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Foreword to special issue celebrating the Umberto Mosco's birthday

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This Special Issue is dedicated to Umberto Mosco in celebration of his eightieth Birthday.

Umberto Mosco is Harold J. Gay Chair Professor of Mathematics at the Worcester Polytechnic Institute from 2005 after 34 years as a faculty member at the University of Rome (including 31 years as a full professor). He is member of the Italian National Academy of Sciences and fellow of the American Mathematical Society. Umberto Mosco developed an international reputation with his groundbreaking research; his outstanding role in Mathematics is well documented by many honours he received: 1996: A. Feltrinelli Prize for Mathematics, Mechanics and Applications, Accademia Nazionale dei Lincei, Rome; 1998: Alexander-von-Humboldt-Stiftung Prize, Bamberg, Germany; 2002: Medaglia d'oro per la Matematica della Accademia delle Scienze detta dei XL, Rome; 2004: Marconi Lecture, Swedish Science Academy.

He published more than 100 papers covering the fields of partial differential equations, convex analysis, control theory, variational problems, and homogenization. His publication list was addressed to as a virtual history of the field of nonlinear analysis. He is well known worldwide for the "Mosco Convergence", which is widely recognized by researchers and students in the areas of analysis, calculus of variations, and partial differential equations and stochastic processes, and which is now a standard concept in variational calculus textbooks. In his recent work he aims to invent a new form of calculus based on fractal curves by forging a link between calculus, discrete mathematics, and analysis.

This special issue follows the "One day Workshop on PDEs" that took place on May 28, 2019 in Rome (Italy). The Workshop offered an overview of recent results connected to functional analysis and theory of the PDEs, highlighting interesting open problems and indicating new directions and perspectives of research with increasing importance in a wide variety of application areas. Some themes, also addressed in the Round Table "Variational structures and Applied Sciences: the Insight of Umberto Mosco", are developed in this Special Issue which collects research papers contributed by highly distinguished mathematicians, friends and students of Umberto Mosco. These contributions cover topics closely connected to the conference themes. In our opinion the content of this issue is very interesting as well as accessible to non-expert readers.

Content of the special issue:

1. A tree structure algorithm for optimal control problems with state constraints. By A. Alla, M. Falcone, L. Saluzzi.

   Abstract. We present a tree structure algorithm for optimal control problems with state constraints. We prove a convergence result for a discrete time approximation of the value function based on a novel formulation in the case of convex constraints. Then the Dynamic Programming approach is developed by a discretization in time leading to a tree structure in space derived by the controlled dynamics, taking into account the state constraints to cut several branches of the tree. Moreover, an additional pruning allows for the reduction of the tree complexity as for the case without state constraints. Since the method does not use an a priori space grid, no interpolation is needed for the reconstruction of the value function and the accuracy essentially relies on the time step h. These features permit a reduction in CPU time and in memory allocations. The synthesis of optimal feedback controls is based on the values on the tree and an interpolation on the values obtained on the tree will be necessary if a different discretization in the control space is adopted, e.g. to improve the accuracy of the method in the reconstruction of the optimal trajectories. Several examples show how this algorithm can be applied to problems in low dimension and compare it to a classical DP method on a grid.

2. T-minima on convex sets and Mosco-convergence. By L. Boccardo, C. Leone.

   Abstract. Half century ago, Umberto Mosco was the "relatore di tesi (tesi about the Mosco-convergence) di laurea" of the first author; a quart of century ago, the first author was the "relatore di tesi di laurea" of the second author. The roots of this paper are the Mosco-convergence of convex sets and the minimization of integral functionals of the Calculus of Variations. We consider integral functionals of the type

   $$J(v) = \int_\Omega j(x; Dv) - \int_\Omega f(x)v(x).$$

   We study the existence of T-minima (infinite energy minima) on convex sets of the Sobolev space $W_0^{1,p}(\Omega)$ and the stability of the T-minima under the Mosco-convergence of the convex sets.

3. Perron-Frobenius, principal eigenvalue, maximum principle: a personal itinerary. By I. Capuzzo Dolcetta.

   Abstract. This review paper dedicated to Umberto Mosco revisits a line of my research starting in the early 70's in the aim of identifying a path connecting dierent concepts which play a role in elliptic pde's, spectral theory and optimal control.

4. A new short proof of regularity for local weak solutions for a certain class of singular parabolic equations. By S. Ciani, V. Vespri.

   Abstract. We shall establish the interior Hölder continuity for locally bounded weak solutions to a class of parabolic singular equations whose prototypes are

   $$u_t = \nabla \cdot \left(|\nabla u|^{p-2} \nabla u \right), \quad \text{for} \quad 1 < p < 2, \tag{0.1}$$

   and

   $$u_t - \nabla \cdot (u^{m-1} |\nabla u|^{p-2} \nabla u) = 0, \quad \text{for} \quad m + p > 3 - \frac{p}{N}, \tag{0.2}$$

   via a new and simplified proof using recent techniques on expansion of positivity and L^1-Harnack estimates.

5. Atomic decomposition for preduals of some Banach spaces. By L. D'Onofrio, C. Sbordone, R. Schiattarella.

   Abstract. Given a Banach space E with a supremum type norm induced by a sequence $\mathcal{L} = (L_j)$ of linear forms $L_j : X \to \mathbb{R}$ on the Banach space X, we prove that if the unit ball $\mathbb{B}_X$ is $\sigma(X, \mathcal{L})$- compact then E has a predual $E_\star$ with an atomic decomposition. We extend results from L. D'Onofrio, K.M. Perfekt, L. Greco, C. Sbordone, R. Schiattarella: Atomic decompositions, two stars theorems, and distances for the Bourgain-Brezis-Mironescu space and other big spaces, Ann. Inst. H. Poincaré Anal. Non Linéaire (2020), where X is assumed a reflexive Banach space.

6. Damage dynamics: a variational approach. By A. Garroni, C. J. Larsen, D. Sarrocco.

   Abstract. In this paper we construct, by means of a variational formulation, the solutions of a problem of elastodynamics which includes the effect of damage for the elastic material. The result is a wave equation with time dependent operators which represents the elastic coefficients of the material undergoing damage. The dynamics that we construct also satisfies a threshold condition with the same threshold value that characterizes the quasi-static evolution of damage (see A.Garroni, C.J. Larsen: Threshold-based quasi-static Brittle Damage Evolution, Arch. Rat. Mech. Anal. (2009)).

7. Interior gradient bounds for nonlinear elliptic systems. By P. Marcellini.

   Abstract. This manuscript is dedicated to Umberto Mosco, with esteem and affection. Umberto was my mentor at the University of Rome, where I completed my four years studies in Mathematics before my PhD program in Pisa. I dedicate to him the article, which is divided in two parts. In the first section I propose some regularity theorems, precisely some interior bounds for the gradient of weak solutions to a class of nonlinear elliptic systems; the title of this manuscript takes its origin from this section. The second part of

the manuscript deals with my first studies in Rome together with Umberto Mosco and with my next studies in Pisa where I met Ennio De Giorgi and where I had the good fortune of assisting to the birth of the G-convergence and the Γ-convergence theories, with some connections with the Mosco's convergence.

Rome, October 28th, 2020.

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