1. Titles and Abstracts of talks

Dagger analytic geometry.
Federico Bambozzi

Abstract. In this talk we discuss a new approach to Berkovich geometry and complex analytic geometry using dagger affinoid algebras, i.e. algebras of overconvergent (namely germs) of analytic functions. We describe how this approach can put on the same footing complex analytic geometry and non-archimedean analytic geometry and the relations between dagger analytic spaces with analytic spaces already present in literature. Finally, we describe some nice properties and possible applications of the theory we developed. If time permits, we will sketch a further develop of this work, in collaboration with Ben-Bassat, in which dagger analytic geometry is recasted as a relative algebraic geometry (in the sense of Toen-Vezzosi) with respect to some suitable closed symmetric monoidal category equipped with a Grothendieck topology.

Asymptotic maximal slope of hermitian line bundles.
Huayi Chen

Abstract. In this talk, I will explain the notion of asymptotic maximal slope of hermitian line bundles, notably its definition and birational nature, and the relation to several subjects in Arakelov geometry such as positivity of hermitian line bundles, Dirichlet property and the behaviour of the associated height function.

G.C.D. estimates in the Green-Griffiths-Vojta conjecture.
Pietro Corvaja

Abstract. In a work of the speaker with Bugeaud and Zannier it was proved that for multiplicatively independent integers $a > b > 1$ the greatest common divisors of $a^n - 1$ and $b^n - 1$ satisfy $GCD(a^n - 1, b^n 1) \ll \exp(en)$. A generalization over number fields was proved soon after, while Noguchi, Winkelmann and Yamanoi proved the natural extension in Nevanlinna theory. Finally, it was generalized to algebraic function fields by Zannier and the speaker. Several applications of these estimates will be presented.
Asymptotic behavior of Green’s currents and related invariants.
Robin de Jong

Abstract. We consider families of compact Riemann surfaces that degenerate over possibly higher dimensional base manifolds. Studying the asymptotic behavior of Green’s currents and related invariants (such as Faltings delta-invariant, Kawazumi-Zhang invariant, ...) is an old subject, with ramifications in and inspiration from physics (perturbative string theory) and asymptotic Hodge theory. We discuss some new precise results in this direction, and stress the Arakelov point of view: the asymptotics are (at least in a growing number of cases) controlled by suitable non-archimedean analogues of the analytic invariants under consideration.

On the concavity of the arithmetic volumes.
Hideaki Ikoma

Abstract. I will talk about the differentiability of the arithmetic volumes along arithmetic R-divisors, and give some equality conditions for the Brunn-Minkowski inequality for arithmetic volumes over the cone of nef and big arithmetic R-divisors.

Arakelov invariants of Belyi curves.
Ariyan Javanpeykar

Abstract. Let $X$ be a curve over a number field. We study invariants of $X$ such as the Belyi degree $d(X)$ and the stable Faltings height $h(X)$. Our main result is an explicit inequality relating these invariants: $h(X) < 10^9 d(X)^7$. We will discuss applications to 1) Szpiro’s small points conjecture, 2) heights of isogenies of hyperbolic curves, and 3) a conjecture of Edixhoven-de Jong-Scheppers on the Faltings height of a branched cover of curves. For example, we prove Szpiro’s small points conjecture for all curves that admit an etale cover to some cyclic cover of $\mathbb{P}^1$.

A tropical approach to non-archimedean Arakelov theory.
Klaus Künnemann

Abstract. We report on joint work with Walter Gubler. Recently Chambert-Loir and Ducros have introduced a theory of real valued differential forms and currents on Berkovich spaces. In analogy to the theory of forms with logarithmic singularities, we enlarge the space of differential forms by so called delta-forms on the non-archimedean analytification of an algebraic variety. We prove a generalization of the Poincaré-Lelong formula which allows us to represent the first Chern current of a formally metrized line bundle by a delta-form. We introduce the associated Monge-Ampère measure $\mu$ as a wedge-power of this first Chern delta-form and we show that $\mu$ is equal to the corresponding Chambert–Loir measure. The star-product of Green currents is a crucial ingredient in the construction of the arithmetic intersection product. Using the formalism of delta-forms, we obtain a non-archimedean analogue at least in the case of divisors. We use it to compute non-archimedean local heights of proper varieties.
Geometric Invariant Theory and Arakelov Geometry.

Marco Maculan

Abstract. Interactions between Geometric Invariant Theory and Arakelov Geometry have been exploited by several authors in the last 25 years (Bost, Burnol, Zhang, Gasbarri, Chen). In this talk we will explain how to use a formula originally due to Burnol in order to deduce results in diophantine approximation, as Roth’s theorem and some more recent variants.

Numerical bounds for the essential minimum of Faltings height on elliptic curves.

Ricardo Menares

Abstract. For a given number field \( K \) and elliptic curve \( E \) defined over \( K \), we denote by \( h_F(E/K) \) the corresponding Faltings height. This height can be put in the context of Arakelov theory of arithmetic surfaces. Indeed, we can consider on the moduli space of (generalized) elliptic curves the line bundle of weight 12 modular forms, endowed with the Petersson metric. This is a log-log singular Hermitian line bundle. Then, the height of an algebraic point is (a fixed multiple of) the Faltings height of a semistable elliptic curve corresponding to it through the modular interpretation.

The essential minimum is defined by
\[
\mu_F^{\text{ess}} := \inf \{ \theta \in \mathbb{R} : \text{the set } \{ E/K : h_F(E/K) \leq \theta \} \text{ is infinite} \}.
\]

We also consider the absolute minimum
\[
\mu_F^{\text{inf}} := \inf \{ h_F(E/K) : E \text{ is defined over } K \}.
\]

Once chosen a normalization, it is not difficult to find explicit and closed expressions for \( \mu_F^{\text{ess}} \). In particular, its numerical value can be approximated with a high degree of accuracy. Even though Faltings height depends on normalization choices, the nonnegative quantity \( \mu_F^{\text{ess}} - \mu_F^{\text{inf}} \) does not. We show that
\[
(1.1) \quad 0 < \mu_F^{\text{ess}} - \mu_F^{\text{inf}} < 10^{-4}.
\]

Although general bounds on the essential minimum are available thanks to S. W. Zhang’s work (suitable generalized by Bost and Freixas-i-Montplet to log-log singular hermitian line bundles), these are less precise than (1.1).

The proof hinges on recent work by Burgos, Philippon and Sombra on the essential minimum of toric heights on toric arithmetic varieties. In our situation, the moduli space is toric (isomorphic to the projective line). Even though the Petersson metric is not toric, we will see that it can be approximated by a toric metric in an appropriate sense.

This is joint work with José Burgos Gil and Juan Rivera-Letelier.

The distribution of Galois orbits of small points on toric varieties.

Martin Sombra

Abstract. Given a toric metrized line bundle on a toric variety over an number field, I will present a formula for the successive minima of the associated height function. As a consequence, we obtain criteria for the equidistribution of the Galois orbits of points of small height in a toric variety.

This is joint work with J. I. Burgos Gil, P. Philippon, and J. Rivera-Letelier.
New inequalities between successive minima on arithmetic surfaces.

Christophe Soulé

Abstract. We generalize a combinatorial result of D. Morrison, who used it to prove the Chow semi-stability of projective curves. We deduce from this result new inequalities between successive minima of the lattice of sections of an hermitian line bundle over an arithmetic surface.

Localization theorem for higher arithmetic $K$-theory.

Shun Tang

Abstract. Soulé, and also Deligne, suggested that one may define the higher arithmetic $K$-theory space as the homotopy fibre of Beilinson’s regulator map. Burgos-Wang’s higher Bott-Chern forms provide a simplicial description of Beilinson’s regulator map, and hence a more concrete definition of the higher arithmetic $K$-groups. In this talk, I will introduce some fundamental theorem for such arithmetic $K$-groups, say the localization theorem, analogue to the same name theorem in Quillen’s algebraic $K$-theory.

Geometric Lang-Vojta conjecture in the projective plane.

Amos Turchet

Abstract. We present a proof of the function field version of Lang-Vojta Conjecture on algebraic hyperbolicity for complements of very generic quartics in the projective plane with at most normal crossing singularities. The proof relies on a reformulation of the problem via moduli spaces of logarithmic stable maps as introduced by Q. Chen and Abramovich.