

Seminario di Algebra & Geometria
Dipartimento di Matematica “Guido Castelnuovo”
SAPIENZA Università di Roma
A.A. 2022/2023

Mercoledì 28 Settembre 2022 - ore 14:00

Hironori Shiga (Chiba University)

K3 hypergeometric modular forms

In 1977 at the Cortona Conference on several complex variables the speaker proposed to make studies on “the K3 modular function”. That is the explicit expression of the inverse of the period map for a family of K3 surfaces.

In this talk, we give a short summary of our study on this direction for 45 years. A recent result is based on the studies by Prof. Salvati-Manni and others. You can see the outline of the talk by two papers by A. Nagano and H. Shiga on Transformation Groups (2022) and Math. Nachrichten (2022).

Mercoledì 5 Ottobre 2022 - ore 14:00

Carlo Mantegazza (Università di Napoli)

Una dimostrazione della disuguaglianza di Penrose riemanniana per mezzo della teoria del potenziale nonlineare

Discuterò la disuguaglianza di Penrose riemanniana in una 3-varietà asintoticamente piatta, con curvatura scalare non negativa, e i punti principali di una nuova dimostrazione per mezzo di una formula di monotonia che vale lungo gli insiemi di livello del potenziale p -capacitario dell'orizzonte di un buco nero.

Lavoro in collaborazione con Virginia Agostiniani, Lorenzo Mazzieri e Francesca Oronzio.

Mercoledì 12 Ottobre 2022 - ore 14:00

Francesco Sala (Università di Pisa)

Algebre di Hall coomologiche, spazi di moduli e gruppi quantici

L'obiettivo del seminario consiste nel dare una panoramica della teoria delle algebre di Hall coomologiche (COHA, in breve) di dimensione due e dei suoi legami, da un lato, con lo studio della topologia di certi spazi di moduli (varietà quiver di Nakajima, spazi di moduli di fasci framed sul piano proiettivo, spazi di moduli di fasci semistabili su superfici proiettive lisce), e, d'altro lato, con i gruppi quantici e le algebre vertice. Verranno discussi in dettaglio due esempi: la COHA del piano affine e la COHA della risoluzione di una singolarità di tipo ADE.

Mercoledì 19 Ottobre 2022 - ore 14:00

Siye Wu (National Tsing Hua University)

Deformation of invariant connections

Suppose a group acts on a manifold and the action can be lifted to a principal bundle over it. We consider invariant connections and their gauge equivalence classes. When the base space is a single group orbit, the invariant connections were studied by H.C.Wang in the 1950s. We show that in general, the space of such connections decomposes according to elements in a group cohomology. We further study the geometry and analysis of the space of invariant connection.

Mercoledì 26 Ottobre 2022 - ore 14:00

Roberto Pagaria (Università di Bologna)

Log-concavità del polinomio cromatico di grafi

Lo scopo del seminario è quello di esporre la recente dimostrazione di June Huh della log-concavità dei coefficienti del polinomio cromatico di un grafo. Quest'anno June Huh è stato premiato con la medaglia Fields per questa dimostrazione e per successivi lavori che usano idee simili.

Il polinomio cromatico di un grafo $P_G(k)$ conta il numero di possibili colorazioni di un grafo con k colori. Negli anni '70 è stato congetturato che i suoi coefficienti formano una sequenza log-concava. L'enunciato della congettura si può dare più in generale per i coefficienti del polinomio caratteristico di un matroide. Queste congetture sono state dimostrate rispettivamente nel 2012 e nel 2018. Le tecniche usate sono sorprendenti e proverò a darne un'idea: costruirò, tramite blow up, una varietà proiettiva e ne studierò l'anello di Chow (che coincide con la coomologia). Infine dal teorema di Hodge-Riemann segue banalmente la disuguaglianza cercata. Nel caso di matroidi la corrispondente varietà non esiste, ma si può comunque definire un anello con le proprietà desiderate e dimostrare la log-concavità.

Mercoledì 2 Novembre 2022 - ore 14:00

Lucas Mason-Brown (University of Oxford)

Unitary Representations of Real Reductive Groups and Conical Symplectic Singularities

One of the most fundamental unsolved problems in representation theory is to classify the set of irreducible unitary representations of a real reductive Lie group. In this talk, I will define a class of such representations coming from filtered quantizations of certain graded Poisson varieties. The representations I construct are expected to form the "building blocks" of all unitary representations.

Mercoledì 9 Novembre 2022 - ore 14:00

Siarhei Finski (CMLS - École polytechnique & CNRS)

On the metric structure of section rings and holomorphic extension theorem

We study the relationship between metric and algebraic structures on section rings of polarized projective manifolds. More precisely, we prove that once the kernel is factored out, the multiplication operator of the section ring becomes an approximate isometry (up to normalization) with respect to the L^2 -norm. We then show that this algebraic property characterizes L^2 -norms and describe some applications of this characterisation.

The semiclassical version of Ohsawa-Takegoshi theorem, describing holomorphic extensions from submanifolds to global manifolds of holomorphic sections of sufficiently large tensor powers, lies at the heart of our approach.

Mercoledì 16 Novembre 2022 - ore 14:00

Rouven Frassek (Università di Modena e Reggio Emilia)

Baxter Q-operators, QQ-systems and the shifted Yangian

I plan to discuss the construction of Baxter Q-operators within the framework of the Quantum Inverse Scattering Method. The method follows the standard procedure for the transfer matrix construction of spin chains that was developed by Faddeev and collaborators but employs Lax matrices with an infinite-dimensional auxiliary space. Those Lax matrices do not fit into the standard Yangian of Drinfeld but can be understood in the context of the shifted Yangian. The Q-operators satisfy a set of functional relations that are known as QQ-relations that can be seen as the successor of the famous Bethe equations. In the case of the Yangian of $su(n)$, the full set of functional relations can be derived from the Q-operators. Beyond $su(n)$

new obstacles arise in the construction of Q-operators which will be discussed in the example of $so(2r)$.

Mercoledì 23 Novembre 2022 - ore 14:00

Gabriella Tarantello (Università di Roma "Tor Vergata")

On a Donaldson functional for CMC-immersions of surfaces into hyperbolic 3-manifolds

We discuss a parametrization for the moduli space of Constant Mean Curvature (CMC) immersions of a closed surface S (orientable and of genus at least 2) into hyperbolic 3-manifolds by elements of the tangent bundle of the Teichmüller space of S .

Namely, by pairs formed of a given conformal structure X on S and a Dolbeault cohomology class of $(0,1)$ -forms valued in the holomorphic tangent bundle of X .

For any such pair, we determine uniquely the pullback metric and the second fundamental form of the immersion by solving the Gauss-Codazzi equations.

The Gauss-Codazzi equations can be viewed as Hitchin's self-duality equations for a suitable nilpotent $SL(2;\mathbb{C})$ -Higgs bundle, and have been handled in this way in case of minimal immersions. However, they correspond also to the Euler-Lagrange equation of a suitable Donaldson functional [see Gonsalves-Uhlenbeck (2007)] and their unique solvability is attained [in collaboration with M. Lucia and Z. Huang (2022)] by showing that such functional admits a global minimum as its unique critical point. Eventually, we can extend such a uniqueness result to more general situations previously treated via the Higgs-bundle approach, including minimal Lagrangian immersions.

In addition, we are able to analyze the asymptotic behavior of those minimizers along a whole ray of cohomology classes and obtain "convergence" in terms of the Kodaira map.

For example in case of genus 2, we are able to catch at the limit "regular" CMC 1-immersions, except in the rare situation where the projective representative of given cohomology class belongs to the image, through the Kodaira map, of the six Weierstrass points of S .

If time permits, we shall mention further recent progress for higher genus obtained in collaboration with S. Trapani.

Mercoledì 30 Novembre 2022 - ore 14:00

Laura Capuano (Università di Roma Tre)

GCD results on semiabelian varieties and a conjecture of Silverman

A divisibility sequence is a sequence of integers d_n such that, if m divides n , then d_m divides d_n . Bugeaud, Corvaja, Zannier showed that pairs of divisibility sequences of the form a^{n-1} have only limited common factors. From a geometric point of view, this divisibility sequence corresponds to a subgroup of the

multiplicative group, and Silverman conjectured that a similar behavior should appear in (a large class of) other algebraic groups.

Extending previous works of Silverman and of Ghioca-Hsia-Tucker on elliptic curves over function fields, we will show how to prove the analogue of Silverman's conjecture over function fields in the case of abelian and split semiabelian varieties and some generalizations. The proof relies on some results of unlikely intersections. This is a joint work with F. Barroero and A. Turchet.

Mercoledì 7 Dicembre 2022 - ore 14:00

Olivier Biquard (Sorbonne Université / IMJ-PRG)

Degenerations of conical Kähler-Einstein metrics

We study the degenerations of conical Kähler-Einstein metrics when the cone angle becomes small. In particular, in the Fano case, we solve a question of Donaldson (2012). Joint work with Henri Guenancia.

Mercoledì 21 Dicembre 2022 - ore 14:00

Giovanni Paolini (Amazon Web Services / Caltech)

The $K(\pi, 1)$ conjecture

Artin groups are a generalization of braid groups, and arise as the fundamental groups of configuration spaces associated with Coxeter groups. A long-standing open problem, called the $K(\pi, 1)$ conjecture, states that these configuration spaces are classifying spaces for the corresponding Artin groups. In the case of finite Coxeter groups, this was proved by Deligne in 1972. In the first part of this talk I will introduce Coxeter groups, Artin groups, and the $K(\pi, 1)$ conjecture. Then I will outline a recent proof of the $K(\pi, 1)$ conjecture in the affine case and further developments in the hyperbolic case. This is joint work with Mario Salvetti and Emanuele Delucchi.

Mercoledì 11 Gennaio 2023 - ore 14:00

Emily Norton (University of Kent)

Temperley-Lieb algebras and p-Kazhdan-Lusztig theory for Hermitian symmetric pairs

Kazhdan and Lusztig introduced their eponymous polynomials for a Coxeter group W in 1979. Shortly thereafter, Lascoux and Schuetzenberger studied Kazhdan-Lusztig polynomials for Grassmannians and showed they admit closed combinatorial formulas (a very special situation). Generalizing the Grassmannian case, which corresponds to a maximal parabolic subgroup in finite type A , when (W, P) is a Hermitian symmetric pair then

Deodhar's parabolic Kazhdan-Lusztig polynomials are unusually tractable and admit closed formulas. Work by Stroppel and co-authors in the 2010s showed that these polynomials arise from diagrammatic algebras related to Temperley-Lieb algebras, while work by Elias and Williamson from the same decade produced Kazhdan-Lusztig polynomials from the diagrammatic category of Soergel bimodules. I will report on joint work with Chris Bowman, Maud De Visscher, Niamh Farrell, and Amit Hazi which links these two strands to show that an oriented version of Temperley-Lieb algebras of type (W,P) controls the Kazhdan-Lusztig theory of Hecke categories of Hermitian symmetric pairs in arbitrary characteristic.

Mercoledì 18 Gennaio 2023 - ore 14:00

Ilaria Mondello (Université de Paris Est Créteil)

Limiti di varietà con un controllo di Kato sulla curvatura di Ricci

Lo studio dei limiti di Gromov-Hausdorff di varietà è cominciato negli anni ottanta grazie a un teorema di precompattezza di Gromov per le varietà la cui curvatura di Ricci è limitata inferiormente. Se in questo caso la teoria di Cheeger-Colding fornisce una descrizione accurata di tali spazi limite, l'ipotesi sulla curvatura è piuttosto restrittiva e non è soddisfatta in molti problemi di analisi geometrica.

In questo seminario presenterò alcuni risultati recenti, ottenuti in collaborazione con Gilles Carron e David Tewodrose, sulla struttura dei limiti di varietà con un controllo integrale di tipo Kato sulla curvatura di Ricci. Mi concentrerò in particolare sul ruolo giocato nelle nostre dimostrazioni da nuove quantità monotone legate al nucleo del calore.

Mercoledì 1 Febbraio 2023 - ore 14:00

Benoît Claudon (IRMAR Université de Rennes 1)

Numerical characterization of torus quotients

In this talk I will explain how to recognize complex tori among Kähler klt spaces (smooth in codimension 2) in terms of vanishing of Chern numbers. It requires first to define Chern classes on singular spaces (a rather unstable notion). On the way, we will establish a singular version of the Bogomolov-Gieseker inequality for stable sheaves and study what can be said in the equality case.

Joint work with Patrick Graf and Henri Guenancia.

Mercoledì 8 Febbraio 2023 - ore 14:00

Frédéric Touzet (IRMAR Université de Rennes 1)

A global Weinstein splitting theorem for holomorphic Poisson manifolds

After reviewing some basic properties of holomorphic Poisson geometry, we will present a decomposition result in the Kähler case: if a compact Kähler Poisson manifold has a

compact symplectic leaf with finite fundamental group, then after passing to a finite étale cover, it splits as the product of the universal cover of the leaf and some other Poisson manifold.

This can be viewed as a global analogue to a theorem due to Alan Weinstein describing local Poisson structures.

Joint work with Stéphane Druel, Jorge Pereira and Brent Pym.

Mercoledì 15 Febbraio 2023 - ore 14:00

Giacomo Cherubini (INDAM / Sapienza Università di Roma)

Teorema delle geodetiche prime

C'è una curiosa analogia tra i numeri primi e le lunghezze delle geodetiche chiuse primitive ("prime") sulla superficie modulare. Nel seminario introdurrò le geodetiche in considerazione e cercherò di motivare tale somiglianza, visibile soprattutto nella loro distribuzione asintotica. Descriverò inoltre una formulazione equivalente che coinvolge il regolatore e il numero di classe di campi quadratici reali, entrambi di grande interesse aritmetico. Infine presenterò una congettura aperta sul risultato ottimale atteso per il termine di errore nel teorema delle geodetiche prime e progressi recenti verso tale risultato.

Mercoledì 22 Febbraio 2023 - ore 14:00

Julie Déserti (Université d'Orléan - CNRS)

Cremona group and regularisable birational maps

This talk deals with the group of birational transformations of the complex projective plane.

After some examples, we will see that this group satisfies some (but not all) properties of linear groups. Finally, we will introduce the notion of regularisable birational maps, i.e. those that are conjugate to an automorphism; we will give criteria allowing to determine if a birational map is regularisable.

Mercoledì 1 Marzo 2023 - ore 14:00

Ursula Ludwig (Universität Münster)

Analytic torsion and the Cheeger-Müller theorem

Analytic torsion is an important secondary spectral invariant of compact Riemannian manifolds. The famous Cheeger-Müller theorem states that for a compact Riemannian manifold equipped with a unitary flat vector bundle the analytic torsion is equal to the topological torsion, and hence a topological invariant. In the first part of this talk I will recall the definition of analytic torsion, the Cheeger-Müller theorem, and how it has been used in the past 10 years e.g. to answer questions motivated from the study of the cohomology of

arithmetic groups. In the second part I will speak about the generalisation of analytic torsion and of the Cheeger-Müller theorem to singular spaces.

Mercoledì 8 Marzo 2023 - ore 14:00

Enrico Fatighenti (Università di Bologna)

Fano varieties of K3 type and their properties

Fano varieties of K3 type are a special class of Fano varieties, which are usually studied for their link with hyperkähler geometry, rationality properties, and much more.

In this talk, we will recap some recent results, obtained jointly with Bernardara, Manivel, Mongardi, and Tauturri, that focus on the explicit construction of examples and the study of their Hodge-theoretical properties.

Mercoledì 22 Marzo 2023 - ore 14:00

Carlo Scarpa (Université du Québec à Montréal)

Kähler metrics and B-fields

Motivated by constructions appearing in mirror symmetry, we consider the problem of finding canonical representatives for a complexified Kähler class on a compact complex manifold. These are complex cohomology classes whose imaginary part is a Kähler class, while the real part is an arbitrary real $(1,1)$ -class. As is often the case in complex geometry, one way to fix a representative of such a class is to impose an elliptic PDE.

In this talk, I will explain why a natural choice of PDE is a coupling of the deformed Hermitian Yang-Mills equation and the constant scalar curvature equation. We will then see how to prove the existence of solutions in some special cases.

Based on arXiv:2209.14157, joint work with Jacopo Stoppa.

Mercoledì 5 Aprile 2023 - ore 14:00

Olivier Schiffmann (Université de Paris-Sud Orsay - CNRS)

Hecke operators on surfaces and the P=W theorem

For any smooth surface S we will introduce an associative algebra acting on the homology of (suitable) moduli spaces (or stacks) of coherent sheaves on S , by elementary modifications at points of S . This algebra includes the algebra of Nakajima operators but also the algebra of multiplication by tautological classes. We will give a presentation of that algebra as well as an application to the proof of the $P=W$ conjecture relating the perverse filtration on the cohomology of moduli spaces of stable Higgs bundles on curves and the weight filtration on the cohomology of character varieties. This is a combination of joint work with T. Hausel, A. Mellit, A. Minets and E. Vasserot.

Mercoledì 12 Aprile 2023 - ore 14:00

Johannes Nicaise (Imperial College London & KU Leuven)

Birational invariance of motivic zeta functions for K-trivial varieties

This talk is based on joint work with Luigi Lunardon. To every smooth and proper variety X with trivial canonical bundle over the field of complex Laurent series $C((t))$, one can attach its motivic zeta function, which measures how the variety degenerates as t goes to 0.

I will show that this motivic zeta function is a birational invariant of X and deduce the birational invariance of the monodromy conjecture for X , the main open problem in this context, which predicts a relation between geometric and cohomological degeneration properties.

The talk will include a gentle introduction to motivic zeta functions and an overview of known results.

Mercoledì 19 Aprile 2023 - ore 14:00

Roberto Fringuelli (SAPIENZA Università di Roma)

On the singular fibers of the Hitchin fibration and a Torelli theorem for moduli spaces of principal bundles

Let G be a complex reductive group and C be a complex smooth projective curve of genus at least two. The moduli space of G -Higgs bundles over C has a natural structure of holomorphic symplectic quasi-projective variety. Furthermore, it admits a proper morphism (the Hitchin morphism) onto an affine space (the Hitchin basis). It turns out that the Hitchin morphism is a Lagrangian fibration whose general fiber is an abelian variety.

In this talk, we provide a description of the locus of the singular fibers of the Hitchin morphism. As an application, we present a proof of a Torelli-type theorem for the moduli space of G -bundles over C .

Mercoledì 26 Aprile 2023 - ore 14:00

Ya Deng (CNRS - IECL Nancy)

Hyperbolicity and fundamental groups of complex quasi-projective varieties

The strong Green-Griffiths-Lang conjecture predicts that a complex quasi-projective variety X is of log general type if and only if there is a proper Zariski closed subset Z of X such that all the holomorphic maps from the punctured disks to X with essential singularity at the origin are all contained in Z .

In this talk I will show that this conjecture holds if the fundamental group of X admits a big and reductive representation into the complex general linear group. The proof is based on non-abelian Hodge theories and Nevanlinna theory.

This work is jointly with Benoit Cadorel and Katsutoshi Yamanoi.

Mercoledì 3 Maggio 2023 - ore 14:00

Robert Friedman (Columbia University)

Deformations of singular Fano and Calabi-Yau varieties

This talk will describe recent joint work with Radu Laza on deformations of generalized Fano and Calabi-Yau varieties, i.e. singular versions of complex manifolds whose curvature is either positive or zero in an appropriate sense. Under the assumption of isolated hypersurface canonical singularities, we extend results of Namikawa and Steenbrink in dimension three and discuss various generalizations to higher dimensions.

Mercoledì 10 Maggio 2023 - ore 14:00

Thomas Peternell (Universität Bayreuth)

Semipositive tangent bundles and canonical extensions

Given a projective complex manifold M with an ample polarization there is canonically associated an affine bundle Z over M . The question I will discuss is under which circumstances Z is an affine variety, or at least Stein. This is related to the global structure of M , specifically to the semipositivity of the tangent bundle. I will explain the main conjectures and recent results (joint work with A. Höring).

Mercoledì 17 Maggio 2023 - ore 14:00

GIORNATA ARITMETICA (organizzatore F. Pellarin)

Per titoli e sommari si veda la pagina web

<https://sites.google.com/uniroma1.it/an-arithmetic-day/home-page>

Mercoledì 24 Maggio 2023 - ore 14:00

(eccezionalmente Aula L)

Sergio Zamora (Max Planck Institute)

Margulis Lemma and RCD spaces

The Margulis Lemma states that in a hyperbolic manifold, the subgroup of the fundamental group generated by small loops around a certain point behaves like an abelian group (more precisely, it is virtually nilpotent).

I will present many generalizations of this result that have been appearing in the last 50 years, including recent work (with Qin Deng, Jaime Santos, and Xinrui Zhao)

where we include RCD spaces, a non-smooth analogue of manifolds with a lower bound on the Ricci curvature.

Mercoledì 7 Giugno 2023 - ore 14:00

Tomoyuki Arakawa (Kyoto University)

4D/2D duality and representation theory

This talk is about the 4D/2D duality by Beem et al., a new connection between quantum field theory and representation theory discovered rather recently.

It associates an algebraic object called VOA to any 4-dimensional superconformal field theory with $N=2$ supersymmetry, and the resulting VOA is conjecturally a complete invariant of the 4-dimensional theory.

I will talk about the rich structure of the representation theory of the VOAs arising from 4-dimensional theory by this duality.

Mercoledì 14 Giugno 2023 - ore 14:00

Martina Lanini (Università di Roma "Tor Vergata")

GKM-Theory for cyclic quiver Grassmannians

After recalling some background on Goresky-Kottwitz-MacPherson (GKM) version of the Localization Theorem for equivariant cohomology, and some of the applications of such a result to (equivariant) Schubert calculus and geometric representation theory, I will explain how it is possible -and why it is desirable- to extend such techniques to the quiver Grassmannian setting. This is joint work with Alex Puetz.

Mercoledì 21 Giugno 2023 - ore 14:00

Emiliano Ambrosi (IRMA Université de Strasbourg)

Riduzione modulo p del problema di Noether

Sia k un campo algebricamente chiuso e V una k -rappresentazione fedele di un l -gruppo G . Il problema di Noether è di comprendere se la varietà V/G è (stabilmente) razionale. Se la caratteristica di k è uguale a zero, Saltman ha costruito dei gruppi G per i quali V/G non è stabilmente razionale, mentre, se la caratteristica di k è uguale a l , Kuniyoshi ha dimostrato che V/G è stabilmente razionale. Vediamo quindi che la geometria di V/G varia con la caratteristica del campo. In questo seminario, dimostriamo che per tutti i gruppi G costruiti da Saltman, non si può interpolare il problema di Noether passando dalla caratteristica 0 alla caratteristica l . Più precisamente, dimostriamo che non esiste una famiglia propria e liscia in caratteristica mista $(0,p)$, la cui fibra speciale (che vive in caratteristica p) e fibra generica (che vive in caratteristica 0) sono entrambe stabilmente birazionali a V/G . La dimostrazione combina i risultati recenti di teoria di Hodge p -adica di Bhatt-Morrow-Scholze, con lo studio

delle forme differenziali in caratteristica positiva. Questo è un lavoro in comune con Domenico Valloni.

Mercoledì 28 Giugno 2023 - ore 14:00

Samuel Grushevsky (Stony Brook University)

Compactifications of moduli of cubic surfaces

The moduli space of smooth complex cubic surfaces can be compactified from the point of view of geometric invariant theory (GIT), and from the point of view of the ball quotient. The Kirwan desingularization resolves the GIT singularities to yield a smooth Kirwan compactification, while the toroidal compactification of the ball quotient is also smooth. We show that these two smooth compactifications are, however, not isomorphic, and study their birational geometry, and related classical algebro-geometric constructions in further detail. Based on joint works with S. Casalaina-Martin, K. Hulek, and R. Laza