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On a New Criterion for Univalent Functions of Order Alpha

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RIASSUNTO – *Si indica con $V_n(A,B,\alpha)$ la classe di funzioni $f(z)=z+\sum_{k=2}^{\infty}a_kz^k$, che siano regolari nel disco unitario $U=\{z:|z|<1\}$ e che verifichino la condizione*

$$\left|\frac{\frac{D^{n+1}f(z)}{z}-1}{[B+(A-B)(1-\alpha)]-B\frac{D^{n+1}f(z)}{z}}\right|<1 \text{ for } z\in U,$$

dove $-1\le B<A\le 1$, $0\le\alpha<1$ e $D^{n+1}f(z)=\frac{z(z^nf(z))^{(n+1)}}{(n+1)!}$. In questo articolo, per mezzo di una relazione di inclusione, si dimostra che le funzioni da $V_n(A,B,\alpha)$ sono univalenti per $z\in U$. Quindi si ottengono operatori che preservano tale classe, stime accurate ed una proprietà di chiusura per tali classi.

ABSTRACT – *Let $V_n(A,B,\alpha)$ be the class of functions $f(z)=z+\sum_{k=2}^{\infty}a_kz^k$ regular in the unit disc $U=\{z:|z|<1\}$ and satisfying the condition*

$$\left|\frac{\frac{D^{n+1}f(z)}{z}-1}{[B+(A-B)(1-\alpha)]-B\frac{D^{n+1}f(z)}{z}}\right|<1 \text{ for } z\in U,$$

where $-1\le B<A\le 1$, $0\le\alpha<1$ and $D^{n+1}f(z)=\frac{z(z^nf(z))^{(n+1)}}{(n+1)!}$. In this paper we show, by an inclusion relation, that the functions from $V_n(A,B,\alpha)$ are univalent for

$z \in U$. Then we obtain a class preserving operators, sharp coefficient estimates and a closure property for those classes.

KEY WORDS – *Regular - Univalent - Hadamard - Coefficient.*

A.M.S. CLASSIFICATION: 30C45 - 30C50

1 – Introduction

Let S be the class of functions $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$ regular in the unit disc $U = \{z: |z| < 1\}$.

If $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$ and $g(z) = z + \sum_{k=2}^{\infty} b_k z^k$ belongs to S, the convolution or Hadamard product of $f(z)$ and $g(z)$ is defined by the power series

$$(f \star g)(z) = z + \sum_{k=2}^{\infty} a_k b_k z^k, \quad z \in U.$$

Let $n \in N_0 = \{1,2,\ldots\}$. The nth order RUSCHEWEYH derivative [1] of $f(z)$, denoted by $D^n f(z)$, is defined by

$$D^n f(z) = \frac{z\,(z^{n-1} f(z))^{(n)}}{n!}.$$

RUSCHEWEYH [10] determined that

$$D^n f(z) = \frac{z}{(1-z)^{n+1}} \star f(z).$$

In [5] GOEL and SOHI have studied the class of those functions of S for which

$$\operatorname{Re} \frac{D^{n+1} f(z)}{z} > \rho, \; 0 \le \rho < 1, \; z \in u.$$

In the present paper we introduce a more general class, namely $V_n(A, B, \alpha)$.

A function $f(z)$ of S belongs to the class $V_n(A, B, \alpha)$ if and only if there exists a function $w(z)$ regular in U and satisfying $w(0) = 0$ and $|w(z)| < 1$ for $z \in U$ such that

$$(1.1) \qquad \frac{D^{n+1} f(z)}{z} = \frac{1 + [B + (A-B)(1-\alpha)]w(z)}{1 + Bw(z)}, \quad z \in U$$

where $-1 \leq B < A \leq 1$ and $0 \leq \alpha < 1$. It is easy to see that the condition (1.1) is equivalent to

$$(1.2) \qquad \left| \frac{\frac{D^{n+1}f(z)}{z} - 1}{[B + (A - B)(1 - \alpha)] - B\frac{D^{n+1}f(z)}{z}} \right| < 1, \quad z \in U.$$

We note that $V_n(A, B, 0) = V_n(A, B)$, is the class of functions $f(z) \in S$, studied by KUMAR [8].

In our first theorem we obtain the basic inclusion relation $V_{n+1}(A, B, \alpha) \subset V_n(A, B, \alpha)$. Since $f(z) \in V_0(A, B, \alpha)$ implies $\operatorname{Re} f'(z) > \alpha$, $0 \leq \alpha < 1$, it follows (cf. [11] p. 6) that the functions of $V_n(A, B, \alpha)$ are univalent in U. Then we obtain class preserving integral operators and sharp coefficient estimates for these classes. We also obtain a sufficient condition in terms of coefficients for a function to be in $V_n(A, B, \alpha)$, when $-1 \leq B < 0$ and we show that the converse of the same need not be true.

Our results generalize many results of CHEN [3], GOEL and SOHI [4], [5], JUNEJA and MOGRA [7] and KUMAR [8].

2 – Preliminary lemmas

LEMMA 2.1. *A function $f(z)$ belongs to $V_n(A, B, \alpha)$, $-1 \leq B < A \leq 1$ and $0 \leq \alpha < 1$, if and only if*

$$(2.1) \qquad \left| \frac{D^{n+1}f(z)}{z} - m \right| < M, \quad z \in U,$$

where

$$(2.2) \qquad m = \frac{1 - [B + (A - B)(1 - \alpha)]B}{1 - B^2} \text{ and } M = \frac{(A - B)(1 - \alpha)}{1 - B^2}.$$

PROOF. First suppose that $f(z) \in V_n(A,B,\alpha)$. Then, by (2.1) and (2.2), we have

$$\frac{D^{n+1}f(z)}{z}-m=\frac{(1-m)+([B+(A-B)(1-\alpha)]-Bm)w(z)}{1+Bw(z)}= \tag{2.3}$$

$$=M\frac{B+w(z)}{1+Bw(z)}=Mh(z).$$

It is clear that the function $h(z)$ satisfies $|h(z)|<1$. Hence (2.1) follows from (2.3).

Conversely, suppose that the condition (2.1) holds.
Then we have

$$\left|\frac{D^{n+1}f(z)}{Mz}-\frac{m}{M}\right|<1.$$

Let

$$g(z)=\frac{D^{n+1}f(z)}{Mz}-\frac{m}{M},$$

then, by (2.3),

$$w(z)=\frac{g(z)-g(0)}{1-g(0)g(z)}=\frac{\frac{D^{n+1}f(z)}{z}-1}{[B+(A-B)(1-\alpha)]-B\frac{D^{n+1}f(z)}{z}}. \tag{2.4}$$

Clearly $w(0)=0$ and $|w(z)|<1$. Rearanging (2.4) we arrive at (1.1). Hence $f(z) \in V_n(A,B,\alpha)$.

NOTE. (i) The condition (2.1) can also be written as

$$\left|\frac{\frac{D^{n+1}f(z)}{z}-\frac{1-[B+(A-B)(1-\alpha)]}{1-B}}{1-\frac{1-[B+(A-B)(1-\alpha)]}{1-B}}-\frac{1}{1+B}\right|<\frac{1}{1+B},\ z\in U.$$

Now as $B \longrightarrow -1$, the above condition reduces to

$$\operatorname{Re}\frac{D^{n+1}f(z)}{z}>\rho,\ \rho=\frac{1-A+\alpha(A+1)}{2},\ z\in U,$$

which is precisely the necessary and sufficient condition for $f(z) \in V_n(A,-1,\alpha)$. Thus, including the limiting case $B \longrightarrow -1$, the results proved with the help of above lemma will hold for $-1 \leq B < A \leq 1$ and $0 \leq \alpha < 1$.

The following lemma is due to JACK [6].

LEMMA 2.2. *If the function $w(z)$ is regular for*

$$|z| \leq r < 1\,, w(0) = 0 \text{ and } |w(z_0)| = \max_{|z|=r} |w(z)|\,,$$

then

$$z_0 w'(z_0) = k w(z_0)\,,$$

where k is a real number such that $k \geq 1$.

3 – Main results

THEOREM 3.1. *Let n_0 be any integer such that $n_0 > n$. Then*

$$V_{n_0}(A,B,\alpha) \subset V_n(A,B,\alpha)\,.$$

PROOF. In order to establish the required result it suffices to show that $V_{n+1}(A,B,\alpha) \subset V_n(A,B,\alpha)$. Let $f(z) \in V_{n+1}(A,B,\alpha)$. Choose a function $w(z)$ such that

$$\frac{D^{n+1}f(z)}{z} = \frac{1+[B+(A-B)(1-\alpha)]w(z)}{1+Bw(z)}\,, \tag{3.1}$$

where $w(0) = 0$ and $w(z)$ is either regular or meromorphic in U. It is easy to verify that

$$z(D^{n+1}f(z))' = (n+2)D^{n+2}f(z) - (n+1)D^{n+1}f(z)\,. \tag{3.2}$$

Differentiating (3.1) and using (3.2) we get

$$\begin{aligned} \frac{D^{n+2}f(z)}{z} - m &= \frac{(1-m)+([B+(A-B)(1-\alpha)]-Bm)w(z)}{1+Bw(z)} = \\ &= \frac{(A-B)(1-\alpha)}{n+2}\frac{zw'(z)}{[1+Bw(z)]^2}\,. \end{aligned} \tag{3.3}$$

Let r^* be the distance from the origin to the pole of $w(z)$ nearest the origin. Then $w(z)$ is regular in the disc $|z| < r_0 = \min(r^*, 1)$. By Lemma 2.2, for $|z| \leq r(r < r_0)$, there is a point z_0 such that

$$z_0 w'(z_0) = kw(z_0)\,, \quad k \geq 1\,. \tag{3.4}$$

From (3.3) and (3.4) we have

$$\frac{D^{n+2}f(z)}{z_0} - m = \frac{N(z_0)}{R(z_0)}\,, \tag{3.5}$$

where

$$\begin{aligned} N(z_0) =&(1-m)(n+2) + [(n+2)([B+(A-B)(1-\alpha)] - Bm) + \\ &+ B(n+2)(1-m) + k(A-B)(1-\alpha)]w(z_0) + \\ &+ B(n+2)([B+(A-B)(1-\alpha)] - Bm)w^2(z_0) \end{aligned}$$

and

$$R(z_0) = (n+2)[1 + 2Bw(z_0) + B^2 w^2(z_0)]\,.$$

Now suppose it was possible to have $M(r, w) = \max_{|z|=r} |w(z)| = 1$ for some $r < r_0 \leq 1$. At the point z_0, where this occurs, we would have $|w(z_0)| = 1$. Then, by using the identities $1 - m = BM$ and $[B + (A - B)(1-\alpha)] - Bm = M$ (cf. (2.2)), we have

$$|N(z_0)|^2 - M^2 |R(z_0)|^2 = a + 2b \operatorname{Re} w(z_0)\,, \tag{3.6}$$

where $a = k(A-B)(1-\alpha)[k(A-B)(1-\alpha) + 2M(n+2)(1+B^2)]$ and $b = 2k(A-B)(1-\alpha)MB(n+2)$.

From (3.6) we have

$$|N(z_0)|^2 - M^2 |R(z_0)|^2 > 0\,, \text{ provided } a \pm 2b > 0\,.$$

Now, in view of the fact $(A-B)(1-\alpha) > 0$, it follows that

$$a + 2b = k(A-B)(1-\alpha)[k(A-B)(1-\alpha) + 2M(n+2)(1+B)^2] > 0\,,$$

$$a - 2b = k(A-B)(1-\alpha)[k(A-B)(1-\alpha) + 2M(n+2)(1-B)^2] > 0\,.$$

Thus, from (3.5) and (3.7) we get

$$\left|\frac{D^{n+1}f(z_0)}{z_0} - m\right| > M .$$

But this is contrary to (2.1). So we can not have $M(r, w) = 1$. Thus $|w(z)| \neq 1$ in $|z| < r_0$. Since $w(0) = 0$, $|w(z)|$ is continuous and $|w(z)| \neq 1$ in the disc $|z| < r_0$, the function $w(z)$ can not have a pole at $|z| = r_0$. Therefore $w(z)$ is regular in U and satisfies $|w(z)| < 1$ for $z \in U$. Hence, from (3.2), $f(z) \in V_n(A, B, \alpha)$.

REMARK. When $A = 1$ and $B \longrightarrow -1$, a result of GOEL and SOHI [5] follows Theorem 3.1.

In our next theorem we study the class preserving integral operators for the classes $V_n(A, B, \alpha)$.

THEOREM 3.2. *Let γ be a real number such that $\gamma > -1$. If $f(z) \in V_n(A, B, \alpha)$, then the function $F(z)$ defined by*

$$F(z) = \frac{\gamma + 1}{z^\gamma} \int_0^z t^{\gamma-1} f(t) dt \tag{3.8}$$

also belongs to $V_n(A, B, \alpha)$.

PROOF. From (3.8) it is easy to verify that

$$z(D^{n+1}f(z))' = (\gamma + 1)D^{n+1}f(z) - \gamma D^{n+1}F(z) . \tag{3.9}$$

Suppose that

$$\frac{D^{n+1}F(z)}{z} = \frac{1 + [B + (A - B)(1 - \alpha)]w(z)}{1 + Bw(z)} , \tag{3.10}$$

where the function $w(z)$ is either regular or meromorphic in U and satisfies $w(0) = 0$.

Differentiating (3.10) and using the identity (3.9) we get

$$\begin{aligned}\frac{D^{n+1}f(z)}{z}-m=&\frac{(1-m)+([B+(A-B)(1-\alpha)]-Bm)}{1+Bw(z)}+\\&+\frac{(A-B)(1-\alpha)}{\gamma+1}\frac{zw'(z)}{[1+Bw(z)]^2}.\end{aligned}\tag{3.11}$$

The required result can be obtained now from (3.11) by using the same technique as applied in (3.3) in the proof of Theorem 3.1.

REMARKS.
(1) When $\alpha=0$, a result of KUMAR [8] follows from Theorem 3.2.
(2) For $n=0$, $\alpha=0$, $A=1$ and $B\longrightarrow-1$, Theorem 3.2 improves a result of BERNARDI [2], who proved it when γ is a positive integer.

In the following theorem we obtain sharp coefficient estimates for the classes $V_n(A,B,\alpha)$.

THEOREM 3.3. *Let* $f(z)=z+\sum_{k=2}^{\infty}a_kz^k$. *If* $f(z)\in V_n(A,B,\alpha)$, *then*

$$|a_k|\le\frac{(A-B)(1-\alpha)}{\delta(n,k)},\quad K=2,3,\dots,\tag{3.12}$$

where $\delta(n,k)=\binom{n+k}{n+1}$. *The result is sharp.*

PROOF. Since $f(z)\in V_n(A,B,\alpha)$, we have

$$\frac{D^{n+1}f(z)}{z}=\frac{1+[B+(A-B)(1-\alpha)]w(z)}{1+Bw(z)},$$

where $w(z)=\sum_{j=1}^{\infty}t_jz^j$ is regular in U, satisfies $w(0)=0$ and $|w(z)|<1$ for $z\in U$. Hence

$$\frac{D^{n+1}f(z)}{z}-1=\left[[B+(A-B)(1-\alpha)]-B\frac{D^{n+1}f(z)}{z}\right]w(z)$$

or

(3.13)

$$\sum_{j=2}^{\infty}\delta(n,j)a_j z^{j-1}=\left[(A-B)(1-\alpha)-B\sum_{j=2}^{\infty}\delta(n,j)a_j z^{j-1}\right]\times\left[\sum_{j=1}^{\infty}t_j z^j\right].$$

Equating corresponding coefficients on both sides of (3.13), we find that the coefficient a_k on the left-hand side of (3.13) depends only on $a_2, a_3, \ldots, a_{k-1}$ on the right-hand side of (3.13). Hence, for $k \geq 2$, it follows from (3.13) that

$$\sum_{j=2}^{\infty}\delta(n,j)a_j z^{j-1}+\sum_{j=k+1}^{\infty}c_j z^{j-1}=$$

$$=\left[(A-B)(1-\alpha)-B\sum_{j=2}^{k-1}\delta(n,j)a_j z^{j-1}\right]w(z),$$

where c_j are some complex numbers. Since $|w(z)|<1$, by using PARSEVAL's identity [9], we get

$$\sum_{j=2}^{k}(\delta(n,j))^2|a_j|^2r^{2(j-1)}+\sum_{j=k+1}^{\infty}|c_j|^2r^{2(j-1)}\leq$$

$$\leq(A-B)^2(1-\alpha)^2+B^2\sum_{j=2}^{k-1}(\delta(n,j))^2|a_j|^2r^{2(j-1)}\leq$$

$$\leq(A-B)^2(1-\alpha)^2+B^2\sum_{j=2}^{k-1}(\delta(n,j))^2|a_j|^2.$$

Letting $r\longrightarrow 1$ on the left-hand side of the above inequality we obtain

$$\sum_{j=2}^{k}(\delta(n,j))^2|a_j|^2\leq(A-B)^2(1-\alpha)^2+B^2\sum_{j=1}^{k-1}(\delta(n,j))^2|a_j|^2.$$

Thus

$$(\delta(n,k))^2|a_k|^2\leq(A-B)^2(1-\alpha)^2-(1-B^2)\sum_{j=2}^{k-1}(\delta(n,j))^2|a_j|^2\leq$$

$$\leq(A-B)^2(1-\alpha)^2.$$

Hence $|a_k| \leq \frac{(A-B)(1-\alpha)}{\delta(n,k)}$.

In order to establish the sharpness we consider the function

$$\frac{D^{n+1}f(z)}{z} = \frac{1+[B+(A-B)(1-\alpha)]z^{k-1}}{1+Bz^{k-1}}, \quad k = 2,3,\dots .$$

Clearly, $f(z) \in V_n(A,B,\alpha)$. It is easty to compute that the function $f(z)$ has the expansion

$$f(z) = z + \frac{(A-B)(1-\alpha)}{\delta(n,k)} z^k + \dots$$

showing that the estimate (3.12) is sharp.

REMARK. Assigning specific values to A, B, α and n, some results of CHEN [3], GOEL and SOHI [4], JUNEJA and MOGRA [7] and KUMAR [8] follow from Theorem 3.3.

Now we obtain a sufficient condition, in terms of coefficients, for a function to be in $V_n(A,B,\alpha)$ when $-1 \leq B < 0$.

THEOREM 3.4. *Let* $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$ *be regular in* U. *If, for* $-1 \leq B < 0$,

$$\sum_{k=2}^{\infty}(1-B)\delta(n,k)|a_k| \leq (A-B)(1-\alpha), \tag{3.14}$$

where $\delta(n,k) = \binom{n+k}{n+1}$, *then* $f(z) \in V_n(A,B,\alpha)$. *The result is sharp. Although the converse need not be true.*

PROOF. Suppose that (3.14) holds. Then, for $z \in U$, we have

$$\left|\frac{D^{n+1}f(z)}{z}-1\right|-\left|[B+(A-B)(1-\alpha)]-B\frac{D^{n+1}f(z)}{z}\right|=$$

$$=\left|\sum_{k=2}^{\infty}\delta(n,k)a_k z^{k-1}\right|-\left|(A-B)(1-\alpha)+B\sum_{n=2}^{\infty}\delta(n,k)a_k z^{k-1}\right|\leq$$

$$\leq\sum_{k=2}^{\infty}\delta(n,k)|a_k|r^{k-1}-[(A-B)(1-\alpha)+B\sum_{k=2}^{\infty}\delta(n,k)|a_k|r^{k-1}]<$$

$$<\sum_{k=2}^{\infty}\delta(n,k)|a_k|-(A-B)(1-\alpha)-B\sum_{k=2}^{\infty}\delta(n,k)|a_k|=$$

$$=\sum_{k=2}^{\infty}\delta(n,k)(1-B)|a_k|-(A-B)(1-\alpha)\leq 0.$$

Hence it follows that

$$\left|\frac{\frac{D^{n+1}f(z)}{z}-1}{[B+(A-B)(1-\alpha)]-B\frac{D^{n+1}f(z)}{z}}\right|<1, \quad z\in U.$$

Therefore $f(z)\in V_n(A,B,\alpha)$. We note that

$$f(z)=z-\frac{(A-B)(1-\alpha)}{\delta(n,k)(1-B)}z^k, \quad k=2,3,\dots,$$

is an extremal function with respect to the above theorem, since for this function

$$\left|\frac{\frac{D^{n+1}f(z)}{z}-1}{[B+(A-B)(1-\alpha)]-B\frac{D^{n+1}f(z)}{z}}\right|=1, \text{ for } |z|=1,$$

and the equality is attained in (3.14).

In order to show that the converse need not be true, we consider the function $f(z)=z+\sum_{k=2}^{\infty}a_k z^k$ defined by

$$\frac{D^{n+1}f(z)}{z}=\frac{1+[B+(A-B)(1-\alpha)]z}{1+Bz}, \; -1\leq B<0, \; z\in U.$$

Then, it is easty to verify that $a_k = \dfrac{(A-B)(1-\alpha)(-B)^{k-2}}{\delta(n,k)}$. But

$$\sum_{k=2}^{\infty}(1-B)\delta(n,k)|a_k| = (A-B)(1-\alpha)\sum_{k=2}^{\infty}(1-B)(-B)^{k-2} >$$

$$> (A-B)(1-\alpha).$$

Hence the converse need not be true.

Lastly we establish a closure property for $V_n(A,B,\alpha)$.

THEOREM 3.5. *If the functions $f(z)$ and $g(z)$ belong to $V_n(A,B,\alpha)$ and $0 \leq \lambda \leq 1$, then the function $F(z)$ given by*

$$F(z) = \lambda f(z) + (1-\lambda)g(z)$$

also belongs to $V_n(A,B,\alpha)$.

PROOF. Since $f(z)$, $g(z) \in V_n(A,B,\alpha)$, by Lemma 2.1, we have

$$\left|\frac{D^{n+1}f(z)}{z} - m\right| < M \text{ and } \left|\frac{D^{n+1}g(z)}{z} - m\right| < M\,,\ z \in U\,, (\text{cf.}(2.2))\,.$$

Therefore

$$\left|\frac{D^{n+1}F(z)}{z} - m\right| = \left|\frac{\lambda D^{n+1}f(z) + (1-\lambda)D^{n+1}g(z)}{z} - m\right| =$$

$$= \left|\lambda\left[\frac{D^{n+1}f(z)}{z} - m\right] + (1-\lambda)\left[\frac{D^{n+1}g(z)}{z} - m\right]\right| \leq$$

$$\leq \lambda\left|\frac{D^{n+1}f(z)}{z} - m\right| + (1-\lambda)\left|\frac{D^{n+1}g(z)}{z} - m\right| <$$

$$< \lambda M + (1-\lambda)M = M\,.$$

Hence $F(z) \in V_n(A,B,\alpha)$.

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