

Seminario di Algebra & Geometria
Dipartimento di Matematica “Guido Castelnuovo”
SAPIENZA Università di Roma
A.A. 2020/2021

Mercoledì 7 Ottobre 2020 - ore 16:30

Jean-Pierre Demailly (Université Grenoble Alpes)

Hermitian-Yang-Mills approach to the conjecture of Griffiths on the positivity of ample vector bundles

Given a vector bundle of arbitrary rank with ample determinant line bundle on a projective manifold, we propose a new elliptic system of differential equations of Hermitian-Yang-Mills type for the curvature tensor.

The system is designed so that solutions provide Hermitian metrics with positive curvature in the sense of Griffiths – and even in the dual Nakano sense. As a consequence, if an existence result could be obtained for every ample vector bundle, the Griffiths conjecture on the equivalence between ampleness and positivity of vector bundles would be settled. Another outcome of the approach is a new concept of volume for vector bundles.

Mercoledì 14 Ottobre 2020 - ore 14:00

Gerhard Röhrle (Ruhr-Universität Bochum)

On a relative version of Serre's notion of G -complete reducibility

We first review some basic results related to Serre's notion of G -complete reducibility for a reductive algebraic group G . We then discuss a relative variant of this concept where we let K be a reductive subgroup of G , and consider subgroups of G which normalise the identity component K° of K . We show that such a subgroup is relatively G -completely reducible with respect to K if and only if its image in the automorphism group of K° is completely reducible in the sense of Serre. This allows us to generalise a number of fundamental results from the absolute to the relative setting. This is a report on recent joint work with M. Gruchot and A. Litterick.

Mercoledì 21 Ottobre 2020 - ore 14:00

Daniel Greb (Universität Duisburg-Essen)

Projective flatness over klt spaces and uniformisation of varieties with nef anti-canonical divisor

I will discuss a criterion for the projectivisation of a reflexive sheaf on a klt space to be induced by a projective representation of the fundamental group of the smooth locus. This

criterion is then applied to give a characterisation of finite quotients of projective spaces and Abelian varieties by Q-Chern class (in)equalities and a suitable stability condition. This stability condition is formulated in terms of a naturally defined extension of the tangent sheaf by the structure sheaf. I will further examine cases in which this stability condition is satisfied, comparing it to K-semistability and related notions. This is joint work with Stefan Kebekus and Thomas Peternell.

Mercoledì 28 Ottobre 2020 - ore 14:00

Ingrid Bauer (Universität Bayreuth)

Rigid compact complex manifolds: recent results, questions and conjectures

The aim of this talk is to give an update on recent achievements and developments on rigid compact complex manifolds. I will start introducing different notions of rigidity and explaining the relations among them. For curves all these notions coincide and the only rigid curve is the projective line. For surfaces rigidity is still quite „rare“, since the only rigid surfaces which are not minimal of general type are Del Pezzo surfaces of degree at least 5 and Inoue surfaces.

In higher dimension the geography of rigid manifolds gets much richer.

Then I will report on an answer to a more than 40 years open question of Morrow and Kodaira, exhibiting an infinite series of rigid but not infinitesimally rigid surfaces of general type.

I will conclude addressing open questions and conjectures.

Mercoledì 4 Novembre 2020 - ore 14:00

Bernhard Hanke (Universität Augsburg)

Scalar positive immersions

As shown by Gromov-Lawson and Stolz the only obstruction to the existence of positive scalar curvature metrics on closed simply connected manifolds in dimensions at least five appears on spin manifolds, and is given by the non-vanishing of the α -genus of Hitchin. When unobstructed we shall realise a positive scalar curvature metric by an immersion into Euclidean space whose dimension is uniformly close to the classical Whitney upper bound for smooth immersions.

Our main tool is an extrinsic counterpart of the well-known Gromov-Lawson surgery procedure for constructing positive scalar curvature metrics. At this point we use the local flexibility lemma proven by Christian Bär and the speaker.

This is joint work with Luis Florit, IMPA (Rio de Janeiro).

Mercoledì 11 Novembre 2020 - ore 14:00

Immanuel Van Santen (Universität Basel)

Embeddings of smooth affine varieties into algebraic groups

This is joint work with Peter Feller. In any category there are the following fundamental problems concerning embeddings from an object Z into another object X :

1. (Existence) Does there exist an embedding of Z into X ?
2. (Uniqueness) Having two embeddings f, g of Z into X , does there exist an automorphism ψ of X such that $g = \psi \circ f$?

In this talk, we will mainly focus on the first problem in the category of affine varieties, where Z is smooth and X is an algebraic group. Amongst other things, we will discuss the following result.

Theorem. For every simple algebraic group G and every smooth affine variety Z with $\dim G > 2\dim Z + 1$, there exists an embedding of Z into G .

The proof is based upon parametric transversality results for flexible affine varieties due to Kaliman. We will also discuss the following result, which implies the optimality of the above existence result up to a possible improvement of the dimension bound by one. Its proof is an adaptation of a Chow-group-based argument due to Bloch, Murthy, and Szpiro for the affine space.

Proposition. For every non-finite algebraic group G and every $d \geq \dim G / 2$, there exists an irreducible smooth affine variety of dimension d that does not admit an embedding into G .

Mercoledì 18 Novembre 2020 - ore 14:00

Jan Draisma (Universität Bern)

The geometry of polynomial functors

A polynomial functor P is a functor from the category of finite-dimensional vector spaces to itself such that for every U, V the map $\text{Hom}(U, V) \rightarrow \text{Hom}(P(U), P(V))$ is polynomial. In characteristic zero, P is a direct sum of Schur functors.

This talk concerns closed subsets of such P , i.e., rules that assign to a vector space V a closed subset $X(V)$ of $P(V)$ such that $P(\phi)X(U)$ is contained in $X(V)$ for every linear map $\phi: U \rightarrow V$.

Quite surprisingly, these behave very much like finite-dimensional affine varieties. For instance, they satisfy the descending chain condition and a version of Chevalley's theorem on constructible sets. I will discuss these results and more. The talk is based on joint work with Arthur Bik, Rob Eggermont, and Andrew Snowden.

Mercoledì 25 Novembre 2020 - ore 14:00

Carolina Araujo (IMPA)

Birational geometry of Calabi-Yau pairs and 3-dimensional Cremona transformations

Recently, Oguiso addressed the following question, attributed to Gizatullin: "Which automorphisms of a smooth quartic K3 surface $S \subset \mathbb{P}^3$ are induced by Cremona transformations of the ambient space \mathbb{P}^3 ?" When $S \subset$

\mathbb{P}^3 is a smooth quartic surface, (\mathbb{P}^3, D) is an example of a Calabi-Yau pair, that is, a pair (X, D) , consisting of a normal projective variety X and an effective Weil divisor D on X such that $K_X + D \sim 0$. The above question is really about birational properties of the Calabi-Yau pair (\mathbb{P}^3, D) .

In this talk, I will explain a general framework to study the birational geometry of mildly singular Calabi-Yau pairs. Then I will focus on the case of singular quartic surfaces $D \subset \mathbb{P}^3$. Our results illustrate how the appearance of increasingly worse singularities in D enriches the birational geometry of the pair (\mathbb{P}^3, D) , and lead to interesting subgroups of the Cremona group of \mathbb{P}^3 .
This is joint work with Alessio Corti and Alex Massarenti.

Mercoledì 9 Dicembre 2020 - ore 14:00

Eric Sommers (University of Massachusetts Amherst)

The geometry of nilpotent varieties via subbundles of the cotangent bundle

Let G be a simple algebraic group with flag variety G/B . The Springer resolution is the moment map from the cotangent bundle of G/B to the (dual of the) Lie algebra \mathfrak{g} of G . The cohomology of the fibers of this map play an important role in the representation theory of G over various fields. Identify the cotangent bundle with the vector bundle $G \times_B \mathfrak{n}$, where \mathfrak{n} is the nilradical of the Lie algebra of B . There are subbundles $G \times_B I$ for each subspace $I \subset \mathfrak{n}$ that is B -stable, and maps $p_I: G \times_B I \rightarrow \mathfrak{g}$. The fibers of p_I are also interesting and their cohomology relates to an intersection cohomology complex on the image of p_I , a nilpotent variety. In this talk we discuss two topics: (1) methods for computing the cohomology of the fibers of p_I ; (2) a vanishing theorem/conjecture for the cohomology of the structure sheaf on these subbundles.

Mercoledì 16 Dicembre 2020 - ore 14:00

Gilles Carron (Université de Nantes)

A Bonnet-Myers Theorem from a spectral assumption

We obtain a finiteness result for the fundamental group of a closed Riemannian manifold (M^n, g) under the assumption that the Schrödinger operator $\Delta_g + (n-2)\rho$ is positive (where at $x \in M$, $\rho(x)$ is the lowest eigenvalue of the Ricci tensor at x). It is a joint work with C. Rose (MPI Leipzig).

Mercoledì 13 Gennaio 2021 - ore 14:00

Karin Baur (University of Leeds)

Cluster structures for Grassmannians

The coordinate ring of the Grassmannian has the structure of a cluster algebra. On the other side, the category of maximal CM modules over a certain infinite dimensional algebra

Is a cluster category associated to this cluster algebra structure. We study this category, in particular in the tame cases. We also show how to associate frieze patterns to these cluster structures.

Mercoledì 20 Gennaio 2021 - ore 14:00

Enrica Floris (Université de Poitiers)

Connected algebraic groups acting on Fano fibrations over \mathbb{P}^1

Let G be a connected algebraic group and X a variety endowed with a regular action of G and a Mori fibre space X/\mathbb{P}^1 whose fibre is a Fano variety of Picard rank at least 2. In this talk I will explain why there is a proper horizontal subvariety of X which is invariant under the action of G , alongside with some applications of this result to the classification of connected algebraic subgroups of the Cremona group in dimension 4. This is a joint work with Jérémy Blanc.

Mercoledì 27 Gennaio 2021 - ore 14:00

Jerzy Weyman (Jagiellonian University in Kraków)

Finite Free Resolutions and opposite Schubert varieties

In the first part of this talk I will give an update on the connection between perfect ideals of codimension 3 and Schubert varieties of exceptional groups (and more generally opposite Schubert varieties for Kac-Moody groups associated to T -shaped graphs T_{pqr}). I will also discuss a parallel theory which points to similar connection between Gorenstein ideals of codimension 4 with n generators and opposite Schubert varieties in homogeneous spaces related to a Kac-Moody group of type E_n .

Mercoledì 3 Febbraio 2021 - ore 14:00

Vikraman Balaji (Chennai Mathematical Institute)

On a "wonderful" Bruhat-Tits group scheme

We make a universal construction of Bruhat-Tits group scheme on wonderful embeddings of adjoint groups in the absolute and relative settings of adjoint Kac-Moody groups. These group schemes have natural classifying properties reflecting the orbit structure on the wonderful embeddings. A theme which is reflected in the work is that the various wonderful embeddings give algebra-geometric realizations of the different kinds of buildings. This shows an intimate relationship between the wonderful embeddings and Bruhat-Tits theory.

Mercoledì 10 Febbraio 2021 - ore 14:00

Alexander Kuznetsov (Steklov Mathematical Institute)

Lefschetz collections and residual categories for Grassmannians

I will try to explain and motivate the notion of Lefschetz (exceptional) collections in derived categories of coherent sheaves and their residual categories and, in particular, its conjectural relation to quantum cohomology. If time permits, I will discuss recent progress in construction of Lefschetz collections for Grassmannians of classical groups. This is joint work with Maxim Smirnov.

Mercoledì 17 Febbraio 2021 - ore 14:00

Mattias Jonsson (University of Michigan)

Filtrations on section rings and non-Archimedean geometry

To any polarized variety (X,L) is associated a section ring R . I will explain the relation between suitable classes of norms on R and functions on the Berkovich analytification of X . Time permitting, I will discuss applications of this to K -stability. This is joint work with S. Boucksom.

Mercoledì 24 Febbraio 2021 - ore 14:00

Claudio Procesi (Sapienza Università di Roma)

A note on the Weingarten function

I will compare work of Formanek on a certain construction of central polynomials with that of Collins on integration on unitary groups. These two quite disjoint topics share the construction of the same function on the symmetric group, which the second author calls Weingarten function. By joining these two approaches we succeed in giving a simplified and very natural presentation of both Formanek and Collins's Theory.

Mercoledì 3 Marzo 2021 - ore 14:00

Richard Wentworth (University of Maryland)

Conformal limits of parabolic Higgs bundles

Gaiotto introduced the notion of a conformal limit of a Higgs bundle and conjectured that these should identify the Hitchin component with the Oper stratum in the deRham moduli space.

In the case of closed Riemann surfaces this result was proven by Dumitrescu, et al., and the limits were shown to exist much more generally by Collier and the speaker.

In this talk I will report on progress in the case of parabolic Higgs bundles, which were the context of Gaiotto's original conjecture.

This is joint work with B. Collier and L. Fredrickson.

Mercoledì 10 Marzo 2021 - ore 14:00

Daniele Alessandrini (Columbia University)

Non commutative cluster coordinates for Higher Teichmüller Spaces

In higher Teichmüller theory we study subsets of the character varieties of surface groups that are higher rank analogs of Teichmüller spaces, e.g. the Hitchin components and the spaces of maximal representations.

Fock-Goncharov generalized Thurston's shear coordinates and Penner's Lambda-lengths to the Hitchin components, showing that they have a beautiful structure of cluster variety.

Here we apply similar ideas to Maximal Representations and we find new coordinates on these spaces that give them a structure of non-commutative cluster varieties, in the sense defined by Berenstein-Rethak.

This is joint work with Guichard, Rogozinnikov and Wienhard.

Mercoledì 17 Marzo 2021 - ore 14:00

Anne Lonjou (Université Paris-Saclay)

Actions of Cremona groups on CAT(0) cube complexes

A key tool to study the plane Cremona group is its action on a hyperbolic space. Sadly, in higher rank such an action is not available.

Recently, in geometric group theory, actions on CAT(0) cube complexes turned out to be a powerful tool to study a large class of groups.

In this talk, based on a common work with Christian Urech, we will construct such complexes on which Cremona groups of rank n act. Then, we will see which kind of results on these groups we can obtain.

Mercoledì 24 Marzo 2021 - ore 14:00

Frédéric Rochon (Université du Québec à Montréal)

L^2 -cohomology of quasi-fibered boundary metrics

Quasi-fibered boundary metrics (QFB metrics) form a class of complete metrics generalizing the class of quasi-asymptotically locally Euclidean metrics introduced by Joyce.

After reviewing what QFB metrics are and presenting natural examples, we will present a new approach to compute the L^2 -cohomology of such metrics.

This is based on a joint work with Chris Kottke.

Mercoledì 7 Aprile 2021 - ore 14:00

Johan Martens (University of Edinburgh)

Sporadic behaviour of quantum representations and rank-level duality

The theory of conformal blocks provides us with projective representations of the mapping class group. These can equivalently also be constructed from the point of view of non-abelian theta functions, via the Hitchin connection. It has been known for some time that, for $SL(2)$, these representations have infinite order, with the exception of some sporadic low levels. We will discuss how some of these sporadic cases can be understood via rank-level duality. This is joint work with Baier, Bolognesi and Pauly.

Mercoledì 14 Aprile 2021 - ore 14:00

Sarah Scherotzke (Université du Luxembourg)

The categorified Grothendieck-Riemann-Roch Theorem

The Chern character is a central construction which appears in topology, representation theory and algebraic geometry.

In algebraic topology it is for instance used to probe algebraic K-theory which is notoriously hard to compute, in representation theory it takes the form of classical character theory. Recently, Toen and Vezzosi suggested a construction, using derived algebraic geometry, which allows to unify the various Chern characters.

We will categorify this Chern character. In the categorified picture algebraic K-theory is replaced by the category of non-commutative motives. It turns out that the categorified Chern character has many interesting applications. For instance we show that the DeRham realisation functor is of non-commutative origin.

Mercoledì 21 Aprile 2021 - ore 14:00

Vito Felice Zenobi (Sapienza Università di Roma)

Higher rho numbers and metrics of positive scalar curvature

The analytic surgery sequence is a long exact sequence of K-theory groups which combines topological information (the K-homology of manifolds), index theoretic information (the K-theory of group C^ -algebras), and secondary index information (the analytic structure group of Higson-Roe). We will see how to give a definition of terms based entirely on algebras of pseudodifferential operators and their K-theory. We use this to systematically develop maps to an exact sequence of non-commutative de Rham homology/cyclic homology. Via pairings with cyclic cohomology classes, this gives rise to new numeric secondary index invariants (higher rho numbers) with explicit formulas and calculation tools due to the compatibility in the whole sequence. We use this for geometric applications. In particular, we derive new information about the moduli space of Riemannian metrics of positive scalar curvature, where we give new lower bounds on the number of its components.*

Mercoledì 28 Aprile 2021 - ore 14:00

Donna M. Testerman (EPFL)

Overgroups of regular unipotent elements in simple algebraic groups

We consider the question of determining reductive overgroups of regular unipotent elements in simple algebraic groups and in particular give a condition which guarantees that the overgroup does not lie in a proper parabolic subgroup of the ambient simple group. This is joint work with Gunter Malle, and follows on work of Suprunenko, Saxl and Seitz, and Testerman and Zalesski.

Mercoledì 5 Maggio 2021 - ore 14:00

Zsolt Patakfalvi (École polytechnique fédérale de Lausanne)

Replacing vanishing theorems in mixed characteristic and the Minimal Model Program for 3-folds over excellent schemes

Kodaira and Kawamata-Viehweg vanishing is frequently used to lift sections of adjoint bundles, a crucial part of many arguments in the classification theory of algebraic varieties, notably in many proofs of the Minimal Model Program. These vanishing theorems fail in mixed characteristic situations, for example for

- 1.) proper, flat schemes over the p -adic numbers, or*
- 2.) proper birational models of mixed characteristic local rings. I present a work that remedies this situation to some extent. In particular, we are able to*
 - a.) show Kodaira and Kawamata-Viehweg vanishing in many situations,*
 - b.) prove the 3-dimensional Minimal Model Program for excellent schemes, and*
 - c.) draw geometric corollaries of point b.) to the existence of the moduli space of stable surfaces in mixed characteristic.*

This is a joint work with Bhargav Bhatt, Linquan Ma, Karl Schwede, Kevin Tucker, Joe Waldron and Jakub Witaszek.

Mercoledì 12 Maggio 2021 - ore 14:00

René Schoof (Università di Roma "Tor Vergata")

Finite flat group schemes over \mathbb{Z}

Finite flat group schemes are important in number theory.

We explain what we do and don't know about their structure over rings of integers of number fields, in particular over \mathbb{Z} .

This is joint work with Lassina Dembele.

Mercoledì 19 Maggio 2021 - ore 14:00

Erwan Rousseau (Aix-Marseille Université)

Numerically special varieties

Campana introduced the class of special varieties as the varieties admitting no maps onto an orbifold of general type. They are also characterized by the non-existence of Bogomolov sheaves which are rank one coherent subsheaves of maximal Kodaira dimension in some exterior power of the cotangent bundle.

Campana has conjectured that one can replace the Kodaira dimension by the numerical dimension in this characterization.

We prove partially this conjecture showing that a projective manifold admitting a rank one coherent subsheaf of the cotangent bundle with numerical dimension 1 is not special.

This is a joint work with J.V. Pereira and F. Touzet.

Mercoledì 26 Maggio 2021 - ore 14:00

Simon Riche (Université Clermont Auvergne)

Perverse sheaves on affine flag varieties, geometry of the dual group, and representations

The description of regular blocks of the category \mathcal{O} of a complex semisimple Lie algebra in terms of perverse sheaves on a flag variety has been a crucial tool for its study, and in particular for the proof of the Kazhdan-Lusztig character formula. This description has a conjectural analogue for representations of reductive groups over fields of positive characteristic, as predicted by Finkelberg-Mirkovic, which involves a category of perverse sheaves on the affine Grassmannian of the Langlands dual group, with coefficients in a field of positive characteristic. In this talk I will present a work in progress with Bezrukavnikov which we expect will lead to a proof of this conjecture. As a step towards this goal, we obtain a description of tilting perverse sheaves on the affine flag variety reminiscent of the corresponding result for usual flag varieties due to Soergel.

Mercoledì 9 Giugno 2021 - ore 14:00

Jean Ruppenthal (Bergische Universität Wuppertal)

Canonical sheaves at isolated canonical singularities

The canonical line bundle and the corresponding canonical sheaf belong to the most important geometric/analytic objects associated to a complex manifold. They play a crucial role e.g. in classification theory, Serre duality or vanishing theorems. If we consider singular varieties instead of smooth manifolds, then there exist various possibilities to generalize the canonical sheaf to that setting. One can consider for example the Grothendieck(-Barlet-Henkin-Passare) dualizing sheaf or the Grauert-Riemenschneider L_2 -sheaf. In this talk, we will discuss another possible generalization, i.e., the sheaf of L_2 holomorphic n -forms with a certain boundary condition at the singular set. This sheaf is essential for L_2 -d-bar-theory on singular spaces, but difficult to understand. We will describe it explicitly for isolated canonical Gorenstein singularities.

Mercoledì 16 Giugno 2021 - ore 14:00

Mihai Păun (Universität Bayreuth)

On extension of twisted canonical forms defined on the central fiber of a Kähler family

We will report on a joint work with J. Cao.

Our main result establishes the extension of twisted canonical forms defined on an infinitesimal neighborhood of the central fiber of a Kahler family under a natural L^2 hypothesis.

Mercoledì 23 Giugno 2021 - ore 14:30

Anna Wienhard (Universität Heidelberg)

Where geometry meets dynamics: groups, entropy and Hausdorff dimension

I will discuss geometric and dynamical properties of actions of discrete groups on Riemannian symmetric spaces.

I will highlight some aspects of the interplay between geometry and dynamics, and present some recent results which generalize theorems of Sullivan, Bridgeman-Taylor, McMullen for convex cocompact subgroups acting on hyperbolic space in the framework of discrete subgroups of Lie groups of higher rank.

Mercoledì 30 Giugno 2021 - ore 14:00

János Kollár (Princeton University)

Deformations of varieties of general type

We prove that small deformations of a projective variety of general type are also projective varieties of general type, with the same plurigenera.