Rend. Mat. Appl. (7).
Volume 42 (2021), 177 – 180

RENDICONTI DI MATEMATICA
E DELLE SUE APPLICAZIONI

Foreword to the Special Issue "Nonlinear Diffusion Problems"

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This Special Issue of Rendiconti di Matematica collects some article devoted to the memory of *Maria Assunta Pozio*, passed away untimely on September 5$^{\text{th}}$, 2018. For many years, Assunta –as everybody used to call her– was our colleague at the Department, bearing an unforgettable contribution of scientific skills, generosity and human warmth.

After graduating in Mathematics at the Sapienza in 1977, Assunta was a researcher at the University of Trento in the period 1978–1981. Later on she moved to the University of Tor Vergata, Rome, where she served first as researcher, then from 1988 as associate professor. Since 1992 her career took place at the Department of Mathematics "Guido Castelnuovo" of Sapienza University, Rome. A selected list of publications authored or co-authored by Assunta is included at the end of this foreword.

In the first period, the research interests of Assunta were addressed to the analytical study of problems arising in populations dynamics (see [15, 16]) and, specifically, in compartmental models motivated by epidemiology (see [8, 12]). In such a setting, she considered models of nonlinear differential equations (with or without spatial diffusion) analyzing the presence of a delay term having a sound biological interpretation. After that, Assunta's scientific activity was mainly devoted to initial-boundary value problems for quasilinear parabolic and elliptic equations, focusing on well-posedness and qualitative properties of classical and weak solutions, keeping an eye wide open at the biomathematical relevance of the results as base element of the much wider domain of theoretical biology.

Her attention was mainly devoted to parabolic reaction-diffusion equations with degenerate diffusion and some (possibly space dependent) zero order term, considering well-posedness, large time behavior, blow-up and other qualitative properties of the solutions (see [1, 6, 7, 10, 11, 13, 14, 17, 18, 19, 22, 23, 24]). Attention to specific reaction-diffusion system was also given (see [20, 21]).

Elliptic equations were mainly regarded as the time-asymptotic limit of their parabolic counterpart, thus considering stability properties as crucial (among others, see [3, 5, 6, 9]). Recently, Assunta, together with her friend and collaborator Catherine Bandle, turned her attention to the case of a destabilized elliptic equation with a perturbative linear term in the form of a Hardy potential, i.e. a multiplier function diverging at the boundary. Here the aim is to describe and quantify

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the delicate competition between the linear term and the nonlinear power-like ones (see [2, 4] and the contribution by C. Bandle in this Special Issue).

Figure 1: A very lovely picture of Assunta, used as background for the poster of the workshop "Nonlinear diffusion problems" held at the Department of Mathematics "Guido Castelnuovo" of Sapienza, Università di Roma (Italy) in September 2019.

Her interest in applied problems was always joint to a thorough search of mathematical rigour. Assunta was a very gifted researcher, whose deep critical sense was combined with an uncommon ability to solve difficult problems by a constructive approach. In particular, these qualities appear in a series of papers on elliptic and parabolic quasilinear equations, in which existence and qualitative properties of solutions to some space-dependent reaction-diffusion equation and its elliptic counterpart were studied by a refined use of monotonicity methods (see [3, 5, 6]).

Her research activity was appreciated in a vast international context: not by chance, many authors of this Issue had been her co-authors.

In parallel with her continuous scientific activity, Assunta was an excellent and dedicated teacher. Her commitment to teaching, as well as her attention to the difficulties encountered by students in the learning process, were almost proverbial in the Department. Equally generous and fruitful was her commitment to solve organization problems and planning issues related to teaching and research, particularly in the management of international scientific projects in which she always had a leading role.

This Special Issue, motivated by the homonymous workshop held at the Department of Mathematics "Guido Castelnuovo" of Sapienza, Università di Roma (Italy) in the period September 11^{th}–13^{th}, 2019, is a modest contribution to Assunta's memory. Her untimely death is still a great loss to those who had the good fortune to appreciate her scientific and human qualities.

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Received: 18 March 2021/Accepted: 18 March 2021/Published online: 28 March 2021

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