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An uniqueness result for body with voids in linear thermoelasticity

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RIASSUNTO: *Il lavoro tratta il problema ai valori iniziali (ed al contorno) per le equazioni della termoelastodinamica lineare dei continui con vuoti. Assunta l'esistenza della soluzione, se ne dimostra l'unicità senza far ricorso né alla legge di conservazione dell'energia né ad alcuna ipotesi di limitatezza dei parametri termoelastici.*

ABSTRACT: *This paper is dedicated to the uniqueness of solution of initial boundary value problem in thermoelasticity of bodies with voids. The proof of theorem is obtained without recourse either to an energy conservation law or to any boundedness assumptions on the thermoelastic coefficients.*

1 – Introduction

The theory of elastic materials with voids is a recent generalization of the classical theory of elasticity. This theory has attracted in the past as nowadays many writers. In fact, in this theory a behavior of porous solids in which the matrix material is elastic and the interstices are void of material, is presented.

The intended applications of this theory are to geological materials, like rocks and soils and, also, to manufactured porous material. In the paper [4] of COWIN and NUNZIATO, the linear theory of elastic materials

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with voids was developed. SACCOMANDI, in [6], has established the equations of the thermoelasticity of materials with voids by considering the effect of dissipation. The linear theory of micropolar bodies with voids was developed by MARIN in [2]. The result presented in what follows are aimed to strengthen some theorems previously available. First, we present the basic equations and conditions in the linear theory of thermoelasticity of bodies with voids, as in [6]. Next, we present a counterpart of BRUN's theorem, [1], in the isotermal theory of elastic bodies and use the latter to prove the uniqueness. Previous studies on uniqueness have been based almost exclusively on the assumptions that the elasticity tensor is positive definite or is strongly eliptic or, others, recourse to an energy conservation law. Exception include a result of the paper by BRUN [1], where an assumption concerning negative definiteness of the initial time derivative of the relaxation tensor is used. Our objective is to obtain the uniqueness without recourse either to an energy conservation law or to any boundedness assumptions on the elastic coefficients. For convenience, the notation and terminology chosen are almost identical to those of [2], [6].

2 – Basic equations

Let the body occupy, at time $t = 0$, a properly regular region B of the euclidian three-dimensional space, bounded by the piece-wise smooth surface ∂B (such that allowing of divergence like theorem). We refer the motion of the body to a fix system of rectangular cartesian axes Ox_i, $i = 1, 2, 3$. We use the summation convention over repeated indices. The subcript j after a comma indicates partial differentiation with respect to x_j. All latin subscripts are understood to range over the integers $(1, 2, 3)$, while the greek indices have the range $1, 2$. A superposed dot denotes the derivative with respect to the t-time variable. The basic equations of the linear thermoelastodynamics of bodies with voids are, [2], [6]

– the equations of motion

$$t_{ij,j} + F_i = \varrho \ddot{u}_i \,; \tag{1}$$

– the equation of energy

$$T_0 \dot{\eta} = q_{i,i} + r \,; \tag{2}$$

– the balance of the equilibrated forces

$$h_{i,i} + L + g = \varrho\kappa\ddot{\sigma}\,; \tag{3}$$

– the constitutive equations

$$\begin{aligned} t_{ij} &= C_{ijmn}e_{mn} + B_{ij}\sigma + D_{ijk}\sigma_{,k} - \beta_{ij}\theta\,,\\ h_i &= D_{mni}e_{mn} + d_i\sigma + A_{ij}\sigma_{,j} - a_i\theta\,,\\ \eta &= \beta_{ij}e_{ij} + a\theta + m\sigma + a_i\sigma_{,i}\,,\\ g &= -\tau\dot{\sigma} - B_{ij}e_{ij} - \xi\sigma - d_i\sigma_{,i} + m\theta\,,\\ q_i &= k_{ij}\theta_{,j}\,; \end{aligned} \tag{4}$$

– the kinetic relations

$$2e_{ij} = u_{i,j} + u_{j,i}, \quad \sigma = \varphi - \varphi_0\,. \tag{5}$$

In these equations we have used the following notations:
t_{ij} – the components of the stress tensor;
ϱ – the constant mass density;
F_i – the components of the body force;
L – the extrinsic equilibrated body force;
h_i – the components of the equilibrated stress;
g – the the intrinsec equilibrated force;
u_i – the components of the displacement;
θ – the temperature variation measured from the reference constant temperature T_0;
κ – the equilibrated inertia;
φ – the volume distribution function which in the reference state is φ_0;
σ – the change in volume fraction measured from the the reference state;
e_{ij} – the kinematic characteristics of strain;
r – the heat supply; q_i-components of the heat flux vector;

The characteristic functions of the material C_{ijmn}, B_{ij}, β_{ij}, D_{ijk}, d_i, a_i, A_{ij}, k_{ij}, ξ, a, m, τ, are assumed to satisfy the symmetry relations:

$$\begin{aligned} &C_{ijmn} = C_{mnij} = C_{jimn}, \quad A_{ij} = A_{ji}\,,\\ &\beta_{ij} = \beta_{ji}, \quad B_{ij} = B_{ji}, \quad D_{ijk} = D_{jik}\,. \end{aligned} \tag{6}$$

and they are continuously differentiable on B. Moreover τ is assumed positive and the extend F_i the extorud heat supply and the extrinsic equilibrated force L are continuous on $B \times [0, \infty)$.

The components of the surface traction t_i, the heat flux q, and the surface equilibrated traction h at the regular points of $\partial B \times [0, \infty)$ are defined by $t_i = t_{ij} n_j$, $q = q_i n_i$ and $h = h_i n_i$ respectively, where we denote by n_i the unit normal on ∂B, pointing towards the exterior of ∂B. Along with (2.1)-(2.5) we shall assume that the following standard initial conditions hold

$$
\begin{aligned}
&u_i(x,0) = \alpha_i(x), \quad \dot{u}_i(x,0) = \beta_i(x), \quad \theta(x,0) = \theta^0(x) \\
&\sigma(x,0) = \sigma^0(x), \quad \dot{\sigma}(x,0) = \sigma^1(x), \qquad x \in B\,,
\end{aligned} \tag{7}
$$

where the functions α_i, β_i, θ^0, σ^0 and σ^1 are prescribed.

Let ∂B_i, $(i = 1, 2, 3, 4, 5, 6)$ be subsets of the surface ∂B so that

$$
\bar{\partial} B_1 \cup \partial B_2 = \bar{\partial} B_3 \cup \partial B_4 = \bar{\partial} B_5 \cup \partial B_6 = \partial B\,,
$$

$$
\partial B_1 \cap \partial B_2 = \partial B_3 \cap \partial B_4 = \partial B_5 \cap \partial B_6 = \phi\,.
$$

To the above equations we adjoin the following boundary conditions

$$
\begin{aligned}
&u_i = \tilde{u}_i \quad \text{on} \quad \partial B_1 \times [0,\infty), \quad t_i = t_{ij} n_j = \tilde{t}_i \quad \text{on} \quad \partial B_2 \times [0,\infty)\,, \\
&\theta = \tilde{\theta} \quad \text{on} \quad \partial B_3 \times [0,\infty), \quad q = q_i n_i = \tilde{q} \quad \text{on} \quad \partial B_4 \times [0,\infty)\,, \\
&\sigma = \tilde{\sigma} \quad \text{on} \quad \partial B_5 \times [0,\infty), \quad h = h_i n_i = \tilde{h} \quad \text{on} \quad \partial B_6 \times [0,\infty)\,,
\end{aligned} \tag{8}
$$

where $\tilde{u}_i$, $\tilde{t}_i$, $\tilde{\theta}$, $\tilde{q}$, $\tilde{\sigma}$ and $\tilde{h}$ are given functions.

In all what follows we use the assumptions

- α_i, β_i, σ^0, σ^1 and θ^0 are continuous functions on B;
- $\tilde{u}_i$, $\tilde{\theta}$ and $\tilde{\sigma}$ are continuous functions on $\partial B_1 \times [0,\infty)$, $\partial B_3 \times [0,\infty)$, $\partial B_5 \times [0,\infty)$, respectively;
- $\tilde{t}_i$, $\tilde{q}$ and $\tilde{h}$ are continuos functions in time and are piece-wise regular on $\partial B_2 \times [0,\infty)$, $\partial B_4 \times [0,\infty)$, $\partial B_6 \times [0,\infty)$, respectively.

By a solution of the mixed initial boundary value problem of the thermoelastic bodies with voids, in the cylinder $B \times [0,\infty)$, we mean an ordered array (u_i, θ, σ) which satisfies eqns. (1)-(5), the initial conditions (7) and the boundary conditions (8).

3 – Uniqueness result

Throughout this section it is assumed that a twice continuously differentiable solution (u_i, θ, σ) exists satisfying the eqns. (1)-(5) and the conditions (7) and (8) on a maximal interval of existence (for instance, see the papers [5] and [6]). First, we establish some estimations and then, as a consequence, the basic theorem which proves the uniqueness for the solution of our problem.

We consider the functions K and U on $[0, \infty)$ defined by

$$K(t) = \frac{1}{2}\int_B \varrho(\dot{u}_i\dot{u}_i + \kappa\dot{\sigma}^2)dV\,, \tag{9}$$

$$\begin{aligned} U(t) = \frac{1}{2}\int_B (&C_{ijmn}u_{i,j}u_{m,n} + 2D_{ijk}u_{i,j}\sigma_{,k} + 2B_{ij}u_{i,j}\sigma + \\ &+ a\theta^2 + 2d_i\sigma\sigma_{,i} + A_{ij}\sigma_{,i}\sigma_{,j} + \xi\sigma^2)dV, \end{aligned} \tag{10}$$

where for convenience, we have omitted the explicit dependence of the functions on their spatial argument x and on the time t. In all that follows where it is possible for simplicity, we shall suppress the spatial argument x or the time variable t. We also define

$$\begin{aligned} G(\alpha,\beta) = &\int_B \Big[F_i(x,\alpha)\dot{u}_i(x,\beta) + L(x,\alpha)\dot{\sigma}(x,\beta) - \frac{1}{T_0}r(x,\alpha)\theta(x,\beta)\Big]dV + \\ &+ \int_{\partial B}\Big[t_i(x,\alpha)\dot{u}_i(x,\beta) + h(x,\alpha)\dot{\sigma}(x,\beta) - \frac{1}{T_0}q(x,\alpha)\theta(x,\beta)\Big]dA, \\ &\forall\, \alpha,\beta \in [0,\infty)\,. \end{aligned} \tag{11}$$

Theorem 1. *If the symmetry relations* (6) *are satisfied, then*

$$U(t) - K(t) = \frac{1}{2}\int_0^t \Big[G(t+s, t-s) - G(t-s, t+s)\Big]ds + \frac{1}{2}R(t) \tag{12}$$

with

$$
R(t)=\int_B\Big\{C_{ijmn}u_{i,j}(2t)u_{m,n}(0)+B_{ij}[u_{i,j}(2t)\sigma(0)+u_{i,j}(0)\sigma(2t)]+ \\
+D_{ijk}[u_{i,j}(0)\sigma_{,k}(2t)+u_{i,j}(2t)\sigma_{,k}(0)]+d_i[\sigma_{,i}(0)\sigma(2t)+ \\
+\sigma_{,i}(2t)\sigma(0)]+a\theta(0)\theta(2t)+A_{ij}\sigma_{,i}(0)\sigma_{,j}(2t)+\xi\sigma(0)\sigma(2t)+ \\
-\varrho\dot{u}_i(0)\dot{u}_i(2t)-\varrho\kappa\dot{\sigma}(0)\dot{\sigma}(2t)\Big\}dV, \qquad t\in[0,\infty)\,. \tag{13}
$$

Proof. From the constitutive equations (4), we obtain

$$
t_{ij}(t-s)\dot{u}_{i,j}(t+s)+h_i(t-s)\dot{\sigma}_{,i}(t+s)+\dot{\eta}(t+s)\theta(t-s)+ \\
-t_{ij}(t+s)\dot{u}_{i,j}(t-s)-h_i(t+s)\dot{\sigma}_{,i}(t-s)-\dot{\eta}(t-s)\theta(t+s)= \\
=\frac{\partial}{\partial s}\Big\{C_{ijmn}u_{i,j}(t+s)u_{m,n}(t-s)+A_{ij}\sigma_{,i}(t-s)\sigma_{,j}(t+s)+ \\
+D_{ijk}[u_{i,j}(t+s)\sigma_{,k}(t-s)+u_{i,j}(t-s)\sigma_{,k}(t+s)]+ \\
+a\theta(t+s)\theta(t-s)\Big\}-B_{ij}\sigma(t+s)\dot{u}_{i,j}(t-s)+ \\
+B_{ij}\sigma(t-s)\dot{u}_{i,j}(t+s)-d_i\dot{\sigma}_{,i}(t-s)\sigma(t+s)+ \\
+d_i\dot{\sigma}_{,i}(t+s)\sigma(t-s)+m\dot{\sigma}(t+s)\theta(t-s)-m\dot{\sigma}(t-s)\theta(t+s)\,. \tag{14}
$$

In view of (1), (2) and (3) , it follows

$$
t_{ij}(t-s)\dot{u}_{i,j}(t+s)+h_i(t-s)\dot{\sigma}_{,i}(t+s)+\dot{\eta}(t+s)\theta(t-s)+ \\
-t_{ij}(t+s)\dot{u}_{i,j}(t-s)-h_i(t+s)\dot{\sigma}_{,i}(t-s)-\dot{\eta}(t-s)\theta(t+s)= \\
=\Big[t_{ij}(t-s)\dot{u}_i(t+s)+h_j(t-s)\dot{\sigma}(t+s)+\frac{1}{T_0}q_j(t+s)\theta(t-s)\Big]_{,j}+ \\
-\Big[t_{ij}(t+s)\dot{u}_i(t-s)+h_j(t+s)\dot{\sigma}(t-s)+\frac{1}{T_0}q_j(t-s)\theta(t+s)\Big]_{,j}+ \\
+F_i(t-s)\dot{u}_i(t+s)+L(t-s)\dot{\sigma}(t+s)+\frac{1}{T_0}r(t+s)\theta(t-s)+ \\
-F_i(t+s)\dot{u}_i(t-s)-L(t+s)\dot{\sigma}(t-s)-\frac{1}{T_0}r(t-s)\theta(t+s)+ \\
+\frac{\partial}{\partial s}[\varrho\dot{u}_i(t-s)\dot{u}_i(t+s)+\varrho\kappa\dot{\sigma}(t-s)\dot{\sigma}(t+s)]-\xi\sigma(t-s)\dot{\sigma}(t+s)+ \\
+\xi\sigma(t+s)\dot{\sigma}(t-s)+d_i\sigma_{,i}(t+s)\dot{\sigma}(t-s)-d_i\sigma_{,i}(t-s)\dot{\sigma}(t+s)+ \\
-B_{ij}\dot{\sigma}(t+s)u_{i,j}(t-s)+B_{ij}\dot{\sigma}(t-s)u_{i,j}(t+s) \tag{15}
$$

Taking into account eqns. (14) and (15), we may write

$$
\begin{aligned}
(16)\quad & \frac{\partial}{\partial s}\Big\{C_{ijmn}u_{i,j}(t+s)u_{m,n}(t-s)+B_{ij}[u_{i,j}(t+s)\sigma(t-s)+\\
&+u_{i,j}(t-s)\sigma(t+s)]+D_{ijk}[u_{i,j}(t+s)\sigma_{,k}(t-s)+\\
&+u_{i,j}(t-s)\sigma_{,k}(t+s)]+d_i[\sigma_{,i}(t+s)\sigma(t-s)+\sigma_{,i}(t-s)\sigma(t+s)]+\\
&+a\theta(t+s)\theta(t-s)+\xi\sigma(t+s)\sigma(t-s)+A_{ij}\sigma_{,i}(t-s)\sigma_{,j}(t+s)\Big\}+\\
&-\frac{\partial}{\partial s}\Big[\varrho\dot{u}_i(t-s)\dot{u}_i(t+s)+\varrho\kappa\dot{\sigma}(t-s)\dot{\sigma}(t+s)\Big]=\\
=&\Big[t_{ij}(t-s)\dot{u}_i(t+s)+h_j(t-s)\dot{\sigma}(t+s)+\frac{1}{T_0}q_j(t+s)\theta(t-s)\Big]_{,j}+\\
&-\Big[t_{ij}(t+s)\dot{u}_i(t-s)+h_j(t+s)\dot{\sigma}(t-s)+\frac{1}{T_0}q_j(t-s)\theta(t+s)\Big]_{,j}+\\
&+F_i(t-s)\dot{u}_i(t+s)+L(t-s)\dot{\sigma}(t+s)+\frac{1}{T_0}r(t+s)\theta(t-s)+\\
&-F_i(t+s)\dot{u}_i(t-s)-L(t+s)\dot{\sigma}(t-s)-\frac{1}{T_0}r(t-s)\theta(t+s)\,.
\end{aligned}
$$

Now, by integrating in (16) on $B\times[0,t]$ and by using the divergence theorem, it results

$$
\begin{aligned}
(17)\quad & \int_0^t\int_{\partial B}\Big[t_i(t-s)\dot{u}_i(t+s)+\frac{1}{T_0}q(t-s)\theta(t+s)+h(t-s)\dot{\sigma}(t+s)+\\
&-t_i(t+s)\dot{u}_i(t-s)-\frac{1}{T_0}q(t+s)\theta(t-s)-h(t+s)\dot{\sigma}(t-s)\Big]dAds+\\
&+\int_0^t\int_B\Big[F_i(t-s)\dot{u}_i(t+s)+\frac{1}{T_0}r(t-s)\theta(t+s)+L(t-s)\dot{\sigma}(t+s)+\\
&-F_i(t+s)\dot{u}_i(t-s)-\frac{1}{T_0}r(t+s)\theta(t-s)-L(t+s)\dot{\sigma}(t-s)\Big]dVds=\\
=&\int_B\Big\{C_{ijmn}u_{i,j}(2t)u_{m,n}(0)+B_{ij}[u_{i,j}(0)\sigma(2t)+u_{i,j}(2t)\sigma(0)]+\\
&+D_{ijk}[u_{i,j}(0)\sigma_{,k}(2t)+u_{i,k}(2t)\sigma_{,k}(0)]+a\theta(0)\theta(2t)+\\
&+A_{ij}\sigma_{,i}(0)\sigma_{,j}(2t)+d_i[\sigma_{,i}(0)\sigma(2t)+\sigma_{,i}(2t)\sigma(0)]+\xi\sigma(0)\sigma(2t)+\\
&-\varrho\dot{u}_i(0)\dot{u}_i(2t)-\varrho\kappa\dot{\sigma}(0)\dot{\sigma}(2t)\Big\}dV-\int_B[C_{ijmn}u_{i,j}(t)u_{m,n}(t)+\\
&+2D_{ijk}u_{i,j}(t)\sigma_{,k}(t)+2B_{ij}u_{i,j}(t)\sigma(t)+a\theta^2(t)+A_{ij}\sigma_{,i}(t)\sigma_{,j}(t)+\\
&+2d_i\sigma_{,i}(t)\sigma(t)+\xi\sigma^2(t)-\varrho\dot{u}_i(t)\dot{u}_i(t)-\varrho\kappa\dot{\sigma}^2(t)]dV\,.
\end{aligned}
$$

Taking into account the notations (9), (10), (11), (13), the relation (17) may be restated such that we arrive at (12). ∎

THEOREM 2. *Let $P(t)$ be the function*

$$P(t)=\int_B\Big[F_i\dot u_i+\frac{1}{T_0}r\theta+L\dot\sigma\Big]dV+\int_{\partial B}\Big[t_i\dot u_i+\frac{1}{T_0}q\theta+h\dot\sigma\Big]dA\,. \tag{18}$$

Then we have the following relations

$$\begin{aligned}2U(t)=U(0)+K(0)+R(t)-\frac{1}{T_0}\int_0^t\int_B k_{ij}\theta_{,i}\theta_{,j}dVds+\\ -\int_0^t\int_B\tau\dot\sigma^2dVds\frac{1}{2}\int_0^t[G(t+s,t-s)+\\ -G(t-s,t+s)+2P(s)]ds\,,\end{aligned} \tag{19}$$

$$\begin{aligned}2K(t)=U(0)+K(0)-R(t)-\frac{1}{T_0}\int_0^t\int_B k_{ij}\theta_{,i}\theta_{,j}dVds+\\ -\int_0^t\int_B\tau\dot\sigma^2dVds-\frac{1}{2}\int_0^t[G(t+s,t-s)+\\ -G(t-s,t+s)-2P(s)]ds\,,\end{aligned} \tag{20}$$

provided that the symmetry relations (6) *hold .*

PROOF. With aid of the constitutive equations (3) and the symmetry relations (6), we can write

$$\begin{aligned}t_{ij}\dot u_{i,j}+h_i\dot\sigma_{,i}+\dot\eta\theta=B_{ij}\dot u_{i,j}\sigma+d_i\dot\sigma_{,i}\sigma+m\dot\sigma\theta+\\ +\frac{1}{2}\frac{\partial}{\partial s}(C_{ijmn}u_{i,j}u_{m,n}+2D_{ijk}u_{i,j}\sigma_{,k}+A_{ij}\sigma_{,i}\sigma_{,j}+a\theta^2)\end{aligned} \tag{21}$$

On the other hand, in view of the equations of motion, (1), the balance of the equilibrated forces, (2), the equation of energy (3) and the geometrical equations (5), it results

$$\begin{aligned}t_{ij}\dot u_{i,j}+h_i\dot\sigma_{,i}+\dot\eta\theta=-B_{ij}u_{i,j}\dot\sigma-d_i\sigma_{,i}\dot\sigma+m\dot\sigma\theta+\\ +F_i\dot u_i+L\dot\sigma+\frac{1}{T_0}r\theta+\Big(t_{ij}\dot u_i+h_j\dot\sigma+\frac{1}{T_0}q_j\theta\Big)_{,j}+\\ -\frac{1}{2}\frac{\partial}{\partial s}(\varrho\dot u_i\dot u_i+\varrho\kappa\dot\sigma^2)-\frac{1}{T_0}k_{ij}\theta_{,i}\theta_{,j}\,.\end{aligned} \tag{22}$$

From (21) and (22), by equalizing their right-hand sides, we are led to

$$
\begin{aligned}
(23)\qquad &\Big(t_{ij}\dot{u}_i + h_j\dot{\sigma} + \frac{1}{T_0}q_j\theta\Big)_{,j} + F_i\dot{u}_i + L\dot{\sigma} + \frac{1}{T_0}r\theta = \\
&= \frac{1}{2}\frac{\partial}{\partial s}(C_{ijmn}u_{i,j}u_{m,n} + 2D_{ijk}u_{i,j}\sigma_{,k} + 2B_{ij}u_{i,j}\sigma + \\
&\quad + A_{ij}\sigma_{,i}\sigma_{,j} + a\theta^2 + \xi\sigma^2 + 2d_i\sigma_{,i}\sigma + \frac{1}{2}\frac{\partial}{\partial s}(\varrho\dot{u}_i\dot{u}_i + \varrho\kappa\dot{\sigma}^2) - \tau\dot{\sigma}^2\,.
\end{aligned}
$$

By integrating in (23) over B, we conclude, with the aid of the divergence theorem and the notations (9), (10) and (18), that

$$
(24)\quad \dot{K}(t) + \dot{U}(t) = P(t) - \frac{1}{T_0}\int_B k_{ij}\theta_{,i}\theta_{,j}dV - \int_B \tau\dot{\sigma}^2 dV - \int_B \tau\dot{\sigma}^2 dV\,.
$$

Integrating in (24) from 0 to t, $t \in [0,\infty)$, we obtain

$$
(25)\qquad
\begin{aligned}
K(t) + U(t) &= K(0) + U(0) + \int_0^t P(t)ds + \\
&- \frac{1}{T_0}\int_0^t\int_B k_{ij}\theta_{,i}\theta_{,j}dV ds - \int_0^t\int_B \tau\dot{\sigma}^2 dV\, ds\,.
\end{aligned}
$$

Now, by adding the relations (25) and (12) we establish the relation (19) and, at last, by subtracting (17) from (25), it follows the relation (20). ∎

Theorem 1 and Theorem 2 form the basis of the following theorem which establishes the uniqueness of solution.

THEOREM 3. *Assume that*

(i) *the symmetry relations* (6) *are valid;*

(ii) ϱ, τ *and* κ *are strictly positive;*

(iii) a *is strictly positive* (*or strictly negative*).

Then the mixed problem of thermoelastodynamics of bodies with voids has at most one solution.

Proof. Assume to the contrary that there exist two solutions, say $(u_i^{(\alpha)}, q^{(\alpha)}, s^{(\alpha)}), \alpha = 1, 2$. We denote their difference by (U_i, Θ, Υ), *i.e.*

$$U_i = u_i^{(2)} - u_i^{(1)}, \Theta = \theta^{(2)} - \theta^{(1)}, \Upsilon = \sigma^{(2)} - \sigma^{(1)} .$$

Because of the linearity of our problem, (U_i, Θ, Υ) is also a solution, but corresponds to null data. Thus, we conclude from (20) that

$$\begin{aligned} &\int_B (\varrho \dot{U}_i \dot{U}_i + \varrho\kappa \dot{\Upsilon}^2) dV + \frac{1}{T_0} \int_0^t \int_B k_{ij} \Theta_{,i} \Theta_{,j} dV\, ds + \\ &\quad + \int_0^t \int_B \tau \dot{\Upsilon}^2 dV\, ds = 0 . \end{aligned} \tag{26}$$

Based on the assumptions (i)-(iii), (26) implies that

$$\dot{U}_i = 0, \dot{\Upsilon} = 0 \; on \; B \times [0, \infty) , \tag{27}$$

and

$$\frac{1}{T_0} \int_0^t \int_B k_{ij} \Theta_{,i} \Theta_{,j} dV ds + \int_0^t \int_B \tau \dot{\Upsilon}^2 dV ds = 0, \quad (0 \leq t < \infty) . \tag{28}$$

With the aid of (27) the relation (28) become

$$\frac{1}{T_0} \int_0^t \int_B k_{ij} \Theta_{,i} \Theta_{,j} dV ds = 0, \quad (0 \leq t < \infty) .$$

Because of the fact that U_i and Υ vanish initially, from (27) we deduce

$$U_i = 0, \Upsilon = 0 \;\; on \;\; B \times [0, \infty) . \tag{29}$$

Taking into account (28), (29), the relation (19) leads to

$$\int_B a \Theta^2 dV = 0 ,$$

such that, since $a > 0$ (or $a < 0$), we conclude that $\Theta = 0$ on $B \times [0, \infty)$. □

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