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The Spectrum of the p-Laplacian on Kähler manifold

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RIASSUNTO – *Si studiano alcune proprietà spettrali del p-Laplaciano su una varietà Kähleriana compatta.*

ABSTRACT – *In this paper we study some spectral properties of the p-Laplacian on compact Kähler manifolds.*

KEY WORDS – *Kähler manifolds - p-Laplacian - Spectrum.*

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1 – Introduction

Let (M, J, g) be an n-dimensional Kähler manifold (all manifolds are assumed to be compact, connected and of complex dimension $n \geq 1$) with complex structure J and Kähler metric g. By Δ^p we denote the Laplacian acting on p-forms on M. Then we have the spectrum for each p:

$$\mathrm{Spec}^p(M,g) = \{0 \leq \lambda_{i,p} \leq \lambda_{2,p} \leq \ldots \longrightarrow +\infty\}.$$

where each eigenvalue is repeated as many times as its multiplicity indicates. It is well known that $\mathrm{Spec}^p(M,g) = \mathrm{Spec}^{2n-p}(M,g)$ and immediately from Hodge theory that $0 \in \mathrm{Spec}^p(M,g)$ if and only if the p-th Betti number $\beta_p \neq 0$ and 0 has multiplicity $\beta_p \neq 0$.

An interesting problem on the spectrum is as follows: "Let (M, J, g) and (M', J', g') be compact Kähler manifolds with $\mathrm{Spec}^p(M, g) = \mathrm{Spec}^p(M', g')$ for a fixed but arbitrary p. Then: is it true that (M, J, g) is of constant holomorphic sectional curvature h if and only if (M', J', g') is of constant holomorphic sectional curvature h' and $h = h'$?".

The answer to the problem is affirmative for $p = 0, 1, 2, \ldots, 6$ and some particular values of n which are determined by a technical argument which is used in all the proofs, namely the condition that some polynomials should be strictly positive, [3], [4], [6]-[8].

In this paper we shall prove that this argument can be extended for each $p \in \mathbb{N}$, $p \geq 1$ and we shall determine most of the values of n for which it applies. To obtain the exact values of n one can use the computer. Moreover, the method which we have used is applicable also to the real case, see [5].

2 – Preliminaries

Let M be a compact Kähler manifold of complex dimension n. If $(\theta^1, \ldots, \theta^n)$ form a local field of unitary coframes, the Kähler metric g and the fundamental 2-form $\emptyset$ are respectively given by:

$$g = \frac{1}{2} \sum (\theta^i \otimes \overline{\theta}^i + \overline{\theta}^i \otimes \theta^i),$$

$$\emptyset = \frac{\sqrt{-1}}{2} \sum \theta^i \wedge \overline{\theta}^i .$$

Let $\Omega^i_j = \sum R^i_{jk1} \theta^k \wedge \overline{\theta}^l$, be the curvature form of M. Then the curvature tensor R is the sensor field with local components R^i_{jkl}. The Ricci tensor $E = (E_i)$ and the scalar curvature τ are given by:

$$E = \frac{1}{2} \sum (R_{ij} \theta^i \wedge \overline{\theta}^j + \overline{R}_{ij} \overline{\theta}^i \wedge \theta^j)$$

$$\tau = 2 \sum R_{ii} .$$

where

$$R_i = 2 \sum R^k_{ikj} .$$

We denote by $|R|$ and $|E|$ the length of R and E respectively. Then for each $p \in \mathbb{N}$, $p \leq 2n$, the Minakshisundarm - Pleijel - Gaffney' formula is given by:

$$\sum_k \exp(-\lambda_{k,p}t) = (4\pi t)^{-n}\left[a_{0,p} + ta_{1,p} + \ldots + t^N a_{N,p}\right] + + 0(t^{N-n+1}), \tag{2.1}$$

as $t \searrow 0$, where we have (see [2]):

$$a_{0,p} = \binom{2n}{p}\int_M dM\,; \tag{2.2}$$

$$a_{1,p} = \frac{1}{6}\left[\binom{2n}{p} - 6\binom{2n-2}{p-1}\right]\int_m \tau dM\,; \tag{2.3}$$

$$\begin{aligned} a_{2,p} = \frac{1}{360}\int_m \Big\{ &\left[5\binom{2n}{p} - 60\binom{2n-2}{p-1} + 180\binom{2n-4}{p-2}\right]\tau^2 + \\ &+ \left[-2\binom{2n}{p} + 180\binom{2n-2}{p-1} - 720\binom{2n-4}{p-2}\right]|E|^2 + \\ &+ \left[2\binom{2n}{p} - 30\binom{2n-2}{p-1} + 180\binom{2n-4}{p-2}\right]|R|^2\Big\} dM\,. \end{aligned} \tag{2.4}$$

Let G and B be the Einstein tensor field and the Bochner curvature tensor field. Their components (G_{ij}), (B_{ijkl}) are respectively given by (see [7]):

$$G_{ij} = E_{ij} - \frac{1}{2n} g_{ij}\tau\,; \tag{2.5}$$

$$
\begin{aligned}
(2.6)\qquad B_{ijkl} = R_{ijkl} - \frac{1}{2n+4}(&E_{jk}g_{il} - E_{jl}g_{ik} + \\
&+ E_{il}g_{jk} - E_{ik}g_{jl} + E_{jr}J_k^r J_{il} - E_{jr}J_l^r J_{ik} + \\
&+ J_{jk}E_{ir}J_l^r - J_{jl}E_{ir}J_k^r - 2E_{kr}J_l^r J_{ij} - \\
&- 2E_{ir}J_j^r J_{kl}) + \frac{1}{(2n+2)(2n+4)}\Big[g_{jk}g_{il} - g_{jl}g_{ik} + \\
&+ J_{jk}J_{il} - J_{jl}J_{ik} - 2J_{kl}J_{ij}\Big]\tau .
\end{aligned}
$$

Then we have:

$$(2.7)\qquad |G|^2 = |E|^2 - \frac{1}{2n}\tau^2 ;$$

$$(2.8)\qquad |B|^2 = |R|^2 - \frac{16}{2n+4}|E|^2 + \frac{8}{(2n+2)(2n+4)}\tau^2 .$$

On the other hand a straightforward computation shows us that for $p \notin \{1,2,3,2n-1,2n\}$:

$$(2.9)\qquad \binom{2n}{p} = \frac{2n(2n-1)(2n-2)(2n-3)}{p(p-1)(2n-p)(2n-p-1)}\binom{2n-4}{p-2}$$

$$(2.10)\qquad \binom{2n-2}{p-1} = \frac{(2n-2)(2n-3)}{(p-1)(2n-p-1)}\binom{2n-4}{p-2}$$

and then the formula (2.4), by means of (2.7), (2.8), (2.9) and (2.10), takes the form:

$$(2.11)\qquad a_{2,p} = c\int_M \left\{4\alpha|B|^2 + \frac{8}{n+2}\beta|G|^2 + \frac{4}{n(n+1)}\gamma\tau^2\right\} dM ,$$

where:

$$c = \frac{\binom{2n-4}{p-2}}{360p(p-1)(2n-p)(2n-p-1)}$$

and

$$\alpha = 8n^4 - n^3(60p+24) + n^2(210p^2 - 30p + 22) - \\ - n(180p^3 - 15p^2 + 6) + 45p^4; \tag{2.12}$$

$$\beta = -4n^5 + n^4(180p+36) + n^3(-450p^2 + 30p - 83) + \\ + n^2(360p^3 - 15p^2 - 210p + 69) + \\ + n(-90p^4 + 105p^2 + 180p - 18) - 90p^2; \tag{2.13}$$

$$\gamma = 20n^6 - n^5(120p+44) + n^4(240p^2 + 180p + 19) - \\ - n^3(180p^3 + 270p^2 + 7) + n^2(45p^4 + 180p^3 - 150p + 21) - \\ - n(45p^4 - 75p^2 - 90p + 9) - 45p^2. \tag{2.14}$$

If p takes one of the values $1,2,3$, $2n-1$, $2n$ then the formula (2.4) becomes:

$$a_{2,p} = c^0 \int_M \left\{ 4\alpha^0 |B|^2 + \frac{8}{n+2}\beta^0 |G|^2 + \right. \\ \left. + \frac{4}{n(n+1)}\gamma^0 \tau^2 \right\} dM \tag{2.15}$$

where:

(i) if $p = 1$ and $n \geq 1$, then

$$c^0 = \frac{1}{360}, \qquad \xi^0 = \frac{\xi\big|_{p=1}}{(2n-1)(2n-2)(2n-3)};$$

(ii) if $p = 2$ and $n \geq 2$, then

$$c^0 = \frac{1}{2 \cdot 360}, \qquad \xi^0 = \frac{\xi\big|_{p=2}}{(2n-2)(2n-3)};$$

while for $p = 2$, $n = 1$

$$c^0 = \frac{1}{2 \cdot 360}, \qquad \alpha^0 = 1, \quad \beta^0 = \frac{5}{2}, \quad \gamma^0 = 6;$$

(iii) if $p = 3$ and $n \geq 2$, then

$$c^0 = \frac{1}{6 \cdot 360}, \qquad \xi^0 = \frac{\xi\big|_{p=3}}{2n-3};$$

(iv) if $p = 2n-1$ and $n \geq 2$, then

$$c^0 = \frac{1}{360}, \qquad \xi^0 = \frac{\xi\big|_{p=2n-1}}{(2n-1)(2n-2)(2n-3)};$$

(v) if $p = 2n$ and $n \geq 2$, then

$$c^0 = \frac{1}{360}, \qquad \xi^0 = \frac{\xi\big|_{p=2n}}{2n(2n-1)(2n-2)(2n-3)}.$$

Here ξ stands for any of the symbols α, β and γ and $\xi\big|_{p=p_0}$ means that in the expression of ξ we take $p = p_0$. Note that α^0, β^0, γ^0 are also polynomials of n in all of the above cases (i)-(v), excepting the situation $p = 2$, $n = 1$.

REMARK 2.1. We conclude that if $(n,p) \neq (1,2)$, the sign of the coefficients of $\int\limits_M |B|^2 dM$, $\int\limits_M |G|^2 dM$, $\int\limits_M \tau^2 dM$ in the formulae (2.11) and (2.15) can be obtained by studying the polynomials α, β and γ. We see that whenever one or the polynomials α, β or γ is non zero for some given $n, p \in \mathbb{N}$, $p \leq 2n$, $(n,p) \neq (1,2)$, it has the same sign as the corresponding before mentioned coefficient even for the cases $p = 1, 2, 3, 2n-1$, or $2n$.

To exploit this, from now on we shall write (2.11) and (2.15) as:

$$\text{(2.16)} \qquad \begin{aligned} a_{2,p} = &\tilde{c} \int\limits_M \Big\{ 4\tilde{\alpha}|B|^2 + \frac{8}{n+2}\tilde{\beta}|G|^2 + \\ &+ \frac{4}{n(n+1)}\tilde{\gamma}\tau^2 \Big\} dM, \end{aligned}$$

where $\tilde{c}, \tilde{\alpha}, \tilde{\beta}$ and $\tilde{\gamma}$ have the appropriate expressions.

3 – Some remarks on the functions $\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}$

In this section we shall present some properties of the functions $\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}$ which will be essential in the following considerations.

THEOREM 3.1. *Let $p, n \in \mathbb{N}^*$, $p \leq 2n$, let α, β, γ be the polynomials given by* (2.12)-(2.14) *and let $\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}$ be the functions introduced by* (2.16) *in Remark 2.1.*

(i) *The function $\tilde{\alpha}(\cdot, p)$ is strictly positive for $p \geq 22$ and for $1 \leq p \leq 21$ it has only the following integer roots:*

$$n = 8 \quad \textit{for} \quad p = 2;$$

$$n = 8 \quad \textit{for} \quad p = 14.$$

(ii) *If $p \geq 100$ and $n \in [0.51p; 0.6198p] \cup [0.6454p; 0.82225p] \cup [1.2755p; 2.2799p] \cup [2.4669p; 42.396p]$, then the polynomials α, β, γ (and the functions $\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}$) are strictly positive.*

(iii)

$$\binom{2n}{p} - 6\binom{2n-2}{p-1} = 0$$

$$\textit{iff } n(2n-1) - 3p(2n-p) = 0$$

$$\textit{iff } n = \frac{u-1}{2}, \quad p = \frac{u-1}{2} \pm v,$$

where u, v are natural roots of the Pell equation $u^2 - 12v^2 = 1$.

The least solutions are $u_0 = 7$, $v_0 = 2$, $u_1 = 97$, $v_1 = 28$, $u_2 = 1351$, $v_2 = 390$, which give:

$$(n; p) \in \{(3;1), (3;5), (48;20), (48;76), (675;285), (675;1065)\}$$

(iv) *if $\binom{2n}{p} - 6\binom{2n-2}{p-1} = 0$, then $\tilde{\gamma}(n, p) < 0$.*

The results of (ii) can be slightly improved if we consider for p a greater lower bound, see the Remark 3.2.

REMARK 3.1. If $p < 100$ the roots of the polynomials α, β and γ can be determined using some numerical methods [1] and the computer. In the Table 1 there are some results.

Table 1

p	the values of n such that $\bar{\alpha}, \bar{\beta}, \bar{\gamma} > 0$			
1	—	—	—	[8,51]
2	1	3,4,7	—	[9,94]
3	—	—	[4,6]	[10,136]
4	2	3	[5,8]	[12,179]
5	—	4	[6,10]	[15,221]
6	3	4,5	[7,12]	[18,264]
7	—	5,6	[8,14]	[20,306]
8	4	6,7	[9,17]	[23,348]
9	—	[6,8]	[10,19]	[25,391]
10	5	[7,9]	[12,21]	[28,433]
20	11,12	[14,17]	[24,43]	[53,857]
40	[21,24]	[26,34]	[50,89]	[102,1705]
60	[31,36]	[39,50]	[75,135]	[150,2553]
80	[41,49]	[52,66]	[101,181]	[199,3401]
100	[51,61]	[65,83]	[126,227]	[247,4249]

We obtain all the values found in [3], [4], [6]-[8], with the exception of $n = 9$ for $p = 4$ (probably a miscalculation in [6]. For $p = 3,4,5,6$ we have found some new values too.

PROOF OF THEOREM 3.1. By remark 2.1., for $(n,p) \neq (1,2)$ it is enough to work with the polynomials α, β, γ. To simplify the calculation we shall take $m = 2n$ and then we have:

$$\alpha(n,p) = \frac{1}{4}\alpha'(m,p)$$

$$\beta(n,p) = \frac{1}{8}\beta'(m,p)$$

$$\gamma(n,p) = \frac{1}{16}(m-2)\gamma'(m,p)$$

where:

$$\alpha'(m,p) = 2m^4 - m^3(30p + 12) + m^2(210p^2 - 30p + 22)-$$
$$- m(360p^3 - 30p^2 + 12) + 180p^4;$$
$$\beta'(m,p) = -m^5 + m^4(90p + 18) - m^3(450p^2 - 30p + 83)+$$
$$+ m^2(720p^3 - 30p^2 - 420p + 138)-$$
$$- m(360p^4 - 420p^2 - 720p + 72) - 720p^2;$$
$$\gamma'(m,p) = 5m^5 - m^4(60p + 12) + m^3(240p^2 + 60p - 5)-$$
$$- m^2(360p^3 + 60p^2 - 120p + 24)+$$
$$+ m(180p^4 - 120p^2 - 360p + 36) + 360p^2$$

Now (i) and (ii) follow from the claims below and by the determination of the roots of $\tilde{\alpha}(\cdot,p)$ for $1 \le p \le 21$ with a computer.

CLAIM 1. For each $x \overset{\text{def}}{=} 2n$ and $p \ge 22$ we have $\alpha'(x,p) > 0$.
Making the substitution $x = p(z + 1)$ we obtain:

$$\alpha'(x,p) = p^4(2z^4 - 22z^3 + 132z^2 - 22z + 2)-$$
$$- p^3(12z^3 + 66z^2 + 66z + 12)+$$
$$+ 22p^2(z^2 + 2z + 1) - p(12z + 12).$$

On the other hand:

$$(3.1) \qquad 2z^4 - 22z^3 + 132z^2 - 22z + 2 = (z^2+1)^2 + (z^2 - 11z + 1)^2 + 7z^2 \ge$$
$$\ge (z^2 + 1)^2 + 7z^2 = z^4 + 9z^2 + 1$$

and

$$(3.2) \qquad p^4 \ge 22p^3, \qquad \text{if} \quad p \ge 22.$$

Using the relations (3.1) and (3.2) we have for each $z \in \mathbb{R}$ and $p \ge 22$.

$$\begin{aligned}\alpha'(p(z+1),p) &\geq 22p^3(z^4+9z^2+1)-p^3(12z^3+66z^2+66z+12)+\\ &+22p^2(z^2+2z+1)-p(12z+12)=\\ &=p^3z^4+p^3z^2[21z^2-12z+2]+\\ &+p^3(11z-3)^2+p^3\left[9z^2-\frac{12}{p^2}+\left(1-\frac{12}{p^2}\right)\right]+\\ &+22p^2(z+1)>0\,.\end{aligned}$$

CLAIM 2. If $p \geq 100$ and $x = 2n \in (-\infty,-p] \cup [1.02p;1.6445p] \cup [2.551p;84.792p]$, we have

$$\beta'(x,p) > 0\,.$$

Let us compute the values of the polynomial β' for $x = kp$.

$$\begin{aligned}\beta'(-p,p) &= 1621p^5-42p^4-757p^3-1302p^2+72p=\\ &=1618p^5+p^4(p-42)+p^3(p^2-757)+p^2(p^3-1302)+72p\,.\end{aligned}$$

and then $\beta'(-p,p) > 0$ if $p \geq 100$. In a similar way we obtain for each $p \geq 100$:

$$\begin{array}{ll}\beta'(0,p)<0 & \beta'(2.513p,p)<0\\ \beta'(p,p)<0 & \beta'(2.551p,p)>0\\ \beta'(1.02p,p)>0 & \beta'(84.792p,p)>0\\ \beta'(1.6445p,p)>0 & \beta'(84.99p,p)<0\,.\\ \beta'(1.6695p,p)<0 & \end{array}$$

Hence for each $p \geq 100$ we can demarcate the roots of the polynomial $\beta'(\cdot,p)$ and then the Claim 2 follows easily.

CLAIM 3. If $p \geq 100$ and $x = 2n \in [0;1.2396p] \cup [1.2908p;4.5598p] \cup [4.9338p;\infty)$ we have

$$\gamma'(x,p) > 0\,.$$

Using the same technique as in the bove claim we obtain that:

$$\gamma'(-p,p) < 0 \qquad \gamma'(1.2908p,p) > 0$$

$$\gamma'(0,p) > 0 \qquad \gamma'(4.5598p,p) > 0$$

$$\gamma'(1.2396p,p) > 0 \qquad \gamma'((3+\sqrt{3})p,p) < 0$$

$$\gamma'((3-\sqrt{3})p,p) < 0 \qquad \gamma'(4.9338p,p) > 0$$

if $p \geq 100$, and then the desired conclusion follows immediately.

The proof of (iii) is a straightforward computation using the fact that, since p cannot be equal to zero or $2n$,

$$\binom{2n}{p} = \frac{2n(2n-1)}{p(2n-p)}\binom{2n-2}{p-1}$$

To obtain (iv) we make the substitution $n = (u-1)/2$, $p = (u-1)/2 \pm v$ in $\gamma(n,p)$ and then we replace v^2 by $(u^2-1)/12$ (see (iii)). The result is $\gamma((u-1)/2,(u-1)/2 \pm \sqrt{(u^2-1)/12}) = -1/8(u-1)(u-2)(u-3)(u^2-6u+53)$ and the least value of u is 7. □

REMARK 3.2. The intervals given in (ii) can be extended by imposing on p a greater lower bound as can be seen from the proof of Claims 2,3. Moreover, for $p \to \infty$ one can obtain asymptotic evaluations of the roots of the polynomials β and γ. The set of values for which $\beta(\cdot,p) > 0$ and $\gamma(\cdot,p) > 0$ is, for large p (with the numerical values truncated after 4 decimals):

$$[0.5059p + 0.1165 + 0_1(p); 0.6339p - 0.7583 - \sqrt{0.1076p - 0.7253} +$$

$$+ 0_2(p)] \cup [0.6339p - 0.7583 + \sqrt{0.1076p - 0.7253} + 0_3(p);$$

$$0.8224p - 1.1738 + 0_4(p)] \cup [1.2753p + 1.8204 + 0_5(p);$$

$$2.3660p + 21.7583 - \sqrt{20.8923p + 673.4753} + 0_6(p)] \cup$$

$$[2.3660p + 21.7583 + \sqrt{20.8923p + 673.4753} + 0_7(p);$$

$$42.3962p - 9.7630 + 0_8(p)].$$

where $\lim_{p\to\infty} 0_i(p) = 0$, $i = 1, 2, \ldots, 8$. (All the numerical values appearing above can, accidentaly, be expressed with rational numbers and square roots).

Since the inference of these asymptotics is quite long, and the improvement is not essential, we don't give the details.

4 – The main results

In this section we shall present the main results of our paper.

THEOREM 4.1. *Let* (M, J, g) *and* (M', J', g') *be compact Kähler manifolds with* $\mathrm{Spec}^p(M, g) = \mathrm{Spec}^p(M', g')$ *for a fixed but arbitrary* $p, p \in \mathbb{N}$, $p \geq 1$, *(which implies* $\dim M = \dim M' = n$*). Then for any* $n \in \mathbb{N}$, $2n \geq p$ *such that the functions* $\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}$ *are strictly positive (hence for all values given by the Theorem 3.1* (ii) *and Remark 3.1),* (M, J, g) *is of constant holomorphic sectional curvture h if and only if* (M', J', g') *is of constant holomorphic sectional curvature* h' *and* $h' = h$.

PROOF. For the proof we shall adopt the standard argument of S. TANNO [7].

It is known that a Kähler manifold has constant holomorphic sectional curvature if and only if $G = 0$ and $B = 0$. Assume that (M', J', g') has constant holomorphic sectional curvature h'. Then the relation (2.16) takes the form:

$$a'_{2,p} = \bar{c} \int_{M'} \frac{4}{n(n+1)} \tilde{\gamma} \tau'^2 dM' .$$

Using the equality of the spectra we obtain:

$$\begin{aligned} &\int_M \left\{ 4\tilde{\alpha}|B|^2 + \frac{8}{n+2}\tilde{\beta}|G|^2 + \frac{4}{n(n+1)}\tilde{\gamma}\tau^2 \right\} dM = \\ &= \int_{M'} \frac{4}{n(n+1)} \tilde{\gamma} \tau'^2 dM' \end{aligned} \tag{4.1}$$

and

$$\int_M \tau dM = \int_{M'} \tau' dM' ; \qquad \int_M dM = \int_{M'} dM' \tag{4.2}$$

because $\binom{2n}{p} - 6\binom{2n-2}{p-1} \neq 0$ by Theorem 3.1 (iv). These imply (since $\tau' =$ constant)

$$\int_M \tau^2 dM \geq \int_{M'} \tau'^2 dM' . \tag{4.3}$$

Therefore from the relations (4.1), (4.3) and our hypothesis we can conclude that (M, J, g) is of constant holomorphic sectional curvature h and moreover $h = h'$. □

THEOREM 4.2. *Let (M, J, g) and (M', J', g') be compact, Einstein, Kähler manifolds with* $\mathrm{Spec}^p(M, g) = \mathrm{Spec}^p(M', g')$ *for a fixed but arbitrary p, $p \in \mathbb{N}$, $p \geq 1$ (which implies* $\dim M = \dim M' = n$*). If $(n, p) \notin \{(8,2),(8,14)\}$ and $n(2n-1) - 3p(2n-p) \neq 0$, then (M, J, g) is of constant holomorphic sectional curvature h if and only if (M', J', g') is of constant holomorphic sectional curvature h' and $h' = h$.*

PROOF. If (M, J, g) and (M', J', g') are Einstein manifolds, then $G = 0$, $G' = 0$ and τ, τ' are constant. The equality of the spectra implies $a_{i,p} = a'_{i,p}$ for $i = 0, 1, 2$.

By Theorem 3.1 (iii), $\binom{2n}{p} - 6\binom{2n-2}{p-1} \neq 0$ and then from (2.2) and (2.3) we obtain that $\tau = \tau'$. Using (2.16), the equality $a_{2,p} = a'_{2,p}$ implies:

$$\int_M \tilde{\alpha} |B|^2 dM = \int_{M'} \tilde{\alpha} |B'|^2 dM' .$$

But $\tilde{\alpha} \neq 0$ by Theorem 3.1 (i), hence $B = 0$ if and only if $B' = 0$. □

As a consequence we obtain the following:

COROLLARY 4.1. *Let (M, J, g) be a compact Kähler manifold and let $(P^n(\mathbb{C}), J_0, g_0)$ be the complex projective space with Fubini-Study metric and assume that* $\mathrm{Spec}^p(M, g) = \mathrm{Spec}^p(P^n(\mathbb{C}), g_0)$ *for a given $p \in \mathbb{N}$, $p \geq 1$. If:*

(a) *the functions $\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}$ are strictly positive (hence for all pairs (n, p) given by the Theorem 3.1* (ii) *and Remark 3.1),*

or

(b) *M is an Einstein manifold and* $(n;p) \notin \{(8;2),(8;14)\}$, $n(2n-1)-3p(2n-p) \neq 0$,

the (M,J,g) *is holomorphically isometric with* $(P^n(\mathbb{C}),J_0,g_0)$.

In other words, in the class of compact Kähler manifolds (respectively in the class of compact, Einstein and Kähler manifolds), $(P^n(\mathbb{C}),J_0,g_0)$ is completely characterized by the spectrum of the p-Laplacian whenever $(n;p)$ fulfils (a) (respectively the restrictions given in (b)).

REMARK 4.1. It is an open problem to decide what happens in the cases when n has a value which is not mentioned in our theorems.

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