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On Some Configurations of Points in a Finite Affine Space AG(n,q)

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RIASSUNTO – *Data una iperquadrica non degenere Q di $AG(n,q)$ con q dispari e primo, per i punti di $AG(n,q)$ non appartenenti a Q viene data la definizione di punto regolare o quasiregolare rispetto a Q e successivamente sono determinati sia il numero sia la configurazione dei punti dello stesso tipo.*

ABSTRACT – *Given a proper hyperquadric Q of $AG(n,q)$ with q odd and prime, for the points not lying on Q we give the definition of regular or quasiregular point with respect to Q and successively we determine the number and the configurations of the points of the same type.*

KEY WORDS – *Finite geometries.*

A.M.S. CLASSIFICATION: 05B25

– Introduction

In this work we determine at first the number and the distribution of the exterior points to a proper hyperquadric Q of the n-dimensional affine space $AG(n,q)$ over a Galois field γ of order q odd and prime. Successively we give the definition of regular or quasiregular point with respect to Q for the points not lying on Q and hence we determine both

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the number and the configuration of the points which are either regular o quasiregular with respect to Q. In this way a pencil $\mathcal{F}$ of hyperquadrics arises, which contains particular classes of hyperquadrics, whose point are of the same type; such hyperquadrics depend on either the pairs of consecutive squares and non squares of γ, or a class of transformations which preserve or change the quadratic character of the elements of γ.

For every hyperquadric of $\mathcal{F}$ there is a group G of affine transformations generated by the symmetries with respect to the hyperplanes intersecting Q and containing its center O. In $PG(n,q)$ the harmonic homologies whose fixed points belong to a secant hyperplane $\bar{\pi}$ through O and whose center is the pole of $\bar{\pi}$ with respect to Q correspond to such symmetries.

Finally it is proved that the type of points of $AG(n,q)$ is an affine invariant.

1 – Subsets of GF(q)

Let q be an odd prime integer, and let $\gamma = GF(q)$ denote the Galois field of order q. As in [3] (see bibliography), we will write $x \in \square$ or $x \in \Delta$ according as x is a non-zero square or non square element in $GF(q)$. Put

(1.1) $$E = \{x : x \in \square\,,\, x-1 \in \square\}$$

(1.2) $$H = \{x : x \in \square\,,\, x+1 \in \square\}$$

(1.3) $$I = \{x : x \in \square\,,\, x-1 \in \Delta\}$$

(1.4) $$L = \{x : x \in \Delta\,,\, x+1 \in \square \cap I\}$$

For such subsets of γ the following properties are true.

I $q \equiv -1 \pmod 4$

a) $|E| = |I| = |H| = |L| = \dfrac{r-1}{2}$, where $r = \dfrac{q-1}{2}$;

b) $x \in \square \cap I \implies \dfrac{1}{x} \in E$; $x \in \square \cap H \implies \dfrac{1}{x} \in H$;

c) $x \in \Box \implies -x \in \Delta$;
d) γ contains $\frac{r-1}{2}$ pairs of consecutive elements of $\Box$ and as many analogous pairs of Δ;
e) γ contains at most two triplets of consecutive elements of either $\Box$ or Δ, only if $q = 11, 19, 23$ (see [9]).

II $q \equiv 1 \pmod 4$
a') $|E| = |H| = \frac{r-2}{2}$, $|I| = |L| = \frac{r}{2}$;
b') $x \in \Box \cap E \implies \frac{1}{x} \in E$; $x \in \Box \cap H \implies \frac{1}{x} \in H$ and $1 \in H$ if $2 \in \Box$;
c') $x \in \Box \implies -x \in \Box$;
d') γ contains $\frac{r-2}{2}$ pairs of consecutive elements of $\Box$ and $\frac{r}{2}$ analogous pairs of Δ.

2 – Subsets of AG(n,q)

Let $(x_1, x_2, \ldots, x_n)$ be the coordinates of a point of $AG(n,q)$ and let $k-1 (k \geq 0)$ be the greatest dimension of the linear spaces contained in a proper hyperquadric of $AG(n,q)$; the equation of such a hyperquadric may be one of the following types (see [3] and [10]).

If $n = 2k$:

$$(2.1) \qquad Q_I) F_I(2k,q) = x_1^2 + \ldots + \delta x_{2k}^2 + 1 = 0 (\delta \in \Box);$$

$$(2.2) \qquad Q_{II}) F_{II}(2k,q) = x_1^2 + \ldots + \delta x_{2k}^2 + 1 = 0 (\delta \in \Delta);$$

$$(2.3) \qquad Q_{III}) F_{III}(2k,q) = x_1 + x_2 x_3 + \ldots + x_{2k-2} x_{2k-1} + x_{2k}^2 = 0 .$$

If $n = 2k - 1$:

$$(2.4) \qquad Q_I) F_I(2k-1,q) = x_1^2 + \ldots + \delta x_{2k-1}^2 + 1 = 0 , (\delta \in \Box);$$

$$(2.5) \qquad Q_{II}) F_{II}(2k-1,q) = x_1 + x_2 x_3 + \ldots + x_{2k-2} x_{2k-1} = 0 .$$

If $n = 2k + 1$:

(2.6) $\quad Q_{III}) F_{III}(2k+1, q) = x_1^2 + \ldots + \delta x_{2k+1}^2 + 1 = 0\,, \quad (\delta \in \Delta)$

(2.7) $\quad Q_{IV}) F_{IV}(2k+1, q) = x_1 + x_2^2 + \ldots + \delta x_{2k+1}^2 = 0\,,$

with $\delta \in \Box$ if $q \equiv -1 \pmod 4$ and $\delta \in \Delta$ if $q \equiv 1 \pmod 4$.

Let $\psi_j(n, q)$ denote the number of the points lying on the proper hyperquadric Q_j of $AG(n, q)$; in particular we have:

$$\psi_I(2k, q) = q^{2k-1} - q^{k-1}\,; \qquad \psi_{II}(2k, q) = q^{2k-1} + q^{k-1}\,;$$

$$\psi_{III}(2k, q) = q^{2k-1}\,;$$

$$\psi_I(2k-1, q) = q^{2k-2} + q^{k-1}\,; \qquad \psi_{II}(2k-1, q) = q^{2k-2}\,;$$

$$\psi_{III}(2k+1, q) = q^{2k} - q^k\,; \qquad \psi_{IV}(2k+1, q) = q^{2k}\,.$$

Regarding the number of the points of $AG(2k, q)$ not lying on a given proper hyperquadric Q it will be denoted by ψ_i or ψ_e according as the corresponding polar hyperplane determines with Q a section of type Q_I in $AG(2k-1, q)$ or Q_{III} in $AG(2k+1, q)$.
In particular for Q_I we have:

(i)

$$\psi(2k, q) = q^{2k} - q^{2k-1} + q^{k-1} = \psi_i(2k, q) + \psi_e(2k, q)\,, \text{ with}$$

$$\psi_i(2k, q) = \frac{1}{2}(q^{2k} - q^{2k-1} + q^k + q^{k-1} - 2)$$

[without counting the center of Q_I] and

$$\psi_e(2k, q) = \frac{1}{2}(q^{2k} - q^{2k-1} - q^k + q^{k-1})\,.$$

The points of former kind belong to $r - 1$ proper hyperquadrics of the same type as Q_I and the asymptotic hypercone Λ_I whose points are $\psi(\Lambda_I) = q^{2k-1} + q^k - q^{k-1} - 1$ and hence $\psi_i(2k, q) = (r-1)\psi_I(2k, q) + \psi(\Lambda_I)$; but the points of second kind belong to r hyperquadrics of the same type as Q_I and hence $\psi_e(2k, q) = r\psi_I(2k, q)$.

For Q_{II} we have:

(ii)

$$\psi(2k,q) = q^{2k} - q^{2k-1} - q^{k-1} = \psi_i(2k,q) + \psi_e(2k,q)\,,\ \text{with}$$

$$\psi_i(2k,q) = \frac{1}{2}(q^{2k} - q^{2k-1} + q^{k} - q^{k-1}) = r\psi_{II}(2k,q)\ \text{and}$$

$$\psi_e(2k,q) = \frac{1}{2}(q^{2k} - q^{2k-1} - q^{k} - q^{k-1} - 2) = (r-1)\psi_{II}(2k,q) + \psi(\Lambda_{II})\,,$$

where the center of Q_{II} is excluded and $\psi(\Lambda_{II}) = q^{2k-1} - q^{k} + q^{k-1} - 1$.

For Q_{III} we have:

(iii)

$$\psi(2k,q) = q^{2k} - q^{2k-1} = \psi_i(2k,q) + \psi_e(2k,q)\ \text{with}$$

$$\psi_i(2k,q) = \psi_e(2k,q) = \frac{1}{2}(q^{2k} - q^{2k-1}) = r\psi_{III}(2k,q)\,.$$

For the points of $AG(2k-1,q)$ not lying on either Q_I or Q_{II} we have:

(i')

$$\psi(2k-1,q) = q^{2k-1} - q^{2k-2} - q^{k-1} = r\psi_{III}(2k-1,q)+$$

$$(r-1)\psi_I(2k-1,q) + \psi(\Lambda_I)\,,\ \text{where}\ \psi(\Lambda_I) = q^{2k-2}\,;$$

and

(ii')

$$\psi(2k-1,q) = q^{2k-1} - q^{2k-2} = 2r\psi_{II}(2k-1,q)\quad \text{respectively}\,.$$

Finally for the points of $AG(2k+1,q)$ not lying on either Q_{III} or Q_{IV} we have:

(i")

$$\psi(2k+1,q) = q^{2k+1} - q^{2k} + q^{k} = r\psi_I(2k+1,q) + (r-1)\psi_{III}(2k+1,q)+$$

$$+\ \psi(\Lambda_{III})\,,\quad \text{where}\ \psi(\Lambda_{III}) = q^{2k}\,,$$

and

(i")

$$\psi(2k+1,q) = q^{2k+1} - q^{2k} = 2r\psi_{IV}(2k+1,q)\,, \text{ respectively.}$$

3 – Regular or quasiregular points of AG(n,q)

Let $F(x_1, x_2, \ldots, x_n) = 0$ be the equation of a proper hyperquadric Q of $AG(n,q)$ and let $\overline{P}$ be a point not lying on Q whose affine coordinates are $(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)$. A line s of $AG(n,q)$, represented by the equations $x_i = \bar{x}_i + l_i t$ with $i = 1, 2, \ldots, n$ and $t \in \gamma$, is intersecting in two distinct points P_1 and P_2, or tangent at $P_1 \equiv P_2$, or exterior to Q, according as the involution represented by $\beta t^2 + 2\alpha t + \overline{F} = 0$ is hyperbolic, parabolic, or elliptic respectively, that is according as $D = \alpha^2 - \beta\overline{F} \in \square$, $D = 0$, or $D \in \Delta$ respectively. Notice that α denotes what arises by substituting the direction parameters l_i of s to x_i in the equation of the polar hyperplane $\bar{\pi}$ of $\overline{P}$ with respect too Q; obviously $\alpha = 0$ means that is parallel to $\bar{\pi}$. Moreover β denotes what arises by substituting the parameters l_i to x_i in the terms of degree of the equation of Q. Finally $\overline{F}$ denotes what arises by substituting $\bar{x}_i$ to x_i in the equation of Q. In particular, if we denote by c what arises by substituting $\bar{x}_i$ to x_i in the terms of greatest degree of the equations of the proper hyperquadrics of $AG(n,q)$, $\overline{F} = c + 1$ or $\overline{F} = c + \bar{x}_1$ according to the equation; moreover either $\overline{F} \in \square$ or $\overline{F} \in \Delta$ according as the point $\overline{P}$ of $AG(n,q)$ not lying on Q belongs or not to a hyperquadric of the same type as Q.

The point $\overline{P}$ is interior or exterior with respect to the affine segment P_1P_2 of the lines intersecting Q according as $(P_1P_2\overline{P}) = \dfrac{t_1}{t_2} = \dfrac{(\alpha - \sqrt{D})^2}{\beta F} \in \square$ or Δ respectively and hence, as it is known, according as (see [3]).

$$(P_1P_2\overline{P})^r = 1 \tag{3.1}$$

or

$$(P_1P_2\overline{P})^r = -1 \tag{3.2}$$

respectively.

If $q \equiv -1$ *(mod 4),* (3.1) *and* (3.2) *become*

$$(3.1') \quad \frac{(\alpha-\sqrt{D})^{2r}}{(\beta\overline{F})^r} - 1 = \sum_{k=0}^{(r-1)/2} \frac{r(r-1)\dots(r-2k)\alpha^{r-2k-1}D^{\frac{2k+1}{2}}}{(2k+1)!} = 0$$

and

(3.2')

$$\frac{(\alpha-\sqrt{D})^{2r}}{(\beta\overline{F})^r} + 1 = \alpha^r + \sum_{k=1}^{(r-1)/2} \frac{r(r-1)\dots(r-2k+1)\alpha^{r-2k}D^k}{(2k)!} = 0$$

respectively.

Both equations are solved with respect to α^2*; exactly, if* $D = 1$*, the solutions of* (3.1') *are* $\beta\overline{F}$ *and the* $\frac{r-1}{2}$ *elements of* E *and the ones of* (3.2') *are* O *and the* $\frac{r-1}{2}$ *elements of* I*.*

If $q \equiv 1$ *(mod 4),* (3.1) *and* (3.2) *become*

$$(3.1'') \qquad \sum_{k=0}^{(r-2)/2} \frac{r(r-1)\dots(r-2k)\alpha^{r-2k-1}D^{\frac{2k+1}{2}}}{(2k+1)!} = 0$$

and

$$(3.2'') \qquad \alpha^r + \sum_{k=1}^{r/2} \frac{r(r-1)\dots(r-2k+1)\alpha^{r-2k}D^k}{(2k)!} = 0$$

respectively.

Also in this case both equations are solved with respect to α^2 *exactly, if* $D = 1$*, the solutions of* (3.1") *are* $\beta\overline{F}$*,* 0 *and the* $\frac{r-2}{2}$ *elements of* E *and in particular* -1 *is a solution if* $q \equiv 1$ *(mod 8) but the ones of* (3.2") *are the* $\frac{r}{2}$ *elements of* I *and in particular* -1 *is a solution if* $q \equiv 5$ *(mod 8).*

With respect to a cordinate system $R[0,l_i](i=1,2,\dots,n)$ of $AG(n,q)$, the equations $\alpha = 0$ and $\alpha^2 = x$ where $x \in E$ or I, may be considered as equations of a hyperplane through O and a pair hyperplanes which are symmetric with respect to O also $\beta + 1 = 0$ may be considered as equation of an irreducible hyperquadric $\overline{Q}$ of $AG(n, q)$. Exactly the following properties are true.

If $n = 2k$ and $\overline{Q}$ is of type Q_I or Q_{II}, such hyperplanes, represented by the equations $\alpha = 0$ and $\alpha^2 = x$ with $x \in E$ or I, determine on $\overline{Q}$ hyperquadrics of type Q_I of $AG(2k-1, q)$ and of type Q_{III} of $AG(2k+1, q)$ or an irreducible hypercone, according as in $\beta+1 = x$, when $\delta \in \Box$, $x \in \Box$ or $x \in \Delta$ or $x = 0$ respectively, and when $\delta \in \Delta$, $x \in \Delta$ or $x \in \Box$ or $x = 0$ respectively. But, if $\overline{Q}$ is of type Q_{III}, the hyperquadrics determined on $\overline{Q}$ are of type Q_I of $AG(2k-1)$ and of type Q_{III} of $AG(2k+1, q)$, according as in $\beta + 1 = x$, when $\delta \in \Box$, $x \in \Box$ or $x \in \Delta$ respectively and viceversa when $\delta \in \Delta$.

If $n = 2k - 1$ or $n = 2k + 1$ and $\overline{Q}$ is of type Q_I or O_{III}, the previous hyperplanes determine on $\overline{Q}$ hyperquadrics, of type, Q_I and Q_{II} of $AG(2k, q)$ or an irreducible hypercone, according as in $\beta + 1 = x$, when $\delta \in \Box$, $x \in \Box$ or $x \in \Delta$ or $x = 0$ respectively, and, when $\delta \in \Delta$, $x \in \Delta$ or $x \in \Box$ or $x = 0$ respectively. But, if $\overline{Q}$ is of type Q_{II} or Q_{IV}, the hyperquadrics determined on $\overline{Q}$ are of type Q_I or Q_{II} of $AG(2k, q)$ respectively when, in $\beta + 1 = x$, $x \in \Box$ and viceversa when $x \in \Delta$.

In every case among the points of such hyperquadrics there is a symmetry with respect to the point O.

Therefore the number of the lines containing a point $\overline{P}$ of $AG(n, q)$ and intersecting Q in two distinct points P_1 and P_2, such that the point $\overline{P}$ is exterior or interior to the affine segment P_1P_2, may be obtained by the following

$$\beta + 1 = \frac{\alpha^2 - D + \overline{F}}{\overline{F}}, \quad (\overline{F} \neq 0), \tag{3.3}$$

which, if $D = 1$ and $\overline{F} = c + 1$, becomes

$$\beta + 1 = \frac{\alpha^2 + c}{c + 1}, \quad (c + 1 \neq 0). \tag{3.3'}$$

For every value of α^2 in γ the (3.3') preserves or changes the quadratic character of c and in this way it is known the type of the hyperquadrics *determined on $\overline{Q}$ by the hyperplanes whose equations are $\alpha^2 = x$ with $x \in \square$ and $\alpha = 0$.*

In fact $\beta + 1 = x$ with $x \in \square$, or $x \in \Delta$ or $x = 0$, according as $\alpha^2 + c$ and $\overline{F}$ are both elements of $\square$ or Δ or not, or $\alpha^2 + c = 0$.

From (3.3), if $D = 0(\alpha^2 = 1)$, we obtain also the number of the lines through $\overline{P}$ and tangent to Q.

According as $\overline{F} + 1$ and $\overline{F}$ are both elements of either $\square$ or Δ or not or $\overline{F} + 1 = 0$, in $\beta + 1 = x$, either $x \in \square$, or $x \in \Delta$ or $x = 0$, respectively.

In the same way, if $\alpha = 0$, we obtain the number of the lines through the point $\overline{P}$, which intersect the hyperquadric Q in two distinct points P_1 and P_2 such that the segment P_1P_2 contains or not $\overline{P}$ and are parallel to the plar hyperplane of $\overline{P}$ with respect to Q.

In $\beta + 1 = x$, $x \in \square$, or $x \in \Delta$ or $x = 0$, a, according as $\overline{F} - 1$ and $\overline{F}$ are both elements of either $\square$ or Δ or not or $\overline{F} = 1$.

By the subdivision of the lines through a point $\overline{P} \notin Q$ which are tangent or intersecting to Q, according as the point $\overline{P}$ is exterior or interior to the before considered segment P_1P_2, we can give the following definitions of regular or quasiregular point with respect to Q.

A point $\overline{P}$ of $AG(n,q)$ is said to be *regular point with respect to a proper hyperquadric* Q, if it is always exterior or interior to the affine segments determines by Q on the lines through $\overline{P}$ which intersect Q in two distinct points and also if it is exterior to half of such segments and interior to the remaining ones. But the point $\overline{P}$ is said to be *quasiregular point with respect to* Q, if it is exterior to half of the before considered segments and interior to the remaining ones except for the lines through $\overline{P}$ and parallel to the polar hyperplane $\overline{\pi}$ of $\overline{P}$ with respect to Q.

Therefore for the regular or quasiregular points of $AG(n,q)$ with respect to a proper hyperquadric Q the following theorems are true.

I) *If $q \equiv -1$ (mod 4), with respect to a hyperquadric of type Q_I or Q_{II}) of $AG(2k,q)$ only the vertex of the asymptotic hypercone Λ of Q_I (or Q_{II}) is regular; but, if $q \equiv 1$ (mod 4), the points of Λ (whose vertex is regular) and the points ($\notin Q_I$ or Q_{II}) of a pair of hyperquadrics associated to two elements of $\square$ when $q \equiv 1$ (mod 8) or two elements of Δ when $q \equiv 5$ (mod 8) are quasiregular.*

II) *If $q \equiv 1$ (mod 4), with respect to a hyperquadric of type Q_1 of $AG(2k-1,q)$ only the points ($\notin Q_I$) luing on a hyperquadric of type Q_{III} are*

quasiregular. The same property is true with respect to a hyperquadric of type Q_{III} of $AG(2k+1,q)$.
With respect to Q_I the vertex of the asymptotic hypercone Λ is always regular but the other points of it are quasiregula if $q \equiv 1 \pmod 4$ and regular if $q \equiv -1$; with respect to Q_{III} the points of Λ are regular if $q \equiv 1 \pmod 4$ and only if $q = 3$ when $q \equiv -1 \pmod 4$.

III) *With respect to a hyperquadric of type Q_{II} of $AG(2k-1,q)$ the points ($\notin Q_{II}$) lying on $2r$ hyperquadrics of the same type as Q_{II} are quasiregular only if $q \equiv 1 \pmod 4$. The same property is true with respect to a hyperquadric of type Q_{IV} in $AG(2k+1,q)$.*

4 – Particular classes of hyperquadrics

All points of $AG(n,q)$, which are either regular or quasiregular with respect to a proper hyperquadric Q, belong to particular classes $\mathcal{F}$ of hyperquadrics of $AG(n,q)$.

In fact by (3.3) there are the following cases:

a) for $\alpha^2 \in E$ and $\overline{F} \in \Box$ or $\alpha^2 \in I$ and $\overline{F} \in \Delta$, if $\beta \in \Box \cap H$,

$$|\mathcal{F}| = \frac{r-1}{2} \text{ when } q \equiv -1 \pmod 4 \text{ and } |\mathcal{F}| = \frac{r-2}{2} \text{ when } q \equiv 1 \pmod 4; \tag{4.1}$$

b) $\alpha^2 \in I$ and $\overline{F} \in \Box$ or $\alpha^2 \in E$ and $\overline{F} \in \Delta$, if $\beta \in \Delta \cap L$,

$$|\mathcal{F}| = \frac{r-1}{2} \text{ when } q \equiv -1 \pmod 4 \text{ and } |\mathcal{F}| = \frac{r}{2} \text{ when } q \equiv 1 \pmod 4. \tag{4.2}$$

c) for $\alpha^2 \in E$ and $\overline{F} = 1$ hence $\beta + 1 \in E$,

$$|\mathcal{F}| = \frac{r-1}{2} \text{ when } q \equiv -1 \pmod 4 \text{ and } |\mathcal{F}| = \frac{r-2}{2} \text{ when } q \equiv 1 \pmod 4. \tag{4.3}$$

d) for $\alpha^2 \in I$ and $\overline{F} = 1$ and hence $\beta + 1 \in I$,

(4.4)
$$|\mathcal{F}| = \frac{r-1}{2} \text{ when } q \equiv -1 \pmod 4 \text{ and } |\mathcal{F}| = \frac{r}{2} \text{ when } q \equiv 1 \pmod 4 .$$

5 – Regulariry of points as affine invariant

In the projective space $PG(n,q)$, obtained by extending the affine space $AG(n,q)$, any proper hyperquadric and the ideal $(n-1)$-dimensional projective subspace, assumed as double hyperplane, determine a pencil $\mathcal{F}$ of hyperquadrics with the same center C and the same ideal hyperquadric.

For every hyperquadric Q of every pencil $\mathcal{F}$ there is a group G of affine transformations which leave C fixed and Q invariant. Such a group G is generated by the symmetries with respect to the hyperplanes through C which are second to Q; such symmetries correspond to the harmonic homologies, whose fixed points belong to a hyperplane through C which is secant to Q and the center is the pole of such hyperplane with respect to Q. G contains also the symmetry with respect to C, which corresponds to the harmonic homology whose center is C and the fixed points belong to the ideal hyperplane. Since, with respect to Q, the hyperplanes through C are conjugate with the ideal hyperplane, the previous hyperplanar symmetries commute with the central symmetry with respect to C and such a symmetry, beside the identity, belongs to the center of G.

It follows that the symmetries with respect to two conjugate hyperplanes, which are second to Q, commute and the product of the symmetries with respect to three hyperplanes through C and second to Q, which are in a pencil $\mathcal{F}$, is equal to the symmetry with respect to a hyperplane of $\mathcal{F}$.

The restriction to the ideal hyperplane of the polarity Φ defined by every hyperquadric Q of $\mathcal{F}$ is the same polarity φ and hence every hyperplanar symmetry commuting with Φ commutes also with φ. Therefore eery hyperquadric of $\mathcal{F}$ determines a group isomorphic to G. It follows that *for the points of $AG(n,q)$ the property of regularity or quasiregularity is an affine invariant.*

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