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Hopf hypersurfaces of D'Atri- and C-type in a complex space form

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Riassunto: *In uno spazio-forma complesso non piatto si assegna una caratterizzazione delle ipersuperficie di Hopf che sono spazi di D'Atri (cioè varietà riemanniane le cui simmetrie geodetiche preservano il volume a meno del segno) o C-spazi (cioè varietà i cui operatori di Jacobi hanno autovalori costanti sulle geodetiche). Questo porta a una classificazione delle ipersuperficie di Hopf che sono naturalmente riduttive, cioè spazi commutativi o debolmente simmetrici.*

Abstract: *We classify all Hopf hypersurfaces in a non-flat complex space form $M^n(c)$ which are D'Atri spaces (that is, Riemannian manifolds all of whose local geodesic symmetries are volume-preserving up to sign) or C-spaces (that is, their Jacobi operators have constant eigenvalues along the corresponding geodesics). This yields a classification of Hopf hypersurfaces which are naturally reductive, g.o., weakly symmetric or commutative spaces.*

1 – Introduction

The study of Riemannian manifolds all of whose local geodesic symmetries are volume-preserving (up to sign) has been started in [9]. Such manifolds generalize locally symmetric spaces and are now called D'Atri *spaces* [26]. Several examples are known and they have been studied ex-

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tensively. We refer to [14] for a survey. Furthermore, in [4], J. BERNDT and the second author generalized in another way the locally symmetric spaces and introduced and studied *C-spaces* as Riemannian manifolds such that for any geodesic the corresponding Jacobi operator has constant eigenvalues along that geodesic. Although the geometry of C-spaces shares a lot of properties with that of the D'Atri spaces, a good understanding of their relation is not yet known. We refer to [3], [5] for a survey.

The main purpose of this paper is to classify all Hopf hypersurfaces in a non-flat complex space form $M^n(c)$, n being the complex dimension, which are D'Atri spaces or C-spaces.

Let $\overline{M}$ be an oriented real hypersurface of a complex space form $M^n(c)$ and let N be a unit normal vector field on $\overline{M}$. Then $\overline{M}$ is said to be a Hopf hypersurface [1] if the structure vector field $\xi = -JN$ is a principal curvature vector field, that is, an eigenvector field of the shape operator field on $\overline{M}$. T. E. CECIL and P. J. RYAN extensively investigated in [7] hypersurfaces which are realized as tubes over certain submanifolds in $\mathbb{C}P^n$ by using their focal maps. Furthermore, in [23] R. TAKAGI classified the homogeneous hypersurfaces of $\mathbb{C}P^n$ into six types. By making use of the results in [7] and [23], M. KIMURA then proved the following [12]

PROPOSITION A. *Let $\overline{M}$ be a Hopf hypersurface in $\mathbb{C}P^n$. Then $\overline{M}$ has constant principal curvatures if and only if $\overline{M}$ is locally congruent to one of the following spaces:*

(A_1) *a geodesic hypersphere of radius r where $0 < r < \frac{\pi}{2}$;*

(A_2) *a tube of radius r over a totally geodesic $\mathbb{C}P^k, 1 \leq k \leq n-2$, where $0 < r < \frac{\pi}{2}$;*

(B) *a tube of radius r over a complex quadric Q_{n-1} where $0 < r < \frac{\pi}{4}$;*

(C) *a tube of radius r over $\mathbb{C}P^1 \times \mathbb{C}P^{\frac{n-1}{2}}$ where $0 < r < \frac{\pi}{4}$ and n $(n \geq 5)$ odd;*

(D) *a tube of radius r over a complex Grassmann manifold $\mathbb{C}G_{2,5}$ where $0 < r < \frac{\pi}{4}$ and $n = 9$;*

(E) *a tube of radius r over a Hermitian symmetric space $SO(10)/U(5)$ where $0 < r < \frac{\pi}{4}$ and $n = 15$.*

Note that the result of R. Takagi implies that all the model spaces mentioned in Proposition A are homogeneous. Moreover, since the principal curvatures are constant, they are isoparametric hypersurfaces. These principal curvatures and their multiplicities are explicitly written down in the table in [24]. (See also Section 2.)

Furthermore, real hypersurfaces of a complex hyperbolic space $\mathbb{C}H^n$ have been investigated in [1], [2], [18], [19], for example. In particular, in [2], J. Berndt classified the Hopf hypersurfaces with constant principal curvatures in $\mathbb{C}H^n$. More precisely, he obtained the following

Proposition B. *Let $\overline{M}$ be a Hopf hypersurface in $\mathbb{C}H^n$. Then $\overline{M}$ has constant principal curvatures if and only if $\overline{M}$ is locally congruent to one of the following spaces:*

- (A_0) *a horosphere;*
- (A_1) *a geodesic hypersphere or a tube over a complex hyperbolic hyperplane $\mathbb{C}H^{n-1}$;*
- (A_2) *a tube over a totally geodesic $\mathbb{C}H^k, 1 \leq k \leq n-2$;*
- (B) *a tube over a totally real hyperbolic space $\mathbb{R}H^n$.*

These model spaces are obviously again isoparametric and they are also homogeneous (see [1]). The principal curvatures and their multiplicities of these hypersurfaces are also given in [2].

In what follows the hypersurfaces of type $(A_1), (A_2)$ in Proposition A and those of type $(A_0), (A_1), (A_2)$ in Proposition B will be called *hypersurfaces of type A.*

In Section 2, we collect some basic facts and then, in Section 3, we prove the

Main Theorem. *A Hopf hypersurface in a non-flat complex space form is a D'Atri space or a C-space, respectively, if and only if it is locally congruent to a hypersurface of type A.*

Several interesting classes of D'Atri and C-spaces are known. More precisely, the following classes of spaces have these properties:

{i}: naturally reductive homogeneous spaces or more general, Riemannian manifolds equipped with a naturally reductive structure;

{ii}: g.o. spaces, that is, Riemannian manifolds all of whose geodesics are orbits of one-parameter subgroups of isometries;

{iii}: weakly symmetric spaces, that is, Riemannian manifolds such that for any pair of points there exists an isometry interchanging these points;

{iv}: commutative spaces, that is, Riemannian manifolds such that the algebra of all isometry-invariant differential operators is commutative.

Here, we have the following inclusion relations: $\{i\} \subset \{ii\}, \{iii\} \subset \{ii\}, \{iii\} \subset \{iv\}$. See [14] for more details and references. We also note that all generalized Heisenberg groups are D'Atri and C-spaces [3]. Furthermore, it has been proved in [20], [21] that a real hypersurface in $M^n(c), c \neq 0$, has a naturally reductive structure if and only if it is locally congruent to a hypersurface of type A. (Note that the proof in [21] can be extended to the case $c < 0$.) Further, it is proved in [6] that the manifolds of type A are weakly symmetric. For $\mathbb{C}P^n$, this result may also be derived from [8], combined with [10]. As a consequence of these remarks and the Main Theorem, we get

Corollary. *Let $\overline{M}$ be a Hopf hypersurface in $M^n(c), c \neq 0$. Then $\overline{M}$ is equipped with a naturally reductive structure or is locally isometric to a g.o. space, a commutative space or a weakly symmetric space, respectively, if and only if it is locally congruent to a hypersurface of type A.*

2 – Preliminaries

Let $(M^n(c), g, J)$ denote a complex space form of constant holomorphic sectional curvature c and let $\overline{M}$ be an orientable, connected real hypersurface. Further, let N be a unit normal vector of $\overline{M}$. For any vector field X tangent to $\overline{M}$ we put

$$JX = \varphi X + \eta(X)N, \quad JN = -\xi \tag{2.1}$$

where φ is a tensor field of type $(1,1)$, η is a one-form and ξ a unit vector field on $\overline{M}$. We also denote the induced metric on $\overline{M}$ by g. Then

(φ, ξ, η, g) determines an almost contact metric structure on $\overline{M}$, that is, we have

$$\varphi^2 X = -X + \eta(X)\xi, \qquad \eta(\xi) = 1, \tag{2.2}$$
$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y)$$

for all tangent vector fields X, Y. Then (2.2) yields

$$\varphi\xi = 0, \quad \eta \circ \varphi = 0, \quad \eta(X) = g(X, \xi)\,. \tag{2.3}$$

The Gauss and Weingarten formulas for $\overline{M}$ are

$$\begin{aligned} \widetilde{\nabla}_X Y =& \nabla_X Y + g(AX, Y)\,, \\ \widetilde{\nabla}_X N =& - AX \end{aligned}$$

for tangent vector fields X, Y and where $\widetilde{\nabla}$ and ∇ denote the Levi Civita connection of $(M^n(c), g)$ and $(\overline{M}, g)$, respectively. A is the shape operator. From (2.1) and $\widetilde{\nabla} J = 0$ we then obtain

$$\begin{aligned} (\nabla_X \varphi)Y &= \eta(Y)AX - g(AX, Y)\xi\,, \\ \nabla_X \xi &= \varphi AX \end{aligned} \tag{2.4}$$

for tangent X, Y. Furthermore, we have the following Gauss and Codazzi equations:

$$\begin{aligned} R(X, Y)Z = \frac{c}{4}\{&g(Y, Z)X - g(X, Z)Y + \\ &+ g(\varphi Y, Z)\varphi X - g(\varphi X, Z)\varphi Y + \\ &- 2g(\varphi X, Y)\varphi Z\} + g(AY, Z)AX - g(AX, Z)AY, \end{aligned} \tag{2.5}$$

$$(\nabla_X A)Y - (\nabla_Y A)X = \frac{c}{4}\{\eta(X)\varphi Y - \eta(Y)\varphi X - 2g(\varphi X, Y)\xi\}\,. \tag{2.6}$$

Here, R is taken with the sign convention $R(X, Y) = [\nabla_X, \nabla_Y] - \nabla_{[X,Y]}$. Using (2.2) and (2.5) we then get for the Ricci tensor Q of type $(1, 1)$:

$$QX = \frac{c}{4}\{(2n + 1)X - 3n(X)\xi\} + hAX - A^2 X \tag{2.7}$$

where $h = \mathrm{tr}\ A$ denotes the mean curvature.

Now, let $\overline{M}$ be a Hopf hypersurface and $c \neq 0$. Then

$$A\xi = \alpha\xi \tag{2.8}$$

where α is constant (see, for example, [2], [11], [17]). Further, let X be a principal curvature vector orthogonal to ξ with principal curvature λ. Then we have

$$AX = \lambda X, \qquad A\varphi X = \frac{\alpha\lambda + c/2}{2\lambda - \alpha}\varphi X\,. \tag{2.9}$$

We note that $\overline{M}$ is of type A if and only if $A\varphi = \varphi A$ [19], [22]. Among the hypersurfaces given in Proposition A and Proposition B, these are also characterized by $\lambda^2 - \alpha\lambda - c/4 = 0$. Moreover, we have

Proposition 1 [24]. *The tangent spaces of the hypersurfaces given in Proposition A may be decomposed as follows:*
for type A: $TM = \mathbb{R}\xi \oplus T_\lambda \oplus T_{-1/\lambda}, \quad A\xi = (\lambda - \frac{1}{\lambda})\xi$;
for type B: $TM = R\xi \oplus T_\lambda \oplus T_{-1/\lambda}, \quad A\xi = \frac{-4\lambda}{\lambda^t - 1}\xi$
where $\lambda > 0$ *for type A and* $0 < \lambda < 1$ *for type B. Further, for type B we have* $\varphi T_\lambda = T_{-1/\lambda}$ [17] .

Next, we consider Riemannian manifolds which are of D'Atri- and C-type. In both cases the curvature satisfies the Ledger conditions of order three and five [14], that is,

$$L_3 : (\nabla_X \rho)(X, X) = 0\,,$$
$$L_5 : \sum_{a,b} R(e_a, X, X, e_b)(\nabla_X R)(e_a, X, X, e_b) = 0$$

where ρ denotes the Ricci tensor of type $(0, 2)$ and $\{e_a\}$ is an orthonormal basis of $T_p\overline{M}, p \in \overline{M}$. Here, $R(X, Y, Z, W) = g(R(X, Y)Z, W)$ and $(\nabla_X R)(Y, Z, U, V) = g((\nabla_X R)(Y, Z)U, V)$. Note that L_3 is equivalent to

$$\mathfrak{S}_{X,Y,Z}(\nabla_X \rho)(Y, Z) = 0$$

where $\mathfrak{S}$ denotes the cyclic sum. This means that ρ is cyclic-parallel or equivalently, ρ is a Killing tensor.

As concerns the Hopf hypersurfaces satisfying the condition L_3, we have

Proposition 2 [15], [16]. *Let $\overline{M}$ be a Hopf hypersurface of $M^n(c)$, $c \neq 0$. Then ρ is cyclic-parallel if and only if*

a) *$M^n(c) = \mathbb{C}P^n$ and $\overline{M}$ is locally congruent to a real hypersurface of type A or to one of type B with special radius r;*

b) *$M^n(c) = \mathbb{C}H^n$ and $\overline{M}$ is locally congruent to a real hypersurface of type A.*

For $c = 4$, the defining equation for the class B in Proposition 2 is

$$\alpha\lambda^2 + 4\lambda - \alpha = 0 \tag{2.10}$$

and the radius r (related to α by $\alpha = 2\cot 2r$) in Proposition 2 is given by

$$2\alpha = 3h \quad (\text{or equivalently, } \alpha^2 = 12(n-1))\,. \tag{2.11}$$

See [15], [16].

Finally, we shall use

Proposition 3 [25]. *A Riemannian manifold (M, g) is locally homogeneous if and only if there exists a tensor field T of type $(1,2)$ on M (called a homogeneous structure) such that with $\overline{\nabla} = \nabla - T$ we have $\overline{\nabla} g = \overline{\nabla} R = \overline{\nabla} T = 0$. Moreover, T is called a naturally reductive structure if $T_X X = 0$ for all tangent vectors X.*

3 – Proof of the Main Theorem

First, let $\overline{M}$ be a Hopf hypersurface which is locally congruent to one of type A. As mentioned in the Introduction, then $\overline{M}$ is equipped with a naturally reductive structure and hence, $\overline{M}$ is a D'Atri space and a C-space.

Conversely, let $\overline{M}$ be of D'Atri- or C-type. Then ρ is cyclic-parallel. So, it follows from Proposition 2 that for $\mathbb{C}H^n$ the hypersurface is locally congruent to one of type A. The proof will be complete if for $\mathbb{C}P^n$ we can exclude the hypersurfaces which are congruent to one of type B.

To do this, we suppose that $\overline{M}$ is such a Hopf hypersurface in $\mathbb{C}P^n$ where we suppose $c = 4$. It follows from Proposition 3 that there exists a homogeneous structure T on $\overline{M}$. Furthermore, since $\overline{M}$ is of D'Atri- or C-type, ρ is cyclic-parallel and hence, we have at $p \in M$:

$$\rho(T_X X, X) = 0$$

for any $X \in T_p\overline{M}$. By polarization, we get

$$\mathfrak{S}_{X,Y,Z}\{\rho(T_X Y, Z) + \rho(T_X Z, Y)\} = 0 \tag{3.1}$$

for $X, Y, Z \in T_p\overline{M}$. Put $X = Y = \xi$ in (3.1) to obtain

$$\rho(T_\xi \xi, Z) + \rho(T_\xi Z, \xi) + \rho(T_Z \xi, \xi) = 0\,. \tag{3.2}$$

Next, using (2.7) in (3.2), we get

$$\begin{aligned} 3g(T_\xi \xi, Z) + hg(T_\xi \xi, AZ) - g(T_\xi \xi, A^2 Z) - \alpha h g(T_\xi \xi, Z) + \\ + \alpha^2 g(T_\xi \xi, Z) = 0\,. \end{aligned} \tag{3.3}$$

Further, we denote by D_λ the eigenspace, orthogonal to ξ, associated to an eigenvalue λ of Q. Assuming $Z \in D_\lambda$, we obtain from (3.3) the relation

$$(\lambda^2 - \lambda h - \alpha^2 + \alpha h - 3) g(T_\xi \xi, Z) = 0\,. \tag{3.4}$$

In what follows we first consider the case $n \neq 2$. Then, taking into account the defining relations (2.10), (2.11), we get $\lambda^2 - \lambda h - \alpha^2 + \alpha h - 3 \neq 0$ and so, for that case, we must have, since T_ξ is skew-symmetric,

$$T_\xi \xi = 0. \tag{3.5}$$

Next, put $Z = Y$ in (3.1). Then we have

$$\rho(T_X Y, Y) + \rho(T_Y Y, X) + \rho(T_Y X, Y) = 0$$

which for $X \in D_\lambda, Y \in D_\mu$ and with (2.7) yields

$$(\lambda - \mu)(h - \lambda - \mu) g(T_Y Y, X) = 0\,. \tag{3.6}$$

Using again (2.10) and (2.11) in (3.6), we obtain

$$g(T_Y Y, X) = 0 \tag{3.7}$$

for $Y \in D_\mu, X \in D_\lambda$ and $\lambda \neq \mu$.

Further, put $X = \xi$ and $Z = Y \in D_\lambda$ in (3.1). Using (3.5) and (2.7), we then get

$$(\lambda^2 - \lambda h - \alpha^2 + \alpha h - 3) g(T_Y Y, \xi) = 0$$

and for the special hypersurfaces in class B we obtain

$$g(T_Y Y, \xi) = 0\,. \tag{3.8}$$

Now, put $X = \xi$ in (3.1) and assume $Y \in D_\lambda, Z \in D_\mu$ for $\lambda \neq \mu$. Using (2.7), we then get

$$\begin{aligned}(\mu^2 - \mu h - \alpha^2 + \alpha h - 3) & g(T_Y Z, \xi) + \\ &+ (\lambda^2 - \lambda h - \alpha h - 3) g(T_Z Y, \xi) + \\ &+ (\mu h - \lambda h - \mu^2 + \lambda^2) g(T_\xi Y, Z) = 0\,.\end{aligned} \tag{3.9}$$

Putting

$$\begin{aligned} a &= \mu^2 - \mu h - \alpha^2 + \alpha h - 3\,, \\ b &= \lambda^2 - \lambda h - \alpha^2 + \alpha h - 3\,, \\ c &= \mu h - \lambda h - \mu^2 + \lambda^2\,, \end{aligned}$$

we see that

$$c = b - a\,. \tag{3.10}$$

So, (3.9) may be rewritten as

$$a g(T_Y Z, \xi) + b g(T_Z Y, \xi) + c g(T_\xi Y, Z) = 0\,. \tag{3.11}$$

Moreover, (2.10) and (2.11) imply $abc \neq 0$ when $n \neq 2$.

In what follows we shall now take into account the Ledger condition L_5. Using the homogeneous structure T and the symmetry properties of R, this condition may be written in the form

$$\sum_{i,j} R(e_i, X, X, e_j) R(e_i, T_X X, X, e_j) = 0 \tag{3.12}$$

where $\{e_i\}$ is an orthonormal basis of T_pM. By polarizing (3.12) we get

$$\begin{aligned}\sum_{i,j}\{R(e_i,X,Y,e_j)+R(e_i,Y,X,e_j)\}\{R(e_i,T_ZV,W,e_j)+\\ +R(e_i,T_ZW,V,e_j)+R(e_i,T_VW,Z,e_j)+\\ +R(e_i,T_VZ,W,e_j)+\\ +R(e_i,T_WV,Z,e_j)+R(e_i,T_WZ,V,e_j)\}=0\,.\end{aligned} \tag{3.13}$$

Next, put $X=V=W=\xi$ in (3.13) and take $Y\in D_\lambda, Z\in D_\mu, \lambda\neq\mu$. Using (2.2), (2.3) and (2.5) in (3.13), a long computation then gives

$$(a+F)g(T_YZ,\xi)+(b+G)g(T_ZY,\xi)+(c+H)g(T_\xi Y,Z)=0 \tag{3.14}$$

where

$$\begin{aligned}F&=-3\alpha\lambda+\alpha^2h^2-\alpha\mu h^2+\alpha\mu^3-2\alpha^4-\alpha^2-\alpha\mu+\alpha^3\mu-\alpha^3\mu^2+\alpha^5\,,\\ G&=-3\alpha\mu+\alpha^2h^2-\alpha\lambda h^2+\alpha\lambda^3-2\alpha^4-\alpha^2-\alpha\lambda+\alpha^3\lambda-\alpha^3\lambda^2+\alpha^5\,,\\ H&=-3\alpha\mu+3\alpha\lambda+\alpha\mu h^2-\alpha\lambda h^2-\alpha\lambda^3+\alpha\mu^3+\alpha\mu-\alpha\lambda-\alpha^3\mu+\alpha^3\lambda+\\ &\quad+\alpha^3\mu^2-\alpha^3\lambda^2\,.\end{aligned}$$

Hence, we have

$$G-F=H\,. \tag{3.15}$$

So, from (3.11) and (3.14) we have

$$Fg(T_YZ,\xi)+Gg(T_ZY,\xi)+Hg(T_\xi Y,Z)=0\,. \tag{3.16}$$

Here, we note that $H\neq 0$ for the considered hypersurfaces. Indeed, if $H=0$, we have

$$\alpha(\lambda-\mu)\{(\lambda+\mu)^2-\alpha^2(\lambda+\mu)+3+\alpha^2-h^2\}=0\,.$$

Taking account of (2.11), we have

$$(\lambda+\mu)^2-\alpha^2(\lambda+\mu)=3+\alpha^2-h^2=0\,. \tag{3.17}$$

Moreover, since in this case $\lambda + \mu = -\frac{4}{\alpha}$ and $h = (n-1)(\lambda + \mu) + \alpha = (h-1)(-\frac{4}{\alpha}) + \alpha$ (see Proposition 1 and (2.10)), (3.17) gives

$$4\alpha^3 + (8n-5)\alpha^2 - 16n(n-2) = 0\,. \tag{3.18}$$

Now, one may check that there is no integer n satisfying (3.18) and (2.11). (We used a computer to check this.) So, we have $H \neq 0$. Then, multiplying (3.10) by H and (3.16) by c and subtracting, we obtain

$$(aH - cF)g(T_Y Z, \xi) + (bH - cG)g(T_Z Y, \xi) = 0\,. \tag{3.19}$$

Further, from (3.10) and (3.15) we also see that $aH - cF = bH - cG$, and so, (3.19) becomes

$$(aH - cF)\{g(T_Y Z + T_Z Y, \xi) = 0\,. \tag{3.20}$$

Finally, in a similar way as for H, one may check that $aH - cF \neq 0$. Hence, we have from (3.20)

$$g(T_Y Z + T_Z Y, \xi) = 0\,. \tag{3.21}$$

So, from (3.5), (3.7), (3.8) and (3.21) we conclude that

$$g(T_X X, \xi) = 0 \tag{3.22}$$

for any vector $X \in T_p M$.

To finish this case, we note that the same reasoning as in [21] now yields $\overline{\nabla}_X \xi = 0$ since $n \geq 3$. Hence, with (2.4) we get

$$T_X \xi = \varphi A X$$

and so, from (3.22) we obtain $g(\varphi AX, X) = 0$. This yields $\varphi A = A\varphi$ which contradicts the fact that $\overline{M}$ is not locally congruent to a hypersurface of type A.

Finally, we consider the case $n = 2$. Since $\overline{M}$ satisfies the conditions L_3 and L_5, $\overline{M}$ is equipped with a naturally reductive structure [13] and hence, is locally congruent to a hypersurface of type A, which is again a contradiction. This completes the proof.

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