Ω).

Next since u is closed, then any vector field $X \in \mathcal{X}M$ which defines an infinitesimal automorphism of u is such that

$$(2.26) \qquad u(X) = c = \text{const} .$$

Take now the Lie derivative of the structure 1-form η with respect to X. One has by (2.2)

$$L_X \eta = d\eta(X) + \eta(X)u - c\eta$$

and by exterior differentiation one gets

$$d^u(L_X \eta) = 0 .$$

hence $L_X \eta$ is as η, d^u-closed.

THEOREM. *Let $M(\Phi, \eta, \xi g)$ be a quasi Sasakian manifold endowed with a (1-c.c)-structure and let $\Omega \in \Lambda^2 M$, $u \in \Lambda^1 M$ and $U = \flat^{-1}(u) \in \mathcal{X}M$ be the associated 1-form and vector field with this structure, respectively. One has the following properties:*

(i) The structure vector field ξ and U define a quasi circular pairing, and U defines an infinitesimal conformal transformation of ξ.

(ii) U is a geodesic and ΦU is a parallel vector field.

(iii) ξ and U are both 1-covariant recurrent, and the vector valued 1-form $F = \xi \wedge U$ is d^{∇^2}-closed.

(iv) The Ricci curvatures of ξ and U are $\mathrm{Ric}(\xi) = -1$, $\mathrm{Ric}(U) = +1$.

(v) The curvature tensor field R of M satisfies

$$R(Z, Z')W + \Phi R(Z, Z')\Phi W + R(Z, Z')U = 0$$

where $Z, Z', W \in xM$ are any vector fields, and the vertical curvature 2-forms of M are exterior recurrent.

(vi) The vector field ΦU defines an infinitesimal automorphism of the (1-c.c.)-structure defined by (η, Ω).

(vii) The Lie derivatives $L_X\eta$, where X is any infinitesimal automorphism of u, are as η, d^u-closed.

3 – Submanifolds lf $M(\Phi, \eta, \xi, g)$

Obviously by (2.2) the horizontal distribution D_h is involutive. Denote then by M_h the *leaf* (hypersurface in M) of the 2m-foliation D_h.

Since ξ is the normal vector field of M_h, it follows at once by (2.5) that the second fundamental form $< dp, \nabla\xi >$ of the immersion $x\colon M_h \to M$ vanishes. hence M_h is a totally geodesic hypersurface.

It should be noticed on behalf of (2.6), that M_h is a *symplectic manifold*, and by (2.9) and (2.12) that U and ΦU are parallel vector fields on M_h. Further since the *symplectic adjoint* $\bar{*}\omega$ of any 1-form ω is expressed by

$$\bar{*}\omega = \omega \wedge \bar{*}\Omega = \omega \wedge \frac{\Omega^{m-1}}{m-1}$$

and the *symplectic codifferential* by

$$\bar{\delta}\omega = \bar{*}d\bar{*}\omega$$

we easly deduce (since $d\flat(U) = 0, d\flat(\Phi U) = 0$) that the dual forms $\flat(U)$, $\flat(\Phi U)$ of U and ΦU respectively are Ω-harmonic (that is *symplectic harmonic*). Denote by $\varepsilon_a = \{V : L_V\Omega = 0\} \in D_h$ the vector space of infinitesimal automorphism of the symplectic vectorial space D_h (we denote the induced elements of M_h by the same letters). Then by (2.24) and (2.25) it follows $\dim \varepsilon_a \geq 2$. Finally by reference to (2.22) it is easly seen that if M_h is a space-form then it is necessarly a *flat* submanifold. Let now $x\colon M_I \to M(\Phi,\eta,\xi,g)$ be an *invariant* [3] submanifold of M, that is

1° ξ is tangent to M_I everywhere on M,

2° ΦZ is tangent to M_I for any tangent vector Z to M_I.

Assume that M_I is of codimension 2ℓ and is defined by

$$\omega^r = 0 \quad , \quad \omega^{r^*} = 0 \quad , \quad r = m+1-\ell \ldots m \quad ; \quad r^* = r+m. \tag{3.1}$$

Since the soldering form dp_I of M_I is

$$dp_I = \omega^i \otimes e_i + \omega^{i^*} \otimes e_{i^*} + \eta \otimes \xi\,; \; (i = 1, \ldots, m-\ell\,;\, i^* = i+m)$$

the *mean curvature vector valued* $2(m-\ell)$*-form* Ⓗ $\in A^{2(m-\ell)}$ (M_I, TM_I) is expressed by

(3.2)

$$\begin{aligned} Ⓗ &= \sum(-1)^{i-1}\omega^1 \wedge \ldots \wedge \hat{\omega}^i \wedge \ldots \wedge \omega^{m-\ell} \wedge \omega^{i^*} \wedge \ldots \wedge \omega^{(m-\ell)^*} \otimes e_i + \\ &+ \sum(-1)^{i^*-1}\omega^1 \wedge \ldots \wedge \omega^{m-\ell} \wedge \omega^{i^*} \wedge \ldots \wedge \hat{\omega}^{i^*} \wedge \ldots \wedge \omega^{(m-\ell)^*} \otimes e_i^* + \\ &+ \omega^1 \wedge \ldots \wedge \omega^{m-\ell} \wedge \omega^{1^*} \wedge \ldots \wedge \omega^{(m-\ell)^*} \otimes \xi . \end{aligned}$$

If σ_I denotes the volume element of M_I, then applying the operator d^∇ to Ⓗ one has

$$d^\nabla Ⓗ = (2(m-\ell)+1)\sigma_I \otimes H \tag{3.3}$$

where $H \in T^\perp_{p_I} M_I$ means the *mean curvature vector field* associated with $x\colon M_I \to M$.

With the help of the structure Eqs. and of (2.21), one gets

$$d^\nabla Ⓗ = 0 \to H = 0$$

which expresses that any invariant submanifold M_I of M is *minimal*.

It should be noticed that this property is similar to that of invariant submanifolds of almost cosymplectic manifolds endowed with a quasi-Sasakian structure [4]. We shall close this section with the following consideration. Let $x: M_A \to M(\Phi, \eta, \xi, g)$ be the immersion of an *anti-invariant* [3] submanifold M_A of M, normal to the structure vector field ξ, and let $T_{p_A}(M_A)$ (resp. $T^{\perp}_{p_A}(M_A)$) be the tangent space (resp. the normal space) at each point $p_A \in M_A$. By definition one has $\Phi T_{p_A}(M_A) \subset T^{\perp}_{p_A}(M_A)$. If we assume that M_A is defined by

$$\omega^{a^*} = 0 \quad \eta = 0$$

(i.e. $\dim M_A = m$) and we suppose that the normal connection $\nabla^{\perp}$ is *flat*, one has $\Theta^{a^*}_{b^*} = 0$. This yiealds

$$\Theta^a_b = 0$$

which proves that M_A is a flat submanifold. It is easly seen that the converse is also true.

THEOREM. *Any (1-c.c.)-quasi Sasakian manifold $M(\Phi, \eta, \xi, g)$ is foliated by geodesic hypersurfaces M_h normal to the structure vector field ξ and any M_h is endowed with a symplectic structure $S_p(m; \mathbb{R})$. If ε_a is the vector space of infinitesimal automorphism associated with $S_p(m; \mathbb{R})$, then $\dim \varepsilon_a \geq 2$, and if M_h is a space-form then necessarly it is a flat hypersurface. Further if M_I is an invariant submanifold of M, then the immersion $x: M_I \to M$ is minimal and if M_A is an anti invariant submanifold of M, then the necessary and sufficient condition in order that M_A be flat is that the normal connection $\nabla^{\perp}$ associated with $x: M_A \to M$, be flat.*

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